ADAPTIVE FILTER WORKING IN SPECTRAL DOMAIN ON THE BASIS OF FAST WALSH-FOURIER TRANSFORMATION (FWFT) USED IN CDMA SYSTEM

S. Jackowski

1. Introduction

Proposed filter can be used in cellular communication system CTDMA (Code Time Division Multiple Access). An assumption is made that the realisation of time division (Time Sharing in TDMA – according to IS-95 standard or IMS Standards) will be made by time frame. From the point of view of further analysis this assumption is not important because introduced solution can be used for any data stream defined by system parameters.

Essential problem is a solution of CDMA code distribution which will be realized by the assignment of predefined number of Walsh function to subset of users connected to certain TDMA time frame to compose (multiply) with spreading pseudorandom stream (PN - Pseudo Noise).

2. Transmitter and receiver structure and signal shape

In the proposed solution of transmitter BPSK manipulation (Binary Phase Shift Keying) is used in both I and Q channels with two different M - streams composed with suitable Walsh functions and with transmitted data. To transmit SS/DS-QPSK signals that can be used in CDMA systems code distribution and synchronization are realized by combining of Walsh functions with transmitted data spread usually by full cycle PN streams. In this way QPSK (Quadrature Phase Shift Keying) modulation is defined by states of both channels (00, 01, 10, 11). In the system to increase signal spectrum spreading instead of linear M - PN streams [1] generated generally over Galois field $GF^*(p^k) = GF^*(p^k) \cap \{0\}$, a use of non-linear de Bruijna M - PN streams [2], called full length PN streams over Galois field $GF^*(p^k)$ is proposed where k is the length of generation shift register. The length of such sequences (ibid) is $K=p^k$ and it differs from lineal PN M – sequences, where $K = p^k - 1$. It results from enabling zero-only state in generating register, whereas it is not possible for lineal M – PN streams. In CTDMA system which will be analysed $p = 2$ (binary case).

Generally (ie. for p>2) instead of Walsh functions, Chresteuson functions of suitable order $K_l^{p^k}$ [3], and calculation in Galois field should be performed modulo p.

Proposal of Bruijn streams use results first of all from the fact that the length of Walsh function of any order r will be always an integer divider of PN stream length. In binary case it gives $2^r | 2^k$ for $k \geq r$. It simplifies acquisition process, synchronization and synchronization following in DS/SS systems.

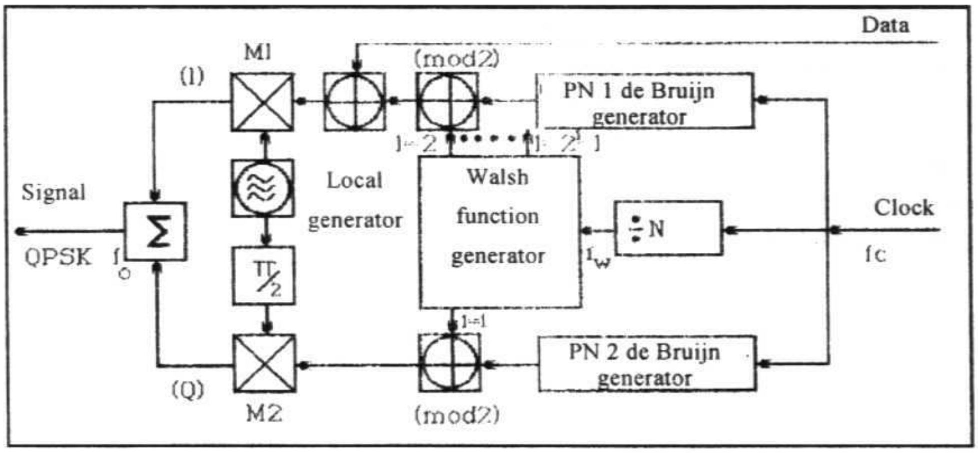

Fig 1. DS/SS - QPSK signal generator

In constructed signal both PN stream and Walsh function will not shift against each other in time as it happens in case lineal M - streams for which $K=2^k-1$ and at last $2^r \mid 2^k - 1$. 2^r is always an even number and $2^k - 1$ is always odd.

