Journal of Regulatory Economics; 18:1 5–32, 2000

Evaluation of a Truthful Revelation Auction in the Context of Energy Markets with Nonconcave Benefits*

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Abstract
We describe a Vickrey-Clarke-Groves auction for supply and demand bidding in the face of market power and nonconcave benefits in which bidders are motivated to bid truthfully, and evaluate its use for power and gas pipeline capacity auctions. The auction efficiently allocate resources if firms maximize profit. Simulations, including an application to the PJM power market, illustrate the procedure. However, the auction has several undesirable properties. It risks being revenue deficient, can be gamed by cooperating suppliers and consumers, and is subject to the information revelation and bid-taker cheating concerns that make single item Vickrey auctions rare.

* Acknowledgments. This work was sponsored by the US Federal Energy Regulatory Commission, with partial support from the National Science Foundation under grant ECS 9696014. Opinions expressed are those of the authors and not of the sponsoring agencies. We are grateful for suggestions by R. Harstad, W. Hogan, S. Stoft, C. Berry, U. Helman, and W. Stewart and the editor and reviewers of this journal.

1. Introduction

A typical feature of energy markets, in both the electricity and gas sectors, is that supply- and demand-side bidders may have nonconvex cost functions or nonconcave benefit functions.[1] They may also possess market power, the ability to manipulate prices. The purpose of this paper is to present and evaluate an auction that can achieve allocative efficiency under such circumstances, that is, to maximize the sum of producer and consumer surplus, or ''social welfare''. This is to be accomplished by designing the auction to be truth revealing; i.e., in such a way that submitting bids that reflect a bidder's true costs and benefits maximizes its profit.

The truthful revelation auction described in this paper is an application of the Vickrey-Clarke-Groves (VCG) procedure to energy markets. In particular, we implement the incentive mechanism of Groves (1973) (which can be viewed as a generalization of those in Vickrey (1961) and Clarke (1971)) in a simultaneous demand-supply auction whose solution is determined by the solution to a (perhaps large scale) mathematical program that can have both integer and continuous variables and may be nonlinear and nonconvex.[2] In theory, when conducted in isolation, our VCG auction will motivate honest bidding, even by suppliers or consumers who have market power; these bids can reflect nonconvexities in cost functions and nonconcavities in benefit functions; and the accepted demand and supply bids will maximize social welfare. The auction elicits honest bids for the same general reason as the Vickrey auction: winning bidders cannot alter their profits by changing their bid, as profit is determined by the bids by others. Therefore, in order to

1 A precise definition of a nonconvex function is as follows. If $C(X)$ is the cost function for some variable X (such as MW generation), then $C(X)$ is nonconvex if there exist some values X_1 and X_2 of X and some multiplier λ between 0 and 1 such that $C(\lambda X_1 + (1-\lambda)X_2) > \lambda C(X_1) + (1-\lambda)C(X_2)$. For instance, if there is a start-up cost for a generating unit, the cost of generating, say, 40 MW will be more than the average of the cost of generating 0 MW and generating 80 MW (i.e., $C(40) > 0.5C(0) + 0.5C(80)$). Nonconcavity is defined as follows. Let $V(X)$ be the benefit function for some variable X (such as MWh purchased or gas pipeline capacity consumed), then $V(X)$ is nonconcave if there exist some values X_1 and X_2 of X and some multiplier λ between 0 and 1 such that $V(\lambda X_1 + (1-\lambda)X_2) < \lambda V(X_1) + (1-\lambda)V(X_2)$. As an example, if a gas marketing firm would receive \$20,000 benefits from acquiring exactly 100,000 mcf/day of pipeline capacity, but would receive no benefits from any amount less than that, then the benefits of receiving 50,000 mcf/day of capacity would be less than the average of the benefits of receiving 0 mcf/day and 100,000 mcf/day ($V(50{,}000) < 0.5V(0) + 0.5V(100{,}000)$).

2 A non-convex mathematical program is defined as an optimization problem in which either:

- the objective function to be maximized is nonconcave (or, if it is to be minimized, it is nonconvex), or
- the feasible region is nonconvex (i.e., there exists a pair of feasible solutions which has an infeasible convex combination).

