Zeno-anti-Zeno crossover via external fields in a one-dimensional coupled-cavity waveguide

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We have studied a hybrid system of a one-dimensional coupled-cavity waveguide with a two-level system inside, which subject to a external periodical field. Using the extended Hilbert space formalism, the time-dependent Hamiltonian is reduced into an equivalent time-independent one. Via computing the Floquet-Green's function, the Zeno-anti-Zeno crossover is controlled by the driven intensity and frequency, and the detuning between the cavity and the two-level system.

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I. INTRODUCTION

Inspired by modern microfabrication technology in photonic crystals[1–5], optical microcavities[6], superconducting devices [7–9], and the realization of the quantum regime in the interaction of atomic-like structures and quantized electromagnetic modes in those system[10–14], considerable attention has been attracted to a family of models for coupled arrays of atom-cavity systems, since they offer a fascinating combination of condensed matter physics and quantum optics. Typically, these models are formed by an array of cavities with each cavity containing one or more atoms[15], where photons hop between the cavities. Recently, due to the potential use for building a quantum switch for routing single-photons in quantum network, systems with one or two atoms inside the array of cavities have been extensively studied to revealed the intriguing features of photon transport in low dimensional environments[16–20]. It is found that the switch is formed by the interference between the spontaneous emission from atoms and the propagating modes in the one-dimensional (1D) continuum. Therefore, the search for a controllable switch is to finding a way to change the spontaneous emission of atoms.

The spontaneous emission results from the inevitable interaction of the atomic system with external influences. Decoherence of a quantum system, with a variety of couplings to a reservoir, has been investigated extensively in theory[21–23]. Basically, there are three ways to suppress or modify the rate of quantum transitions in a system. One is to engineer the state of the reservoir, as well as the form of the system-reservoir coupling[24–26]. Obviously, the coupled-cavity waveguide (CRW) meets this condition for its advantage of addressability of individual sites, extremely high controllability, and the great degree of flexibility in their geometric design. Two is to involve the quantum inference between multiple transition pathways of internal states, for example, the electromagnetically induced transparency technique[27–29]. The third way is to applies a succession of short and strong pulses, or measurement to the quantum system[30–35].

Atoms have been particularly interested for acting as a quantum node in the extended communication networks and scalable computational devices, specially artificial atoms. Atoms in a time-varying field has been investigated long time before. However, the multiphoton resonance and quantum interference have been experimentally demonstrated in a strongly driven artificial atom[36] until recently, which is important to superconducting approach of quantum computation, for instance, decreasing the time required for each gate operation[37–40].

Floquet theory is developed by Floquet centuries ago, it is a theory about the solutions of linear differential equations with periodic coefficients[41]. Later, it is applied to the two-level system (TLS) with the time-dependent problem, which is discussed by Autler and Townes[42]. And then such kind of the periodic time-dependent problem is reformulated as an equivalent time-independent infinite-dimensional Floquet matrix[43], which is done by introducing the composite Hilbert space of square integrable and time-periodic wave functions. Now Floquet formalism is used as a theoretical tool to investigate time dependent phenomena[44]. In this paper, we study a two-level system (e.g. a flux qubit) interacting with a cavity which together with other cavities constructs a one-dimensional (1D) coupled-cavity waveguide (CRW). The CRW is modeled as a linear chain of sites with the nearest-neighbor interaction. Obviously, the dynamic of the TLS is irreversible, i.e. once the TLS is initially in its excited state, it never returns to its initial state spontaneously. Therefore, the CRW with a TLS inside is a typical system with a discrete state coupled to continua of states, which means the TLS is subject to decay. In order to control the decay rate of the TLS, an external periodical field is applied to the TLS through diagonal coupling. It is well known that periodic coherent pulses can either inhibit or accelerate the decay into its reservoir, however, such modification is based on off-diagonal time-dependent couplings between the TLS and the external field. In a atomic experiment, the off-diagonal couplings is caused by a transverse field perpendicular to the polarizing field, which is used to polarize the atomic spins. Then the spins of atoms and photon are aligned. However, diagonal couplings between the driving field and the TLS means that the driving field is parallel to the polar-

