Comment on "Comment on Supersymmetry, PT-symmetry and spectral bifurcation

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In "Comment on Supersymmetry, PT-symmetry and spectral bifurcation" [1], Bagchi and Quesne correctly show the presence of a class of states for the complex Scarf-II potential in the unbroken PT-symmetry regime, which were absent in [2]. However, in the spontaneously broken PT-symmetry case, their argument is incorrect since it fails to implement the condition for the potential to be PT-symmetric: $C^{PT}[2(A-B)+\alpha]=0$. It needs to be emphasized that in the models considered in [2], PT is spontaneously broken, implying that the potential is PT- symmetric, whereas the ground state is not. Furthermore, our supersymmetry (SUSY)-based 'spectral bifurcation' holds *independent* of the $sl(2)$ symmetry consideration for a large class of PT-symmetric potentials.

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The primary goal of the paper "Supersymmetry, PT-symmetry and spectral bifurcation" [2] was to analyze the condition for spontaneous PT-symmetry breaking for a wide class of potentials.
The condition for the complex Scarf-II potential [4] to be PT-symmetric, under suitable parameterization, came out to be,

$$C^{PT}[2(A-B)+\alpha]=0. \tag{1}$$

In the unbroken PT-symmetry regime, it was found that $C^{PT}=0$, and the corresponding superpotential was,

$$W(x)=Atanh(\alpha x)+iBsech(\alpha x), \tag{2}$$

yielding the potential,

$$V_{-}(x)=-\left[A(A+\alpha)+B^2\right]sech^2(\alpha x)+iB(2A+\alpha)sech(\alpha x)tanh(\alpha x). \tag{3}$$

When PT-symmetry is spontaneously broken, $C^{PT}\neq 0$, meaning $A=B-\frac{\alpha}{2}$. This results in a unique potential,

$$V_{-}(x)=-\left[2A(A+\alpha)-2(C^{PT})^2+\frac{\alpha^2}{4}\right]sech^2(\alpha x)+i\left[2A(A+\alpha)+2(C^{PT})^2+\frac{\alpha^2}{2}\right]sech(\alpha x)tanh(\alpha x), \tag{4}$$

corresponding to two *different* superpotentials,

$$W^{\pm}(x)=\left(A\pm iC^{PT}\right)tanh(\alpha x)+\left[\pm C^{PT}+i\left(A+\frac{\alpha}{2}\right)\right]sech(\alpha x), \tag{5}$$

representing two disjoint sectors of the Hilbert space with normalizable wave-functions.
We agree with Bagchi and Quesne that in the unbroken PT-symmetry regime, a further symmetry in the parameter space yields another normalizable set of wavefunctions having different spectrum, which owes its origin to an underlying $sl(2)$ symmetry [3].

Bagchi and Quesne [1] further demonstrate that, when PT-Symmetry is spontaneously broken, the $sl(2)$ symmetry of the potential is realized through the exchange,

$$\mathcal{A}+\frac{\alpha}{2}\leftrightarrow\mathcal{B}, \tag{6}$$

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where $\mathcal{A} = A \pm iC^{PT}$ and $\mathcal{B} = B \mp iC^{PT}$, resulting yet again in two disjoint sectors in the Hilbert space. The corresponding ground-state energies are $-\mathcal{A}^2$ and $-\mathcal{B}^2$. It needs to be emphasized that in the models considered in [2], PT is spontaneously broken, implying that the potential is PT- symmetric, whereas the ground state is not. Hence, the condition given in Eq.(1), holds in both the broken and unbroken sectors. For spontaneously broken PT-symmetry, we have $C^{PT} \neq 0$ [2]. This yields $A + \frac{\alpha}{2} = B$, which reduces the parametric $sl(2)$ exchange to,

$$C^{PT} \leftrightarrow -C^{PT}. \tag{7}$$

Then, the ground state energies of both the sectors turn out to be $-(A \pm iC^{PT})^2$. Therefore, the $sl(2)$ transformation, when PT-symmetry is broken, merely relates the bifurcated sectors of the Hilbert space, obtained already through the application of SUSY [2]. Hence, both the sectors of the Hilbert space for unbroken PT-symmetry, under $sl(2)$ symmetry, maps to the same pair of sectors when PT-symmetry is spontaneously broken. This is the reason why, despite overlooking the $sl(2)$ algebra, the present authors obtained the complete complex-conjugate spectra. Furthermore, it was correctly found out in [2] that $C^{PT} \neq 0$ is the sole parametric criterion for broken PT-symmetry, resulting in the spectral bifurcation.

Our method was applied to a number of other potentials, tabulated in [2], which do not satisfy the $sl(2)$ algebra. In each case, the spectral bifurcation was present for $C^{PT} \neq 0$. This further shows that Bagchi and Quesne's approach does not lead to the SUSY-parametric criterion of spontaneous breaking of PT-symmetry. Further, the two superpotentials when PT-symmetry is preserved, maps *independently* to the same pair of superpotentials when PT-symmetry is broken, which is not clear in [1, 3].

References

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