Nonconvex cost functions result in nonconvex mathematical programs, but nonconvex MPs can arise for other reasons. Examples include mathematical programs with integer variables or nonlinear equality constraints. In general, local optima for such optimization problems may not be globally optimal, which can make solving such problems difficult or practically impossible. Furthermore, market clearing prices may not exist.

avoid the possibilities of losing when winning would be profitable or winning when losing would be better, the bidder has an incentive to bid its true costs or benefits.[3]

Our motivation for considering the VCG auction is the existence of strong nonconvexities in energy markets that pose difficulties for more traditional market mechanisms. For cost functions, start-up costs and minimum run levels for generating units are examples of nonconvexities. Nonconvex cost functions cause problems for single price auctions, defined as an auction in which a bidder submits a fixed per unit bid for the commodity it wants to sell. In particular, nonconvexities mean that there may be no equilibrium market clearing price in which (a) all winning bids are paid the same price per unit of output, (b) the total energy supplied equals the amount demanded, and (c) no bidder can increase its profit by changing the amount it supplies, given the market clearing price (Johnson and Svoboda 1996).[4]

Recognition of such nonconvexities in power dispatch influenced the California power market restructuring. Initial proposals involved a UK-style ''Poolco'' auction in which all power suppliers would submit bids to the independent system operator, who would then use a unit commitment model to minimize the aggregate cost (as represented by the bids) of meeting loads. However, bidders would be paid only the marginal system energy cost in each period, and consequently observers expected that bidders would game their bids (e.g., inflating them to ensure recovery of fixed costs) (Johnson and Svoboda 1996). Another

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3 There is an extensive literature on auctions theory, most of it devoted to models of single, isolated auctions. For an early general survey, see Engelbrecht-Wiggans (1980), and for a survey of game theoretic models of single, isolated auctions see McAfee and McMillan (1987). Handbook chapters on bidding have been published by Wilson (1992) and Rothkopf (1994). Yet the theoretical auction literature has to be read critically. It is important to understand the effects of the many simplifying assumptions that must be made in order to obtain a solvable game theoretic model. See Rothkopf and Harstad (1994) for a discussion of this and, in particular, Rothkopf et al. (1990) as well as Engelbrecht-Wiggans and Kahn (1991) and Rothkopf and Harstad (1995) for discussions of the reasons for the nonuse of the truth revealing Vickrey auctions.

Recently, the literature has begun to deal with auctions involving multiple items with nonconvex costs or nonconcave values. Some work has been motivated by the FCC spectrum auctions (Ausubel et al. 1997; Rothkopf et al. 1998) and other work by electricity auctions (Rothkopf et al. 1990; Johnson et al. 1997; Elmaghraby and Oren 1999). Bushnell and Oren (1994) deal specifically with the issue of truthful bidding in electric power auctions; their auction for independent power producers is designed to elicit truthful revelation of operating costs, but not fixed operating costs or capital costs. McGuire (1997) proposes a VCG auction for unit commitment, while MacKie-Mason (1994) proposes one for power system operation under transmission constraints, but neither provides numerical examples or critiques.

4 A simple example illustrates this. Say demand is 75 MW for one hour and there are two 50 MW generating units available. Unit A has a start-up cost of $500 and a variable cost of $10/MWh. Unit B has a start-up cost of $750 and a variable cost of $20/MWh. There is no single spot price that will elicit exactly 75 MW of supply if the firms are maximizing profit. If the price is less than $20/MWh, no plant will operate. At a price between $20 and $35/MWh, exactly 50 MW of supply results (from just Unit A). Above $35/MWh, Unit B finds it worthwhile to incur its start-up cost, and it too operates at 50 MW, leading to a total supply of 100 MW. $35/MWh is not an equilibrium price, because if output from either of the units is constrained to be less than 50 MW (so that the total is just 75 MW), then that unit will not be maximizing its profit (since both units have variable costs well below $35/MWh). Indeed, if Unit B was so constrained, it would be losing money, and would instead choose to shut down. Johnson et al. (1997) explore some practical consequences of this ''duality gap'' for power auction design.

concern arose because large-scale unit commitment models are hard to solve to full optimality in a reasonable amount of time (Johnson et al. 1998), and there are many near-optimal solutions that can have different allocations of generation among firms. As a result, there is a concern that the auctioneer might choose solutions that systematically discriminated against some firms in favor of others (Johnson et al. 1997). These concerns, among others, lead to sweeping changes to the California proposal, in essence allowing a bilateral trading system between sellers and buyers to exist in parallel with a modified central auction that was to be somewhat similar to the iterative FCC auction (although that auction design has not been fully implemented due to software problems).

Nonconcavities in benefit functions can cause similar problems. As an example of the latter, gas pipeline capacity sometimes needs to be auctioned (according to Federal Energy Regulatory Commission (1998) order) among potential users whose demands are ''lumpy'' (all-or-nothing). The result is a combinatorial problem of what mix of lumpy bids to accept. In that case, single price auctions can easily fail to identify optimal combinations of bids, and can motivate extensive ''gaming'' of bids.