Block diagram of DS-SS generator is presented in Fig 1.

In presented system in-phase I branch is used to send the information, whereas quadrature branch Q sends Walsh function with the same number for all system (here conventionally $l = 1$) to perform bit synchronization.

One should pay attention to the fact that different variants of solutions are possible here i.e. in one section of Walsh function constancy one or several de Bruijn streams (according to N) can be suited. Walsh functions and PN streams can have the same or different length etc. In analysed system collection of information can take place with speed of Walsh function bit generation or one information bit can contain all period of Walsh function (or several, a dozen or so) designating subset of subscribers. Selected variant in case of adaptive filter used depends on time possibilities of FWFT realization. A common feature of all the variants is the use of inversion manipulation by calculating of sums mod 2 (Fig 1). Before manipulation of I and Q components information is transformed by isomorphism

$$f:\begin{cases}0 \rightarrow 1\\ 1 \rightarrow -1\end{cases} \tag{1}$$

transforming binary streams to NRZ (Non Return to Zero) signal. It makes possible to perform BPSK manipulation and in consequence QPSK one.

At the choice of de Bruijn PN streams very important is also the fact that for lineal M - streams number of different streams resulting from number of primitive polynomials over Galois field GF (p^k) is:

$$M_1 = \Phi\left(p^k - 1\right)/k, \tag{2}$$

where:

$\Phi(n)$ - Euler function [4],

$$\Phi(n) = p_1^{k_1-1} p_2^{k_2-1} \ldots p_r^{k_r-1}(p_1-1)(p_2-1)\ldots(p_r-1)$$

$n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$ - canonical distribution of natural number n,

and for de Bruijn streams at suitable generation [5] this number is [5]

$$M_2 = p^{p^K - k} \tag{3}$$

reaching strong inequality M1<<M2 for the same k. The use of different spreading streams for different