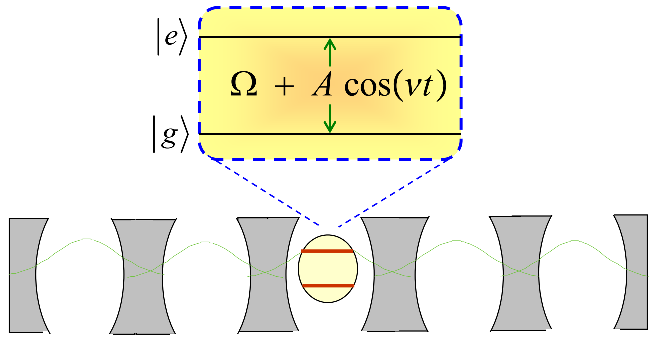

FIG. 1. (Color online) Schematic illustration of the model, where a two level system is inside a 1D coupled cavity waveguide. The two level system is driven by external forces that are periodic in time with period ν.

izing field[45]. Here, the spontaneous emission of a TLS is studied, the Zeno-anti-Zeno crossover is investigated via Floquet formulation.

The paper is organized as follows. To set our consideration, in Sec. II, we state the Hamiltonian for 1D CRW with a TLS inside one of the cavity. in Sec. III, we first give a brief outline of the Floquet theorem, then present the Floquet representation of the system we consider. In Sec. **??**, we calculate the amplitude for the TLS in its excited state in the one quantum subspace, and the quantum Zeno and anti-Zeno crossover is investigated. Conclusions are summarized at the end of the paper.

II. DESCRIPTION OF THE MODEL.

Consider a TLS subject to an harmonically transverse driving with frequency ν and intensity A. The lower and upper eigenstates of the two-level system are described by notation $|e\rangle$ and $|g\rangle$ respectively, which is separated by energy Ω without external fields. Such kind of TLSs can been experimentally realized using superconducting circuits[36, 45]. The Hamiltonian describing the TLS is

$$H_A = (\Omega + A\cos\nu t)\,\sigma_z/2, \tag{1}$$

where σ_z describes the atomic inversion. $\Omega + A\cos\nu t$ is the energy splitting. The TLS interacts with a quantized electromagnetic field of a 1D waveguide, which is constructed by coupling of cavities in an array. Due to the overlap of the spatial profile of the cavity modes, photon hops between neighbouring cavity. Introducing the creation and annihilation operators of the cavity modes, $a_j^\dagger$ and a_j, the CRW is modeled as a linear chain of sites with the nearest-neighbor interaction. The Hamiltonian of the 1D CRW can be written as

$$H_W = \sum_j \omega_c a_j^\dagger a_j - \xi \sum_j \left(a_{j+1}^\dagger a_j + h.c.\right), \tag{2}$$

where ξ is the hopping energy between adjacent cavities and ω_c is the eigenfrequency of each cavity. The TLS is inside one of the cavity in the 1D CRW, which is labeled as the zeroth cavity. In the rotating wave approximation (RWA), the interaction between the TLS and the zeroth cavity is described by a Jaynes-Cummings Hamiltonian

$$H_I = g\left(\sigma_- a_0^\dagger + h.c.\right). \tag{3}$$

The operators σ_+ and σ_- are the usual raising and lowering operators of the TLS. The Hamiltonian governing the system is

$$H = H_A + H_W + H_I. \tag{4}$$

By employing the Fourier transformation

$$a_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} a_k, \tag{5}$$

the Hamiltonian H in the Hilbert space of configuration is now given by in the momentum space

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \frac{1}{2}\left[\Omega + A\cos(\nu t)\right]\sigma_z + \frac{g}{\sqrt{N}} \sum_k \left(a_k^\dagger \sigma_- + h.c.\right), \tag{6}$$

where the dispersion relation

$$\varepsilon_k = \omega_c - 2\xi\cos k \tag{7}$$

describes an energy band of width 4ξ (the lattice constant is assumed to be unity). The second-quantization operators $a_k/a_k^\dagger$ annihilate/create one photon in the kth mode of the 1D CRW. The periodical boundary condition is used to obtain the Hamiltonian H in Eq.(6).