These examples motivated our exploration of whether the VCG auction could be a practical way of achieving truthful revelation and efficiency in energy markets with nonconcave benefits, nonconvex costs, and market power. We start by describing how the proposed auction works (section 2). We then present three examples that illustrate the process. The first example is an auction of gas transmission capacity to gas marketers, in which each bidder requires a fixed amount of capacity, and receives no benefit if it receives less than that amount (section 3). The second is a power auction involving suppliers only, based on the PJM power market (section 4). The third example involves both demand and supply bids for power (section 5). A joint supply-demand auction raises the possibility that collusion between suppliers and consumers could distort the market. Numerical simulations show that this possibility can become important when both supplier and consumer each control 20% or more of the market.[5]

A critique of our application of the VCG auction then follows (section 6). Although in theory it achieves allocative efficiency, the auction will generally require subsidies that can cause distortions elsewhere in the economy. Further, properties of the Vickrey auction that make it unpopular in single item auctions are shared by the VCG auction. We conclude the paper by recommending that the shortcomings of VCG auctions be taken seriously by auction designers attracted by the efficiency benefits of truthful revelation (section 7). An appendix contains numerical simulations that quantify the magnitude of the subsidies required by the auction under a range of assumptions, and show how consumer-supplier collusion can cause the auction to fail to maximize social welfare.

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5 Such concentrations are common on the supply side in many markets, but less so on the demand side. However, large power marketers, such as Enron, could conceivably achieve such market shares in the future, and distribution-only utilities, such as New England Electric System, approach that size now in regional markets.

2. Statement and Properties of the VCG Auction

The operation of the VCG auction is simple, and consists of three steps: bidding, bid acceptance, and payment determination.

In the first step, the auctioneer collects bids B_i from potential consumers and suppliers i of the commodity in question, $i = 1, 2, \ldots, I$. More than one commodity at a time can be auctioned (such as 24 hours of power). Bids may take any form, and include any constraints. For instance, suppliers may specify start-up costs, minimum up times and running levels, and maximum ramp rates. Consumers may specify minimum takes and total bids that are convex functions of the amount provided. Hence, B_i should be viewed as a vector of both price and constraint information. Let $\underline{B}$ be the set of all bids $\{B_i, i = 1, 2, \ldots, I\}$, and $\underline{B}_{-i}$ be the set of all bids excluding firm i (i.e., $\{B_j, \forall j \neq i\}$).

In the second step, bid acceptance, the auctioneer accepts those bids that maximize net social welfare, equal to the value of the winning demand bids minus the costs represented by the winning supply bids. This process can be formulated as an optimization problem whose objective function is the sum of accepted demand bids minus the sum of accepted supply bids. The problem's constraints includes those stated by the bidders, the stipulation that total supply equals total demand for each commodity, and any other constraints imposed by the physical system (e.g., maximum flows, Kirchhoff's laws). The objective function is separable in the bidders i, but may be quite complex and nonconcave if the bids of consumers are nonconcave or those of suppliers are nonconvex. In this paper, we assume that this optimization problem can be solved sufficiently close to optimality; note, however, that this can be problematic for large-scale unit commitment problems in electric power (Johnson et al., 1998).[6] After solving the optimization problem, the bidders must produce or consume the amounts determined by the auctioneer. The primal solution chosen by the auctioneer is designated as $\underline{X}(\underline{B}) = \{X_i(\underline{B}), i = 1, 2, \ldots, I\}$, with $X_i(\underline{B})$ representing the primal variables for bidder i; the solution, of course, is a function of all the bids $\underline{B}$.

In the third and final step, payment determination, the auctioneer pays each supplier the amount the supplier bid for the amount of supply accepted by the auctioneer, *plus* the improvement in social welfare that results from accepting that bid in Step 2. That improvement is calculated as the difference between the optimal objective function of two optimization models: the full optimization model of Step 2, including all bidders, and the same optimization model, but with all bids by the supplier in question excluded. The latter model never yields higher social welfare, and so this improvement will usually be positive and is never negative.

Meanwhile, this step results in each demand bidder paying the auctioneer the amount that the consumer bid for the amount of consumption that the auctioneer accepted, *minus* the improvement in social welfare that results from accepting that bid. That improvement is calculated in exactly the same manner as for the supplier, equaling the difference between the optimal objective function value of the optimization model of Step 2 with all

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6 A reviewer pointed out that systematic departures from exact optimality may kill the truthful revelation result, if a bidder is aware of and takes advantage of them.

bidders, and the same optimization model, except excluding all bids by the consumer in question.