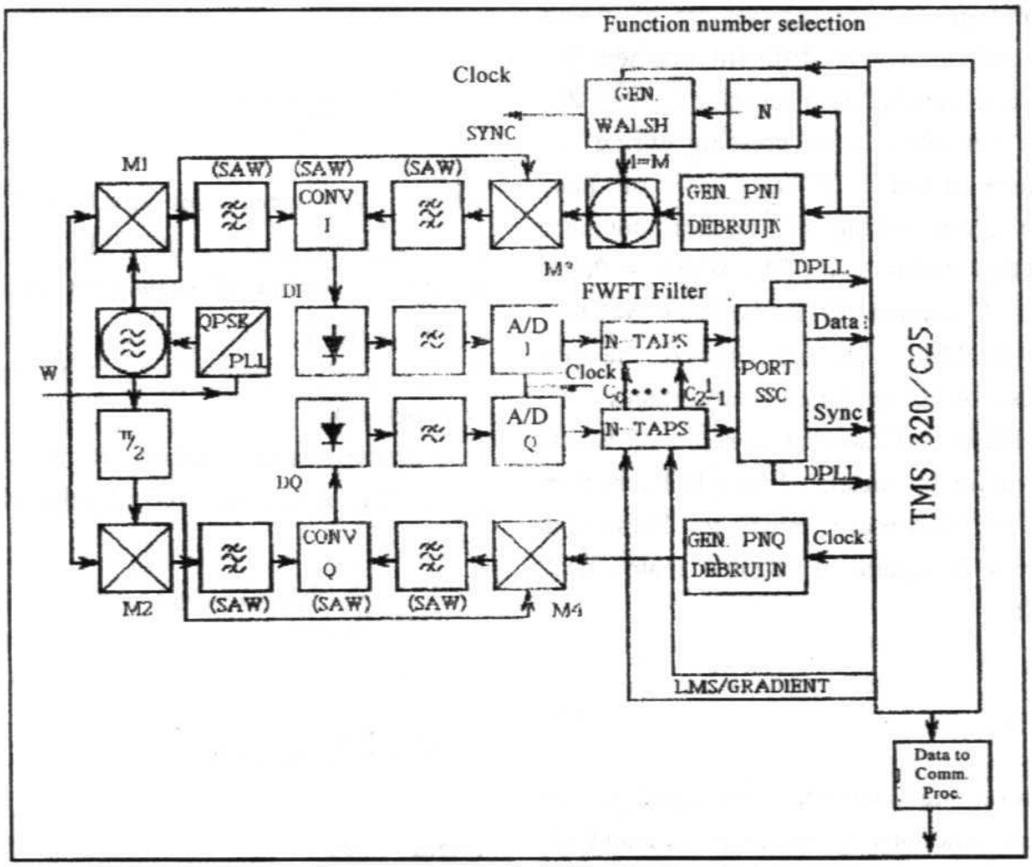

Fig 2. Decorrelators and adaptive filters in receiver for DS/SS/QPSK signals

cells or subscribers groups can grant to system additionally features of SSMA (Spraed Spectrum Multiple Access) system. In systems with information protection it is more difficult to recover PN streams because lineal equivalent of non-linear de Bruijn stream many times exceeds in general its length. It makes practically useless Berlekamp - Massey algorithm [4] (etc.) to recover the structure of stream generator feedbacks.

De Bruijn streams may possess a little worse characteristics of auto- and crosscorrelation function than lineal M – streams giving always quasi-orthogonal choice (ie. outside main maximum $R(0)=2^k-1$ of autocorrelation function $R(m) = -1$ for dla $m \neq 0 \pmod{2^k-1}$). From a large number of possible streams given by (4) one can choose considerable number of such strings for which $|R(\max)| \leq R(0)/8$ [6].

In receiving systems one can use correlators based on surface acoustic waves (SAW) [7].

Functional diagram of receiver is presented in Fig 2.

Synchronization and control processes can be supported by digital signal processor (DSP) in real time.

3. Fast Walsh - Fourier transform (FWFT) and functional schema of adaptive filter

One can arrange full, orthogonal set of Walsh function, one can arrange in different manners [8] e.g according to increasing zero crossing ie. pseudo-frequency as a number of zero crossing during determination range of length 2^L (L - order of function set) what gives sorting of function numbers according to Gray binary code [9], Walsh – Paley (W-P), Walsh – Kaczmarz (W-K) whether Walsh – Hadamard (W-H) (ibid). We will analyze the last example because of its application advantages.

One can achieve W-H sorting in result of Walsh functions generation by means of simple Kronecker product [10] of Hadamard matrices. Presented Hadamard matrix is square orhtogonal matrix 2x2 dimension (ibid)

$$[H_2]=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}. \qquad (4)$$

Matrix representing complete, orthogonal set of 2n order Walsh functions is received in result of operation

$$[H_{2n}]=\bigotimes_{i=1}^{n}[H_2]_i \qquad (5)$$

where $\otimes_n$ - symbol of n-ary simply Kronecker product.

For example, for $[H_8]$ we have $[H_{2x4}]$:

$$[H_8]=\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1\\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1\\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1\\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1\\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1\end{bmatrix} \qquad (6)$$

From (6) results that in this matrix both its verses and columns are Walsh functions. Walsh functions with regard on dot multiplication are alternating abel group G_L and it results that one can use generalized spectral transformations GODA (Generalized Orthogonal Developments Algorithms) [10] both for p= = 2 and for p > 2.

This means that a pair of Laplace' and – Galois (L-G) (ibid) transforms exists

$$L(g)=\sum_{n=0}^{\infty}[E^{-gn}]\times y^T(n), \qquad (7a)$$

$$y(n)=\sum_{g=-n^g}^{n^g}[E^{gn}]\times L(g), \qquad (7b)$$

where L(g) - generalized spectrum L - G;
y(n) = [y(0), y(1),..., y(g-1)] - vector of signal samples - algebra of signals on abel group G_L of order $|g|$ [11];
$[E^{-gn}] = [E^{gn}]^T$ - pair of transformation matrices (eg of type(6)).