It can be found that the number of quanta, which is defined by operator $\mathcal{N} = \sum_k a_k^\dagger a_k + \sigma_z$, is conserved in this system, i.e. $\mathcal{N}$ commutes with Hamiltonian H. Therefore, if we have one quantum in the initial state, the state vector evolves restrictedly in the one-quantum space. We introduce the states $|\bar{k}\rangle = a_k^\dagger |0g\rangle$ which describes that there are one excitation in the kth mode of the CRW while the TLS stay in its ground state, and $|\bar{e}\rangle = |0e\rangle$ which denotes that the TLS has been flipped to its excited state while the CRW is in the vacuum state. The orthonormal basis set $\{|\bar{k}\rangle, |\bar{e}\rangle\}$ spans the one-quantum space. Therefore Hamiltonian H is rewritted as

$$H = \frac{1}{2}\left[\Omega + A\cos(\nu t)\right]\left(|\bar{e}\rangle\langle\bar{e}| - \sum_k |\bar{k}\rangle\langle\bar{k}|\right) + \sum_k \varepsilon_k |\bar{k}\rangle\langle\bar{k}| + \frac{g}{\sqrt{N}} \sum_{\bar{k}} \left(|\bar{k}\rangle\langle\bar{e}| + h.c.\right) \tag{8}$$

in the one-quantum subspace. Hamiltonian H in Eq.(8) describes a single discrete state is coupled to a continuum when the periodical-driven field is absent, which means the discrete state is subject to decay. Consequently, the excited state of the TLS is an unstable state.

III. FLOQUET FORMULATION FOR DR ATOM INSIDE A 1D WAVEGUIDE.

Since the external driving field is strictly per time, Hamiltonian in Eq.(8) is a periodic fun time, i.e. $H(t) = H(t+T)$ with $T = 2\pi/\nu$ be period. To study our problem, it is necessary sider solutions of Schrödinger equation with a t riodic Hamiltonian. In this section, the theoretic that we used is the Floquet representation of qu mechanical systems. A brief outline of this met been given in appendix.

We now employ the Floquet-state nomencla $|\alpha n\rangle = |\alpha\rangle \otimes |n\rangle$, where n is the Fourier index from $-\infty$ to ∞, $\alpha = \bar{e}, \bar{k}$ is the system index. deal with Hamiltonian H_A in Eq.(1). In the space, the time parameter is another degree of of the system. Hence, the periodical function co an operator with the expression

$$\cos(\nu t) = \frac{1}{2}\sum_n \left(|n+1\rangle\langle n| + h.c.\right).$$

In the one quantum subspace, Hamiltonian H_A – be separated into two segments $H_e + H_g$ with

$$H_e = \sum_n \left(\frac{\Omega}{2} + n\nu\right)|\bar{e}n\rangle\langle\bar{e}n| + \sum_n \frac{A}{4}\left(|\bar{e}n+1\rangle\langle n| + h.c.\right)$$

$$H_g = \sum_{kn}\left(-\frac{\Omega}{2} + n\nu\right)|\bar{k}n\rangle\langle\bar{k}n| - \sum_{kn}\frac{A}{4}\left(|\bar{k}n+1\rangle\langle\bar{k}n| + h.c.\right)$$

It can be diagonalized by the following transform

$$|\bar{e}n\rangle = \sum_m J_{n-m}(-\chi/2)|\bar{e}\phi_m\rangle \quad (11a)$$

$$|\bar{k}n\rangle = \sum_m J_{n-m}(\chi/2)|\bar{k}\phi_m\rangle \quad (11b)$$

where $J_n(x)$ is the Bessel function of the first kind and $\chi = A/\nu$. In terms of states in Eq.(11) we reduce the solution of a periodic time-dependent Hamiltonian H to the problem of diagonalizing the time-independent Floquet Hamiltonian $H_F = H_0 + H_1$ with

$$H_0 = \sum_m E_m^{\bar{e}}|\bar{e}\phi_m\rangle\langle\bar{e}\phi_m| + \sum_{km} E_m^{\bar{k}}|\bar{k}\phi_m\rangle\langle\bar{k}\phi_m| \quad (12a)$$

$$H_1 = \sum_{kmm'}\frac{gJ_{m-m'}(\chi)}{\sqrt{N}}\left(|\bar{k}\phi_m\rangle\langle\bar{e}\phi_{m'}| + h.c.\right). \quad (12b)$$