More generally, the payment $t_i(\underline{B})$ made by the auctioneer to bidder i is defined as:

$$\begin{aligned} t_i(B) &= -V_i(X_i(\underline{B}), B_i) + [\Sigma_j V_j(X_j(\underline{B}), B_j) - \Sigma_{j \neq i} V_j(X_j(\underline{B}_{-i}), B_j)] \\ &= \Sigma_{j \neq i} V_j(X_j(\underline{B}), B_j) - \Sigma_{j \neq i} V_j(X_j(\underline{B}_{-i}), B_j), \end{aligned} \quad (1)$$

where $V_j(X_j, B_j)$ is the estimated value received by bidder j if j's primal variables equal X_j. For supply bidders, V_j is the negative of the cost function. The V_j in (1) is ''estimated'' because it is the value that the auctioneer obtains treating B_j as if it were the true parameters of j's value function. The first sum in the second line of (1) is the optimal social welfare found by the auctioneer under bids $\underline{B}$, minus the value received by bidder i. In the second sum, $X_j(\underline{B}_{-i})$ is the solution that the auctioneer gets if i is omitted (i.e., the second of the two optimizations in the payment calculation); we assume that such a solution can always be calculated.[7] This definition of $t_i(\underline{B})$ also applies to bidders who simultaneously bid both supply and demand (perhaps for delivery or receipt at different times or places).[8]

Under certain assumptions, an auction based on such a payment scheme is truth revealing (Groves 1973). That is, a supplier will find that bidding its true cost function is profit maximizing, while consumers will maximize consumer surplus by bidding their true benefit functions. These strategies are weakly dominant, in that bidding truthfully is optimal for a given party not only in equilibrium, but even if other bidders do not adopt that strategy. This result applies to any form of cost or benefit function, as long as values are private (each bidder i's value function V_i depends only upon its primal variables X_i).

Other than private values, the main assumption underlying this result is that each bidder

7 Such solutions might not exist, however, for power markets if one or more of the bidder's facilities are ''must-run'' plants that are sited in such a way that their output is required to maintain system reliability. To avoid a windfall payment to such irreplaceable facilities, the system adopted in California could be used. That system subjects must-run facilities to cost-based regulation, requiring that they be operated when the system operator decides they are needed. An alternative approach to constructing a feasible solution is to assume that there is some perfectly elastic fictional source of supply at some (relatively high) price. This would also have the benefit of reducing the payment made to firms possessing market power. One danger in this approach is that there may be periods of time when the market clearing price would be above the price for the fictional supply; as a result, bidders might have an incentive to bypass the auction.

8 Our payment scheme is a special case of a more general incentive-compatible Groves (1973) payment:

$$t_i(B) = \Sigma_{j \neq i} V_j(X_j(\underline{B}), B_j) - H_i(\underline{B}_{-i}),$$

where H_i is an arbitrary function. In power auctions, it may be useful, for example, to define a H_i that prevents unusually high payments from being made if withdrawal of a supply bidder results in high loss of load (for example, the ceiling price discussed in footnote 5, *supra*). Alternatively, H_i could include a fixed payment (e.g., a customer charge) to help cover the costs of running the auction, including deficits borne by the auctioneer when payments to suppliers exceed revenues from consumers. However, such payments may motivate bidders withdraw from the auction; this violates the participation constraint assumption of the VCG auction and could yield other inefficiencies.

believes that if it changes its bid, the other bidders will not change theirs in this auction (Nash assumption) or future auctions. This assumption may seem naive, but it may be a better approximation here than in other market games, especially for one-time auctions. The reason for this is as follows. Because of the relatively complex nature of the payment calculations, the link between one entity's bid and what other bidders receive is not as direct or as obvious in the VCG auction as in other market games (such as first price or classic Vickrey-second price auctions). If two bidders tried to cooperate, profit-maximizing departures from honest bidding would not be obvious, and any such departure would be accompanied by more obvious decreases in sales and purchases. This may weaken the incentive to cheat by colluding, particularly in one-time auctions. On the other hand, in repeated auctions, the players may learn more sophisticated strategies despite the complexity of the auctioneer's calculations. For instance, in power market auctions, daily repetition could facilitate collusion (Rothkopf 1999).

The truthful revelation result can be proven by contradiction as follows (Mas-Colell et al. 1995). Let the true parameters of i's value function be B_i^T, and assume that there exists some $B_i \neq B_i^T$ such that i's profit is increased by lying:

$$V_i(X_i(\underline{B}_{-i}, B_i), B_i^T) + t_i(\underline{B}_{-i}, B_i) > V_i(X_i(\underline{B}_{-i}, B_i^T), B_i^T) + t_i(\underline{B}_{-i}, B_i^T) \qquad (2)$$

where profit equals the firm's true valuation of its solution minus its payment. By substituting the definition (1) of t_i in both sides of the above inequality, some algebra yields:

$$\begin{aligned} &V_i(X_i(\underline{B}_{-i}, B_i), B_i^T) + \Sigma_{j\neq i} V_j(X_j(\underline{B}_{-i}, B_i), B_j) \\ &\quad > V_i(X_i(\underline{B}_{-i}, B_i^T), B_i^T) + \Sigma_{j\neq i} V_j(X_j(\underline{B}_{-i}, B_i^T), B_j). \end{aligned} \qquad (3)$$