Because Walsh function set in matrix form is a group (7a, b) one can record otherwise

$$Y_L=\frac{1}{|g|}\sum_{g\subset G_L}y(g)\chi_l{}^*(g), \qquad (8a)$$

$$y(g)=\sum_{l\in G_L}Y_L\,\chi_g(l) \qquad (8b)$$

where
Y_L - generalized spectrum (eg Walsh - Galois);

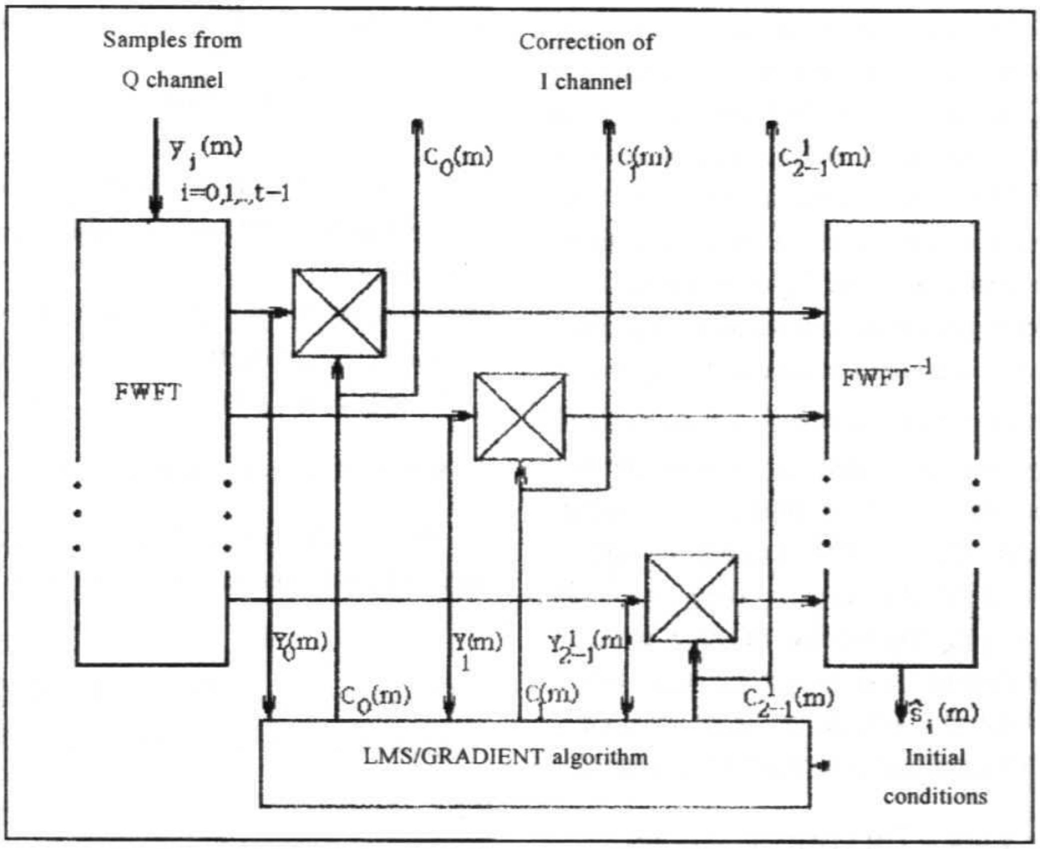

Fig 3. Structure of adaptive filter based on FWFT

y(g) - algebra of signals [11] over group G_L;
$\chi_i(g)$, $\chi_g^*(l)$ - characters of group representation G_L [3],
or directly for Walsh function

$$Y_L = 2^{-L} \sum_{2^L-1=0}^{2^L-1} y(i)\, w_L(i)\,, \tag{9a}$$

$$y(i) = \sum_{L=0}^{2^L-1=0} Y_L\, w_i(l)\,, \tag{9b}$$

where Y_L - spectrum W-H, y (i) - streams of samples (block) lengths 2^L,
$w_L(i)$, $w_i(l)$ - Walsh functions in W-H order.