Here, $E_m^{\bar{e}}$ ($E_m^{\bar{k}}$) is the eigenvalue of $H_e(H_g)$ with

$$E_m^{\bar{e}} = \Omega/2 + m\nu, \quad (13a)$$

$$E_m^{\bar{k}} = \varepsilon_k - \Omega/2 + m\nu. \quad (13b)$$

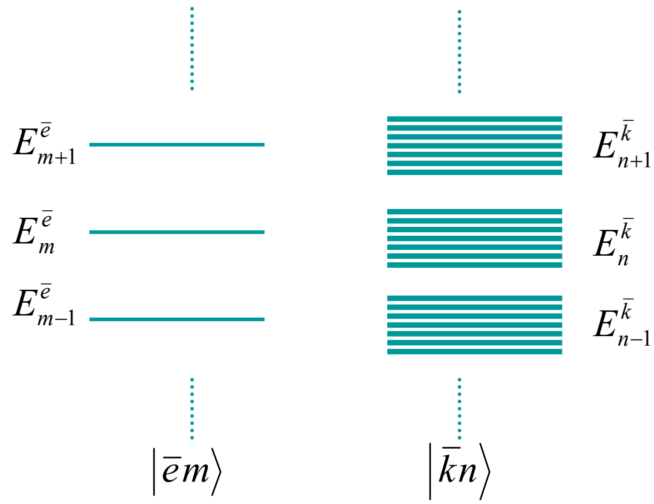

FIG. 2. (Color online) Schematic Quasienergy diagram of the Hamiltonian H_e (a) and H_g (b) in Eq.(10) under the condition $2\xi < \nu$.

The eigenvectors of Hamiltonian H_e and H_g are coupled via the nonzero coupling strength g. Figure **??** shows the energy diagram of Hamiltonian H_e and H_g in Eq.(10) when $2\xi < \nu$. When the eigenvalues $E_m^{\bar{e}}$ and $E_n^{\bar{k}}$ are close to each other, i.e. $E_m^{\bar{e}} \approx E_n^{\bar{k}}$, Floquet Hamiltonian H_F is reduced to the following form

$$H_R = H_{R0} + H_{R1} \quad (14)$$

where

$$H_{R0} = E_m^{\bar{e}}|\bar{e}\phi_m\rangle\langle\bar{e}\phi_m| + \sum_k E_n^{\bar{k}}|\bar{k}\phi_n\rangle\langle\bar{k}\phi_n| \quad (15a)$$

$$H_{R1} = \sum_k \frac{gJ_{n-m}(\chi)}{\sqrt{N}}\left(|\bar{k}\phi_n\rangle\langle\bar{e}\phi_m| + h.c.\right) \quad (15b)$$

This approximation seems equivalent to the rotating wave approximation (RWA) which is traditionally used in the field of atomic field to neglect the counter-rotating term of the harmonic driving, however this approximation is different from the RWA, which is valid only for amplitudes of the driving field small compared to the energy difference between the atomic states and breaks down in the strong field.

The above discussion shows that if the TLS is initially in its excited state, it will emit a photon as a result of the interaction with the radiation field of the CRW, photons will gain or loss energy quantum $\hbar\omega$ due to the periodical modulation. Hence the state of the emitted photon is characterized by the set of energy $E_q = \Omega - q\hbar\omega$ with $q = n - m$. Once E_q equals to the energy ε_k of the CRW, photons go to the CRW. However, when ratio of the intensity to the frequency of the modulation is equal to the roots of the Bessel function $J_q(\chi)$, this process is prevented due to the decoupling of the TLS and CRW as one can see in Eq.(15b). When the modulation is absent, the index q vanishes (i.e. $q = 0$). The behavior of the emitted photon is determined by whether Ω is equal to ε_k or not. Here, we only give a intuitively discussion, more details will present in the next section.