The right side is the social welfare of solution $\underline{X}(\underline{B}_{-i}, B_i^T)$ when evaluated under bids B_{-i}, B_i^T, while the left side is the welfare of $\underline{X}(\underline{B}_{-i}, B_i)$, also evaluated using bids B_{-i}, B_i^T. However, such an inequality cannot be true since, by the definition of optimization, the welfare of $\underline{X}(\underline{B}_{-i}, B_i^T)$ evaluated using bids B_{-i}, B_i^T must be at least as high as for any other feasible solution, including $\underline{X}(\underline{B}_{-i}, B_i)$. This is because the auctioneer obtains $\underline{X}(\underline{B}_{-i}, B_i^T)$ by maximizing $\Sigma_i V_i$ over all feasible $\underline{X}$, using B_{-i}, B_i^T to evaluate the solutions.

The VCG auction has several features that differentiate it from other auctions. The first is truthful revelation: even if a supply bidder possesses a large amount of market power, it will not be profitable for it to raise its bid above its cost in order to restrict its output in an effort to increase the price it receives. The payment calculation is designed so that net revenue cannot be increased by such a strategy. The argument is symmetric for large consumers; there is no incentive for them to decrease their bids and restrict consumption in order to lower prices.

A second feature differentiating the VCG mechanism is that, in general, bidders will not necessarily pay or be paid an identical amount per unit (although bidders with the same bid will be paid the same). Basically, in order to motivate bidders to disclose their true costs, supply bidders who have more market power will be paid more per unit than other suppliers. Similarly, larger consumers will often have to pay less per unit than smaller ones. In contrast, most energy auctions result in the same price for the same commodity, although discriminatory auctions are often used in many other contexts.

The third and final feature is that, in general, a simultaneous supply-demand auction in which all supply and demand is bid into the auction will not necessarily be revenue sufficient. That is, the auctioneer will, in many cases, have to make more payments to suppliers than it receives from consumers if any bidder possesses significant market power. For instance, numerical simulations in the appendix show that if a supplier or consumer controls one-third of the market, then under the cost and demand assumptions made there, the shortfall can amount to as much as about 5% of the auctioneer's collections from buyers.

Our version of the VCG auction is a generalization of the Vickrey auction, in that it reduces to the classic Vickrey second-price auction if one item is being auctioned off and only potential buyers are submitting bids. The payment scheme can also be viewed as generalizing the exercise of perfect (first degree) price discrimination by a monopolist or monopsonist, in that the scheme pays a bidder the entire increment in surplus resulting from its participation in the market. If there is only one supplier (or consumer) bidding, the payment reduces to that gained by a perfectly discriminating monopolist (or monopsonist). In the extreme case of a bilateral monopoly in which both consumption and supply are bid, the auctioneer is in the unhappy position of paying the supplier an amount equal to the integral of the demand curve while having the buyer only pay the integral of the supply curve; the auctioneer's net loss then equals the social welfare gain from the auction.[9]

In contrast, under effective competition (many buyers and sellers), the result of an auction involving both demand and supply side bidding is the pure competition solution. The payment scheme reduces to all consumers paying one price, equal to the price received by sellers. This solution can also occur when buyers and sellers are not atomistic if supply or demand is perfectly elastic in a sufficiently large neighborhood of the solution.[10]

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9 Of course, our use of supply and demand functions in this paragraph and market clearing prices in the next implies that we are assuming convex cost functions and concave benefit functions (which would allow those functions and prices to exist); however these assumptions are not needed, in general, for our VCG auction.

10 An example of a set of sufficient conditions for such a solution is as follows:

(1) The auction is only for one time period. Each consumer has a constant per unit marginal benefit up to some upper bound for its quantity demanded; each supplier has a constant per unit marginal cost for quantity supplied up to some upper bound.

(2) In equilibrium, there are rejected supply bids with the following characteristics: (a) they are from suppliers who have no accepted bids; (b) their cost per unit equals the equilibrium price; and (c) the quantity of rejected bids is at least as large as maximum quantity actually provided by any one supplier.

(3) In equilibrium, there are accepted bids for supply that are simultaneously: (a) from suppliers who have no rejected bids; (b) have a cost per unit equal to the equilibrium price; and (c) whose quantity supplied is at least as large as the maximum quantity actually purchased by any one consumer.

Basically, the second and third conditions state that supply is perfectly elastic in some neighborhood of the solution. Under these conditions, no individual bidder has market power, in that their complete

3. Example of VCG Demand Auction: Auctioning Gas Transmission Capacity

This example illustrates how the VCG auction operates when only consumers submit bids. Section 4 demonstrates supply bidding, while section 5 gives an example of simultaneous demand and supply bidding.