The use of suitable factorization matrix that means representation able group of Walsh functions as a straight product of its cyclic subgroups [12] leads to FWFT algorithm with butterfly type graph [10, 13] at full model GODA [10]. In case of sparse transform matrix GODA it leads to fast transformation in using set of Haar functions (ibid). It suggests the possibility of using these functions to adaptive filters synthesis in spectral domain with similar properties. Computational complexity FWFT at p=2 and given L is determined in [10] as 2^{2L} of multiplications and $2^L(2^L-1)$ of addings. One should notice that multiplications in Walsh transformation rely only on change of sign of numbers and in signal processor they are executed in a single order cycle as the addings are.

In Fig 3 structure of adaptive filter realized in spectral domain is presented. This structure is also a structure of program realized by DSP. Both input and output blocks are predetermined external RAM memory areas working as alternative buffers ie. when straight or inverse FWFT are calculated on one block of samples then the second area is recorded (readed) and vice versa. The conclusion is that all the calculations on the previous block of data have to be finished before recording (reading) the next block. For realized CDTMA system one assumed size of Walsh function set $N=64=2^L$ and so L=6 and Walsh spectrum contains N=64 components. One assumed also that adaptive algorithm will be of LMS type and recursive estimation of corrective coefficients CL will be done using gradient method [14]. Filter will process block of data from quadrature channel (Fig 2), but it will work out corrections for in-phase channel (Fig. 3), where it would be difficult to make because of transmitted information. Elaboration of adaptation algorithm is based on training sequence method [14].

For assumed frequency of Walsh function generation f_w =192 kbps function period T_w = 0.3(3) ms.

For example in [15] one gave subroutine of recursive calculations for N=64 corrective coefficients $c_L(n)$ l=0, 1,..., 63 realized in DSP language during $T_0 \cong 19.2$ ms that leaves sufficient time reserve to calculate four FWFT transforms (forward and inverse in Q and I channels) at computational complexities given previously and $T_w = 0.3(3)$ ms.

As standard functions to calculations of adaptation error in LMS algorithm standard Walsh spectra Ψ_L, $\overline{\Psi}_L$ are used. They are stored in external DSP ROM memory and calculated for synchronization Walsh function number l=1 in W-H order transmitted in Q channel (Fig 1). This function is one of Rademacher function that can be used to obtain Walsh functions [3]. Recovering of time scale to control A/D converter and synchronization DPLL (Digital Phase Locked Loop) in intelligent serial ports SCC (Serial Communication Controller) are used.

4. Analysis of LMS - GRADIENT algorithm for realization of adaptive filter using FWFT

Let as assume that on input of FWFT block (Fig. 3) are blocks of samples after converting of autocorrelation maximums for de Bruijn streams obtained in SAW converters (Fig 2)

$y_L(i) = m(i)w_L(i)s(i) + n(i)$ for $i = 0,1,..2^L - 1$ (10)

$L = 0,1...2^L - 1$

where $\mu(i)$ - factor representing multiplicative fading;

$w_L(i)$ - L-th Walsh function synchronizing or designating;

$n(i)$ - samples of gaussian noise;

$s(i)$ - value of correlation maximum.

Assuming that following samples $\mu(i)$, $n(i)$ are statistical independent and that $\mu(i)$ and $n(i)$ are mutually independent we can calculate according to (9a) Walsh spectrum of samples blocks length 2^L

$$ {}_mY_l = 2^{-L}\sum_{i=0}^{2^L-1} {}_m y(i)\, w_l(i) \quad \text{for} \quad l=0,1..2^L-1 \quad (11)$$

where ${}_mY_l$ – Walsh spectrum of m-th block of samples;

${}_my(i)$ - m - block of samples;

$w_l(i)$ - l-th Walsh function.