IV. ZENO-ANTI-ZENO CROSSOVER

Time evolution of standard quantum theory assumes two principles: the continuous unitary evolution without measurement, and the projective measurement. Quantum Zeno effect (QZE) or anti-Zeno effect (AZE) is a phenomenon related to projective measurements, which says that repeated observations prolong or shorten of a lifetime of an unstable state. However, slowdown or speedup of the decay of an unstable state is also possible without measurements or observations[47, 48].

We now investigate the lifetime of the periodically driven TLS in interaction with the CRW when the hopping energy and the driven frequency are chosen such that $2\xi < \nu$. An excited state of the TLS in one quantum subspace of this system will evolves into a superposition of itself and the states in which the atom is unexcited and has released a photon into the CRW. With the half width of the band smaller than the driven frequency, the dynamic of the TLS is major governed by Hamiltonian in Eq.(14) with $m = 0$. In terms of the Green function, the probability for finding an initial excited TLS still in the excited state reads $P_e = \left|\oint dE e^{-iEt} C_e(E)\right|^2$. Here we have defined

$$C_e(E) = \langle e\phi_0| (E - H_R)^{-1} |e\phi_0\rangle \tag{16}$$

which is the Fourier-Laplace transform of the amplitude $C_e(t)$ for the TLS in its excited state at arbitary time. Since the number of the cavities in the CRW is larger, the coupling between the TLS and the CRW is small. Therfore, Hamiltonian H_{R0} can be regarded as the unperturbed part, while Hamiltonian H_{R1} is treated as a perturbation part. Via Dyson's equation, we compute the Floquet-Green's functions up to the second nonvanishing order in the coupling strength $g/\sqrt{N}$. Then the amplitude reads

$$C_e(E) = \frac{1}{E - E_0^{\bar{e}}}\left[1 + \frac{\tilde{g}_n(E)}{E - E_0^{\bar{e}}}\right] \tag{17}$$

where $g_n(E)$ is the Fourier–Laplace transform of $g_n(t)\, e^{-i(\omega-\Omega/2+n\nu)\tau}$

$$\begin{aligned}\tilde{g}_n(E) &= \int_0^\infty dt g_n(\tau)\, e^{-i(\omega-\Omega/2+n\nu)\tau} e^{iEt-\eta t} \\ &= \sum_k \frac{g^2}{N}\frac{J_n^2(\chi)}{E - E_n^{\bar{k}}}.\end{aligned} \tag{18}$$

The inverse Fourier–Laplace transform of $g(E)$ yields the memory function

$$g_n(t) = \frac{g^2}{N} J_n^2(\chi) \sum_k e^{it2\xi\cos k} \tag{19}$$

or reservoir response function[34, 35], which depends on the quasiexcitation in the N modes of the CRW and characterizes the spectrum of the reservior. Comparing with the memory function $\Phi(t)$ in the absence of modulation, an extra factor $J_n^2(\chi)$ has been involved. Obviously, by setting $n = 0$ and $A = 0$, $g_n(t)$ reduces to $\Phi(t)$.

The inverse Fourier-Laplace transform of Eq.(17) yields the time evolution of the amplitude $C_e(t)$

$$C_e = e^{iE_0^{\bar{e}}t}\left[1 - t\int_0^t d\tau\left(1 - \frac{\tau}{t}\right) g_n(\tau)\, e^{-i(\Delta+n\nu)\tau}\right], \tag{20}$$

where detuning $\Delta = \omega_c - \Omega$. The probability for finding the TLS in its excited state reads

$$P_e \simeq \exp(-Rt)\,. \tag{21}$$

Here the decay rate is the overlap of the modulation spectrum $f_n(\omega)$ and the reservoir coupling spectrum $g_n(\omega)$

$$R = 2\pi\int_{-\infty}^{+\infty} d\omega f_n(\omega)\, g_n(\omega)\,, \tag{22}$$

where $f_n(\omega)$ and $g_n(\omega)$ is the Fourier transform of functions $f_n(\tau)$ and $g_n(\tau)$. The function $f_n(\tau)$ is defined as

$$f_n(\tau) = \left(1 - \frac{\tau}{t}\right) e^{-i(\Delta+n\nu)t}\Theta(t-\tau)\,, \tag{23}$$

where $\Theta(x)$ is the Heaviside unit step function, i.e., $\Theta(x) = 1$ for $x \geq 0$, and $\Theta(x) = 0$ for $x < 0$. Comparing with the form factor induced by the frequently measurement, an extra factor $e^{-in\nu t}$ has been introduced by the modulation. However one can find that when the modulation is absent ($n = 0$), the modulation spectrum reduces to the measurement-induced level-broadening function in Ref.[34].