This auction involves 100 units of pipeline capacity, for which four gas marketers are bidding. Table 1 shows the amount of capacity requested by the marketers, and their valuation of that capacity, in $/unit. Partial acceptance of bids is not allowed; each marketer bids on an ''all or nothing'' basis. Thus, for example, bidder A would pay up to $700 for its 70 units of capacity, but is not willing to pay anything for just 69 units. These are the types of preferences that have been expressed by some actual purchasers of natural gas pipeline capacity in cases before the Federal Energy Regulatory Commission (1998).

The three steps of the VCG auction would proceed as follows:

(1) *Bid Submission*. Bidders A through D would submit their bids, here consisting of a quantity and total payment. For example, A might bid $500 for the 70 units it requires. This bid is below its maximum willingness to pay ($700); however, as discussed below, A can do no better than to bid precisely its willingness to pay.

(2) *Bid Acceptance*. The auctioneer accepts those bids that would maximize the aggregate benefits to the bidders. In mathematical programming parlance, this is termed a ''knapsack'' optimization problem. If, for instance, the bidders each submitted bids equal to its maximum willingness to pay, then the accepted bids would be A's and C's. This would result in 90 units being utilized and a total benefit of $860 (= $700 for A plus $160 for C). No other feasible combination of bids would have a higher value. For example, if A and D were accepted instead, then the entire capacity (100 units) would be allocated, but total benefits would only be $850.

(3) *Payment Determination*. Each bidder's payment would equal the amount it bid minus the decrease in total benefits that would result if its bid was left out of the auction process. For instance, consider bidder C. If it bid its true willingness to pay of $160, then its payment would be $150, calculated as:

- $160 (its bid) *minus*

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withdrawal from the auction would not alter the equilibrium price. The payment per unit by each consumption bidder whose bid is accepted is the equilibrium price, which equals (a) the bidder's marginal benefit minus (b) the loss in welfare if it withdraws (this loss equaling its marginal benefit minus price). Similarly, each supply bidder is paid the equilibrium price, which equals (a) the bidder's marginal cost, plus (b) the loss in welfare if the bidder withdraws (the loss being calculated as the cost of the marginal supplier—equal to the price—minus the bidder's marginal cost). This pure competition solution can occur even if there are as few as two supply bidders or two consumption bidders.

Table 1. Bidder Capacity Requests and Valuations, Pipeline Auction

Bidder	Units Required	Willingness to Pay (Benefit), $/unit	Total Valuation, $
A	70	10	700
B	40	9	360
C	20	8	160
D	30	5	150

- $10, the difference between the optimal system benefits including C's bid ($860, resulting from accepting A and C) and the optimal benefits if C was excluded (in which case the maximum benefits that can be realized is $850, obtained by accepting A's and D's bids).

In this situation, C's surplus is $\$160 - \$150 = \$10$, the difference between its valuation of the 20 units and what it pays. Turning to the other winning bidder, A will be asked to pay $510, calculated as its bid of $700 *minus* $190, the latter being the decrease in optimal total benefits if A's bid is excluded. Optimal total benefits with A is $860; without A, the bids by B, C, and D would instead be accepted, leading to a total demand of 90 units and a total benefit that is $190 less ($\$670 = \$360$ from B, $160 from C, and $150 from D). Note that A's surplus is $190, considerably more than C's. The reason is that A's presence adds more to the total social benefit than C's; i.e., taking away A would lower the auctioneer's objective by more than taking away C.

There are three notable results of this VCG auction. First, the winning bidders do not pay a single price; each pays a different amount per unit. C pays $150 total, or $75/unit, while A pays $510 total, or $72.8/unit. Bidder A pays less, even though it values the capacity more ($10 per unit, versus C's $8/unit). The reason is A's large size: its removal from the auction would shrink total benefits than the removal of any other bidder. This larger impact can be interpreted as a measure of its market power.

The second notable result is the VCG auction induces truthful revelation of costs and values by the bidders. Given the bids that the other players submit, bidding one's true valuation is optimal. (Although we do not illustrate it here, bidding one's true evaluation is optimal even if other bidders lie.) Let's consider each bidder in turn, starting with A. No bid gives A a higher surplus than bidding its true valuation. If A bids any amount more than $510 (including its true valuation of $700), it will win and pay just $510.[11] If A drops

11 For instance, if A bids $550, its payment will be calculated in Step 3 as $510 in the following manner:

- its bid of $550, *minus*
- the auctioneer's estimate of how much A's absence would lower the total surplus, which equals $40. This is obtained as follows. Given the bid of $550 by A and bids by B, C, and D reflecting their true valuations, Step 2 winds up choosing A and C, yielding a total surplus of $\$550 + \$160 = \$710$. If A is not considered, B, C, and D would instead be chosen, giving a surplus of $670, which is $40 less.

its bid below \$510, however, it will lose, and earn zero surplus. So the maximum surplus it can obtain is \$190, which it can earn by simply bidding its true valuation.