In algorithm LMS - GRADIENT succeeding correlational coefficient $c_l(m)$ for l -th component of Walsh spectrum [14] according to recursive equation

$$c_l(m+1) = c_l(m) + \alpha\varepsilon_l(m)_m Y_l, \qquad (12)$$

where $c_l(m+1)$, $c_l(m)$ - successor and predecessor of corrective coefficient of Walsh spectrum l -th component;

$\varepsilon_l(m)$ - error of adaptation for m-th block;

α - convergence coefficient of adaptive algorithm.

In considered case

$$\varepsilon_l(m) = \beta\Psi_l - c_l(m)\Psi_l(m). \qquad (13)$$

After substitution (13) to (12) elimination of indexes l (conditions for all components are identical)

$$\begin{aligned} C(m+1) &= c(m) + \alpha[\beta\Psi - c(m)\Psi(m)]\Psi(m) = \\ &= c(m) + \alpha\beta\Psi\, Y(m) - c(m)Y^2(m). \end{aligned} \qquad (14)$$

Where β – as in (13) pattern scale coefficient.

Because c(m) are random variables we count average values for both sides (14):

$$\begin{aligned} &\langle C(m+1)\rangle = \\ &= \langle c(m)\rangle + \alpha\beta\Psi\langle Y(m)\rangle + \langle c(m)Y^2(m)\rangle. \end{aligned} \qquad (15)$$

Substituting Y(m) form (14) we have:

$$\begin{aligned} &\rangle C(m+1)\langle = \\ &= \rangle c(m)\langle + \alpha\beta\Psi \Bigg\rangle 2^{-L}\left(\sum_{i=0}^{2^L-1}\mu(i)\,w_1(i)\,s(i)\,w_k(i) + \sum_{i=0}^{2^L-1} n(i)\,w_j(i)\right)\Bigg\langle \\ &+ \rangle c(m)\langle\rangle Y^2(m)\langle = \\ &= \rangle c(m)\langle + m_\mu + \rangle c(m)\langle \Bigg\rangle \sum_{i=0}^{2^L-1}\mu^2(i) + \sum_i\sum_k(.)\Bigg\langle \end{aligned}$$

(16)

Where $w_j(i)$, $w_k(i)$ – Walsh functions;

$\sum\sum(.)$ equal $2^L \cdot m_2$ – because of orthogonal relation $(w_j(i), w_k(i)) = \begin{cases} 0 & \text{for } j \neq k, \\ 2^L & \text{for } j = k. \end{cases}$

The same orthogonal condition causes that in single sum $\sum_{i=0}^{2^L-1}\mu^2(i)$ only expressions with scalar product $(w_j(i), w_k(i)) = \begin{cases} 0 & \text{for } j \neq k \\ 2^L & \text{for } j = k \end{cases}$ remain giving after averaging $\sum_{i=0}^{2^L-1}\mu^2(i)=2^L\mu m_2$, where μm_2 second order moment of probability distribution of random variable representing multiplicative fading.

Similarly $2^L nm_2 = 2^L\sigma_n^2$ as for gaussian noise $\langle n(i)\rangle = 0$ ie. average value is equal to zero and second order moment is equal to variance (central moment).

We have now

$$\langle C(m+1)\rangle = \langle c(m)\rangle\left(1+\mu m_2+n\sigma^2\right)+\alpha\beta\Psi. \qquad (17)$$

Using Z-transform to (17) [16] and putting $\gamma = \alpha\beta\Psi$ and $\Gamma = 1+\mu m_2 + n\sigma^2$ we obtain Z transform C(z) equal

$$C(z)=\frac{z[z-(1+\gamma)]}{(z-\Gamma)}. \qquad (18)$$

After inverse transform we have

$$\langle c_1(0)\rangle = 1. \qquad (19)$$

What gives

$$\langle c(m)\rangle = \frac{1}{2\pi i}\int_{c=(|z|=1)} C(z)z^{m-1}\,dz =$$
$$=\frac{1}{2\pi i}\int_C \frac{z^m[z-(1+\gamma)]}{(z-1)(z-\Gamma)}dz = \sum_k \underset{z=z_k}{\mathrm{res}}\, C(z). \qquad (20)$$