The expression of functions $f_n(\tau)$ in Eq.(23) and $g_n(\tau)$ in Eq.(19) allows us to calculate the decay rate as

$$R = \frac{tg^2}{N} J_n^2(\chi) \sum_k \sin c^2 \frac{(\Delta - 2\xi\cos k + n\nu)\, t}{2}, \tag{24}$$

where $\sin c = \sin x/x$. It shows that the decay rate R is determined by: 1) the parameters of the driving field, i.e. the driven intensity A and frequency ν; 2) the detuning Δ between the cavity and the TLS; 3) the modulation time t; 4) the number N of cavities in the 1D waveguide. But only frequency ν, intensity A, and the detuning Δ can be adjusted experimentally.

We first consider the long-time dynamics of the periodically driven TLS. As time $t \to \infty$, the decay rate in Eq. (24) is a sum of Dirac delta functions

$$R = \frac{g^2}{N} J_n^2(\chi) \sum_k \delta(\Delta - 2\xi\cos k + n\nu) \tag{25}$$

Eq.(25) shows that depending on whether the matching condition

$$\Delta - 2\xi\cos k + n\nu = 0 \tag{26}$$

is satisfied, the TLS is either (i) frozen to its initial excited state or (ii) the TLS moves to the lower state and stays in it for ever. Case (ii) appears at the matching condition, but case (i) occurs when the transition energy of the TLS is out of resonance with the nth energy band of the CRW. When the number of cavities in the CRW is infinity, the states in the reservoir are continuum, one can replace the sums over k by integrals. Therefore, the properties of CRW are described by the reservoir spectral density

$$\rho(\omega) = N^{-1}\sum_k \delta(\omega + 2\xi\cos k) \tag{27}$$

$$= \begin{cases} 0 & 2\xi < |\omega| \\ \infty & 2\xi = |\omega| \\ \frac{2/\pi}{\sqrt{4\xi^2-\omega^2}} & 2\xi > |\omega| \end{cases}.$$

And the Fourier transformation of the reservoir response function reads

$$g_n(\omega) = g^2 J_n^2(A/\nu)\,\rho(\omega). \tag{28}$$

The behavior of the TLS is determined by whether the transition energy of the TLS is inside or outside the energy band of the CRW. When $\varepsilon_{k=0} \le \Omega - n\nu \le \varepsilon_{k=\pi}$, the TLS is in its ground state and the single quantum stays in the modes of the CRW. The decay rate reads $R = 2\pi g_n(\Delta + n\nu)$. It is the extension of the golden rule rate to the case of a time-dependent coupling. When the TLS is out of resonance with the CRW, the TLS remains in its excited state. However, all the above matching conditions doesn't matter when the ratio of intensity A to frequency ν is chosen such that $J_n(A/\nu) = 0$. And at those points where the driving intensity A and frequency ν satisfy $J_n(A/\nu) = 0$, the decay is completely suppressed. Actually, the occurrence of the complete suppression at $J_n(A/\nu) = 0$ is caused by the decoupling between the TLS and the CRW. Notice that the subscript n of the Bessel function is determined by the ratio of the energy difference between the CRW and the TLS to the frequency ν, and the zeroes of the Bessel function $J_n(x)$ with different n appear in different argument x. Therefore, one can switch on or off the coupling between the TLS and the CRW by adjusting the detuning Δ for a given intensity A and ν which satisfy $J_n(A/\nu) = 0$.