B, on the other hand, is not a winner in the auction if it bids its true valuation of \$360 for 4 units. Its surplus is therefore zero. B would not win for any bid less than \$550, if A, C, and D bid their true valuations. If B bids more than \$550, the auctioneer will pick B, C, and D, and the Step 3 payment that B would have to make would be \$550.[12] As a result, B's surplus would be $-$\$190. Hence, B earns its maximum surplus (\$0) by bidding truthfully.

Turning to C, it is a winner in the auction if everyone bids its true valuation. For any bid over \$150, C would pay exactly \$150, and earn a surplus of \$10. If it bids less than \$150, the auctioneer chooses A and D instead, and C's surplus falls to zero. Once again, bidding the truth is optimal. Finally, bidding honestly also optimal for D. D is not chosen by the auctioneer in Step 2 if all parties bid their true benefits. D would have to raise its bid to over \$160, at which point the auctioneer would choose A and D rather than A and C. D's payment would be calculated in Step 3 as \$160, meaning that its surplus would be its valuation of \$150 minus \$160, or $-$\$10. Again, the bidder would be best off bidding its true valuation of \$150 and, in this case, not being chosen.

In general, bidding anything other than a bidder's true valuation cannot increase the bidder's surplus, and might shrink it. In particular, if the bidder would win if it bid its true valuation, then raising its bid does not change its surplus, while lowering the bid will either not alter its surplus (if it still wins) or will cause the surplus to fall to zero (if the bid is so low that the bidder loses). On the other hand, if the bidder would lose if it bid its true value, then lowering its bid will not change anything, while raising the bid will either not alter the bidder's surplus (if it still loses) or will cause the surplus to go negative (if it bids so high that it wins).

The third notable result of this VCG auction is its allocative efficiency, which is not a general characteristic of a first-price auction in which each winning bidder pays the amount it bid. For instance, in the case of the simple example above, a first-price auction would not yield a Nash equilibrium in pure strategies. That is, there is no single set of bids from the four bidders in which each party has no motivation to alter its bid when it assumes that the other bidders will not change theirs. The only equilibrium is a mixed strategy equilibrium, in which each party chooses a bid from a probability distribution. In that case, there is generally a positive probability that an inefficient solution (such as A + D or B + C + D) will result. First-price auctions can also be inefficient even in markets in

12 Let's say that B bids total amount X which is greater than \$550. In Step 2, the auctioneer picks B, C, and D, and calculates an apparent total surplus of X + \$160 + \$150 = X + \$310. In Step 3, B's payment will be:

- its bid of X, *minus*
- the decrease in apparent total surplus if B is not considered, which equals X-\$550, calculated as follows. The apparent surplus of X + \$310 would decrease to \$860 (resulting from selecting A and C rather than B, C, and D) if B is excluded. X + \$310 > \$860 because we assumed X > \$550. Thus, the decrease is X + \$310 − \$860 = X − \$550.

Therefore, the total payment is X − (X − \$550) = \$550, and is independent of X as long as X > \$550.

which, as is common, uncertain information leads participants to adopt pure strategies (Harstad et al. 1996).

4. Example of a Supply-Only VCG Auction: Power Auction in the PJM Power Pool

The above example illustrates how the VCG auction can be used to elicit truthful bids from consumers. This section presents an application to supply-side bidding, and illustrates the extent to which market power can influence bidder payments. The application is to the largest power pool in the world, the Pennsylvania-Jersey-Maryland (PJM) interconnection. PJM presently has a free market in generation, in which suppliers would provide bids to an independent system operator.

We simulate the application of the VCG auction by considering an hour in which demand is 40,000 MW, a bit less than 80% of PJM's installed capacity. Data on the capacity and variable costs for 151 generating units in the system are drawn from US Department of Energy and proprietary data bases. Assuming (a) truthful bidding of costs, (b) least cost dispatch, and (c) no transmission constraints, the marginal cost of supply becomes $41/MWh and no single utility serves more than 18% of the load. The Hirschman-Herfindahl Index (sum of squared percentage market shares) is then 1300, far below the 2500 level used by the US Department of Justice to signal possible concentration problems.