Where z_k - poles C(z)

$$\langle c(m)\rangle = Z^{-1}C(z) =$$
$$=\lim_{z\to 1}(z-1)C(z)+\lim_{z\to\Gamma}(z-\Gamma)C(z). \qquad (21)$$

It gives at last

$$\langle c(m)\rangle = \Gamma^m\left(1+\frac{\gamma}{1-\Gamma}\right)-\frac{\gamma}{1-\Gamma}=$$
$$=\begin{cases}1 & m=0\\ \Gamma^m - \dfrac{\gamma}{1-\Gamma} & m=1,2,\ldots\end{cases} \qquad (22)$$

or

$$\langle c(m)\rangle = \begin{cases}1 & m=0,\\ \Gamma^m - \gamma\left(\sum_{n=0}^{m}\Gamma^n\right) & m=1,2,\ldots\end{cases} \qquad (23)$$

because $\sum\Gamma^n = \frac{1-\Gamma^m}{1-\Gamma}$ is partial sum of geometrical sequences.

Value $\langle c(0)\rangle = 1$ results from proprieties of Z-transformation [17] $Z[f(n+1)] = z[F(z)-f(0^+)]$ that we used in calculating (19) under the assumption that initial values all of correction coefficients $\langle c_1(0)\rangle = 1$.

5. Conclusions

Presented structure and basic properties of adaptive filter is another version of similar filter proposed in [14] realized basing on DFFT (Discrete Fast Fourier Transform). Filter using FWFT is especially useful for discreet signals processing if they are Walsh or quasi-Walsh type. They do not possess defects of filters realized in spectral domain using FFT. Use of FFT causes spectral aliasing that enforce enlargement of block length by addition of trailing zeroes. It increases time of calculations and entire elimination of computational errors requires up to five or even seven both forward and inverse transformations [14]. It limits transmission data rate and sometimes eliminates work in real time.

It is also possible (it results from point 3) to realize this type filter using orthogonal Haar functions but we did not analyse this method because in proposed system code distribution uses Walsh functions directly. Modifying described filter by realization of additional FWFT transformation of error sequence of m-th block one can obtain realization filters for rejection or expanding narrowband signals from wideband signals or disturbances [17].

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CDMA SISTEMOJE NAUDOJAMAS ADAPTUOJAMAS FILTRAS, VEIKIANTIS SPEKTRO ZONOJE PO GREITOS VALŠO-FOURERIO TRANSFORMACIJOS

S. Jackowski

S a n t r a u k a

Pateikta realizavimo idėja ir parametrų analizė adaptuojamo filtro, išlyginančio radijo kanalo amplitudžių dažnumų charakteristikas perduodant signalus DS/SS (Direct Sequence/Spraded Spectrum).

Modifikuotas filtras dar gali būti panaudotas mažo dažnio trukdžiams eliminuoti plataus dažnio perdavimo kanalu. Siūloma panaudoti filtrą po greitos Valšo-Fourerio transformacijos (FWFT - Fast Walsh-Fourier Transformation).

Analizuojamas filtras gali būti panaudotas ryšio sistemoje CTDMA (Code Time Division Multiple Access).

STEFAN JACKOWSKI

D. Sc. is a professor of Radom University of Technology. He graduated from electronics department of Military University of Technology in Warsaw, where he also received Ph. D. degree and made habilitation dissertation. In Military University of Technology he worked on adaptive radio systems and signal recognition and jamming. Since 1993 he works in Radom University of Technology as the head of Electronics and Telecommunications Section. He is a well known specialist in spread spectrum signals area, especially in CDMA applications.