We now consider the dynamics of the periodically driven TLS with a finite time. The investigation starts with Eq.(24). Obviously, the decoupling between the TLS and the CRW still happens when the ratio of the driven intensity A to frequency ν meets a root of the Bessel function $J_n(x)$. This decoupling preserves the population of the TLS. Consequently, QZE appears in this system. For $J_n(A/\nu) \neq 0$, one can still have QZE because the decay rate R grows as t increases when equation (26) is satisfied. However, When the transition energy Ω is out of resonance with the energy band of the CRW for any k, i.e. equation (26) is not satisfied for any k, the decay rate is roughly a descending function of t. Consequently, the AZE occurs. Hence, for a given

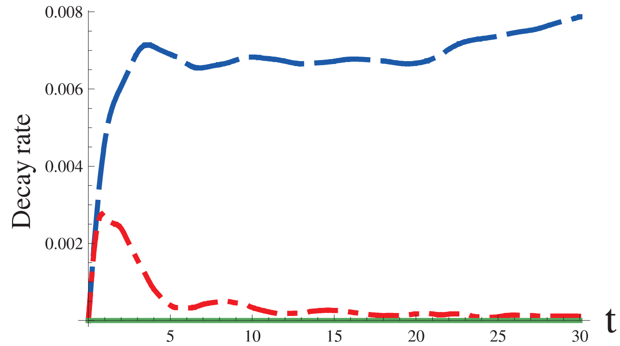

FIG. 3. (Color online) The decay rate of the TLS as a function of time t with t in units of ξ^{-1}. The coupling strength $g = 0.25$ and the number of cavities $N = 41$, $\Delta = 1, A/\nu = 1$ for blue dashed line, $\Delta = 3, A/\nu = 1$ for red dot-dashed line, $\Delta = 3, A/\nu = 2.4$ for green solid line. All parameters are in units of ξ.

driven source, the Zeno-Anti-Zeno crossover can be adjusted by the detuning between the TLS and the CRW. For a given detuning Δ, one can switch the occurrence of the QZE and AZE by controlling the driven intensity and frequency. In Fig.3, we plot the decay rate as a function of time t with the coupling strength $g = 0.25\xi$ and the number of the resonators $N = 41$. For the green solid line, the detuning $\Delta = 3$, the ratio of the intensity to frequency $A/\nu = 2.4$. which satisfies the $J_0(A/\nu) = 0$, therefore the decay rate is vanished. For the red dot-dashed line, the transition energy of the TLS is outside the band of the CRW, hence, anti-Zeno effect appears. For the blue dashed line, transition energy inside the band, consequently, quantum Zeno effect occurs. Therefore, one can switch quantum Zeno effect to Anti-Zeno effect by tuning the transition energy of the TLS or the intensity and frequency of the driven field.

As for the case with infinite cavities and finite time, the important factors are the the width and center of the spectrums, which determine the appearance of Zeno and anti-Zeno effect. We denote th width and center of the modulation spectrum as $\Delta_f = t^{-1}$ and $\omega_f = \Delta + n\nu$. The width and center of the reservoir coupling spectrum $g_n(\omega)$ read $\Delta_g = \sqrt{2}\xi g J_n(A/\nu)$ and $\omega_g = 0$, respectively. When $\Delta_f \gg \Delta_g, \omega_f$, the effective decay rate $R \sim 2\pi f_n(\Delta + n\nu)$, which grows with t, consequently, the quantum Zeno effect generally occurs. When $\Delta_f \ll |\omega_f - \omega_g|$ and ω_f is significantly detuned from the center ω_g of $g_n(\omega)$, the effective decay $R \sim 2\pi g_n(\Delta + n\nu) \ll 2\pi g_n(\omega_g)$. As a result, the effective decay R is a increasing function of the width Δ_f, which leads to the acceleration of decay, i.e. the anti-Zeno effect.

V. CONCLUSION

In summary, we have considered a CRW with a period-driven TLS inside one of the resonators. The spontaneous emission of the TLS is investigated in the one-quantum subspace via the Floquet representation of this system. Via computing the Floquet-Green's function up to the second nonvanishing order in the coupling strength, the amplitude for the TLS still in its excited state is obtained. It is found that the band structure of the CRW in the dispersion relation of the electromagnetic field gives rise to crossover of the quantum Zeno effect and anti-Zeno effect, which can be controlled by the driven intensity A and frequency ν, the detuning Δ between the cavity and the TLS. One important effect induced by the diagonal couplings between the driving field and the TLS is the strong dynamical suppression of decaying at suitable values of applied ac field.

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Appendix A: Floquet formulation

Here, we give a brief outline of this method. According to Floquet's theorem[41, 43, 44], the time-dependent Schrödinger equation exists Floquet-state solutions of the form

$$\left|\Psi_{\alpha}(t)\right\rangle = e^{-i\varepsilon_{\alpha}t}\left|\Phi_{\alpha}(t)\right\rangle, \tag{A1}$$

where the Floquet state $\left|\Phi_{\alpha}(t)\right\rangle = \left|\Phi_{\alpha}(t+T)\right\rangle$ is a function with the same period as the driving field, ε_{α} is a real-valued energy function and is called the quasi-energy. The Floquet states obey the eigenvalue equation

$$\left(H - i\frac{\partial}{\partial t}\right)\left|\Phi_{\alpha}(t)\right\rangle = \varepsilon_{\alpha}\left|\Phi_{\alpha}(t)\right\rangle. \tag{A2}$$

The operator on the left hand of the above equation gives the Floquet Hamiltonian

$$H_F \equiv H - i\frac{\partial}{\partial t}, \tag{A3}$$

which is time-independent. Floquet Hamiltonian operates on an extended Hilbert space $\mathcal{R} = \mathcal{H}\otimes\mathcal{T}$, which made up of the Hilbert space $\mathcal{H}$ of the system and the temporal space $\mathcal{T}$ of time-periodic functions. The temporal part can be spanned by the orthonormal set of functions $\langle t\,|m\rangle = \exp(im\nu t)$, where $m = 0, \pm1, \pm2, \cdots$ is the Fourier index, and

$$\langle n\,|m\rangle = \frac{1}{T}\int_0^T \exp\left[i(m-n)\nu t\right]dt = \delta_{mn}. \tag{A4}$$

The composite Hilbert space $\mathcal{R}$ is called Floquet or Sambe space[43]. In Floquet space, the Floquet Hamiltonian is linear and Hermitian, and the Floquet states provide a complete basis with the scalar product defined as

$$\langle\langle\Phi_{\alpha}(t)\mid\Phi_{\beta}(t)\rangle\rangle = \frac{1}{T}\int_0^T \langle\Phi_{\alpha}(t)\,|\Phi_{\beta}(t)\rangle\,dt. \tag{A5}$$

The matrix elements of the time-evolution operator $U_{\alpha\beta}(t,t_0)$ propagates the state $|\alpha\rangle$ at time t_0 to the state $|\beta\rangle$ at time $t > t_0$ according to the time-dependent Hamiltonian H. In the Floquet representation, $U_{\alpha\beta}(t,t_0)$ is related to the Floquet Hamiltonian

$$U_{\alpha\beta}(t,t_0) = \sum_n \langle\beta n|\exp\left[-iH_F(t-t_0)\right]|\alpha 0\rangle\, e^{in\omega t} \tag{A6}$$

and is interpreted in Ref.[43] as "the amplitude that a system initially in the Floquet state $|\alpha 0\rangle$ at time t_0 evolves to the Floquet state $|\beta n\rangle$ at time t according to the time-independent Floquet Hamiltonian H_F, summed over n with weighting factors $\exp(in\omega t)$". In experiment, the probability to go from the initial state $|\alpha\rangle$ to the final state $|\beta\rangle$ is the time-averaged transition probability between $|\alpha\rangle$ and $|\beta\rangle$

$$P_{\alpha\beta} = \sum_n \left|\langle\beta n|\exp\left[-iH_F(t-t_0)\right]|\alpha 0\rangle\right|^2. \tag{A7}$$

The matrix elements $U_{\alpha\beta}(t,t_0)$ is related to the Floquet-Green's functions via the Cauchy integral formula $U_{\alpha\beta}(t,t_0) = \oint e^{-iEt}G_{\alpha\beta}dE$ with the Floquet-Green's functions[46]

$$G_{\alpha\beta} = \sum_n e^{in\omega t}G_{\alpha\beta}^{[n]}. \tag{A8}$$

The right hand of equation (A8) is the Fourier expansion of $G_{\alpha\beta}$ with the Fourier coefficients

$$G_{\alpha\beta}^{[n]} = \langle\beta n|\left(E - H_F\right)^{-1}|\alpha 0\rangle. \tag{A9}$$

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