Yet there is still significant market power. This occurs for two reasons. First, the market supply curve rises steeply in the region of 40,000 MW, with marginal cost climbing from $30/MWh at 35,000 MW to over $80/MWh at 45,000 MW. Second, there is effectively no short-run price elasticity, for few consumers even see real-time prices. As figure 1 shows,

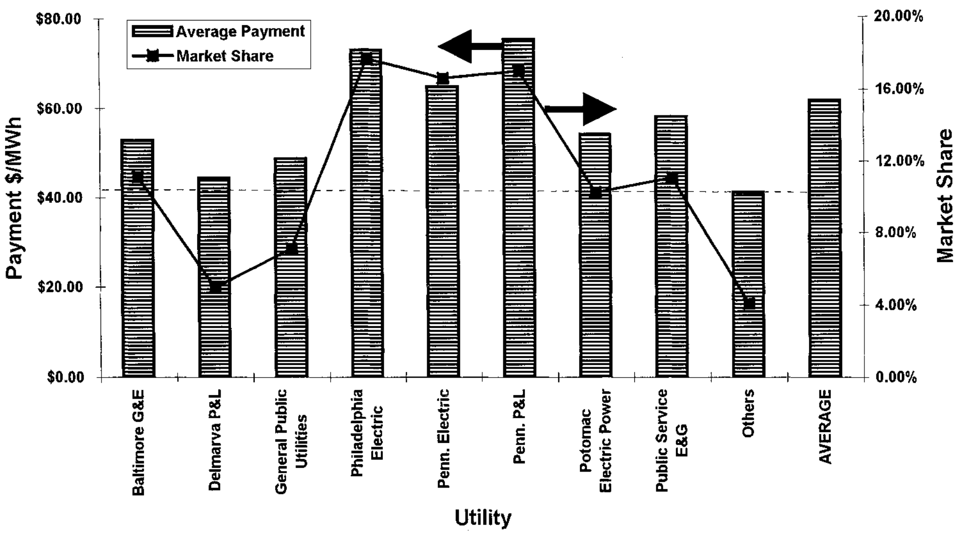

Figure 1. Average VCG payment relative to competitive price (dotted line), Pennsylvania-Jersey-Maryland power pool.

even utilities with market shares of 10% gain payments under the VCG system well in excess of the competitive price of $41/MWh (dotted line). The correlation of market share with payment size is obvious. The average payment to producers required to ensure truthful revelation is over $60/MWh, 50% higher than the competitive price. This price rise would be moderated somewhat if there were significant demand elasticity or bidders from outside the PJM region, but not greatly. It would also be much smaller for off-peak demand periods, but still significant. Price increases of this magnitude would undoubtedly be unacceptable to consumers, who would probably find little comfort in the thought that such prices would elicit truthful supply bids.

A notable feature of the VCG auction's outcome is that the fruits of market power accrue just to its owner, who receives a higher price than smaller producers. In contrast, in an auction that yields a single price for the commodity, all producers, small and large, benefit equally from the exercise of market power by larger bidders. This short run difference could have implications for long run stability of oligopolies, as the VCG auction may not encourage as much entry.

5. Example of a Simultaneous Supply-Demand VCG Auction

5.1. The Case of Separate Consumers and Suppliers

This subsection illustrates the VCG auction for the situation in which there are nonconvex costs and both supply and demand-side bidders, but no bidder simultaneously bids for supply and consumption. In section 5.2, we examine a more general case in which such simultaneous bids are allowed.

As in section 3, a one-period auction is considered. Let there be 3 demand-side bidders i, $i = A, B$, and C with the following characteristics: each demands any amount up to 1 unit of electricity, whose value is constant per unit. A's value per unit is 6, B's is 4, and C's is 1. Let there be a large number of producers j, each capable of producing 1 unit of electricity at a marginal cost of $j/2$ per unit and a fixed (start-up cost) of $j/2$. (E.g., if producer $j = 3$ produces 0.4 units, its total cost of production would be $3/2 + 3/2*0.4$, or 2.1.) Thus, the market supply and demand curves are as shown in figure 2. (The vertical arrows on the supply curve are impulses representing fixed costs incurred in order to make the next unit of supply available.)

If each bidder bids its true benefit (consumers) or cost (suppliers), the optimal total quantity Q is 2, A and B consume their maximum amount of power, and producers 1 and 2 generate up to their capacity. Any market price in the range [2,3] would produce this outcome. Social welfare (SW) is 7 $(= 6 + 4 - 1 - 2)$.

The full value of its consumption is an optimal bid for each consumer, while its actual fixed and marginal costs are an optimal bid for each supplier. Under these bids, the resulting payments and profits are shown in table 2.

As an illustration of the calculations, consider consumer A. If A is excluded from the bidding, then the social welfare falls to 3 (just one unit is supplied, with the only surplus resulting from B consuming the first unit whose value is 4, but whose cost is 1). Consumer

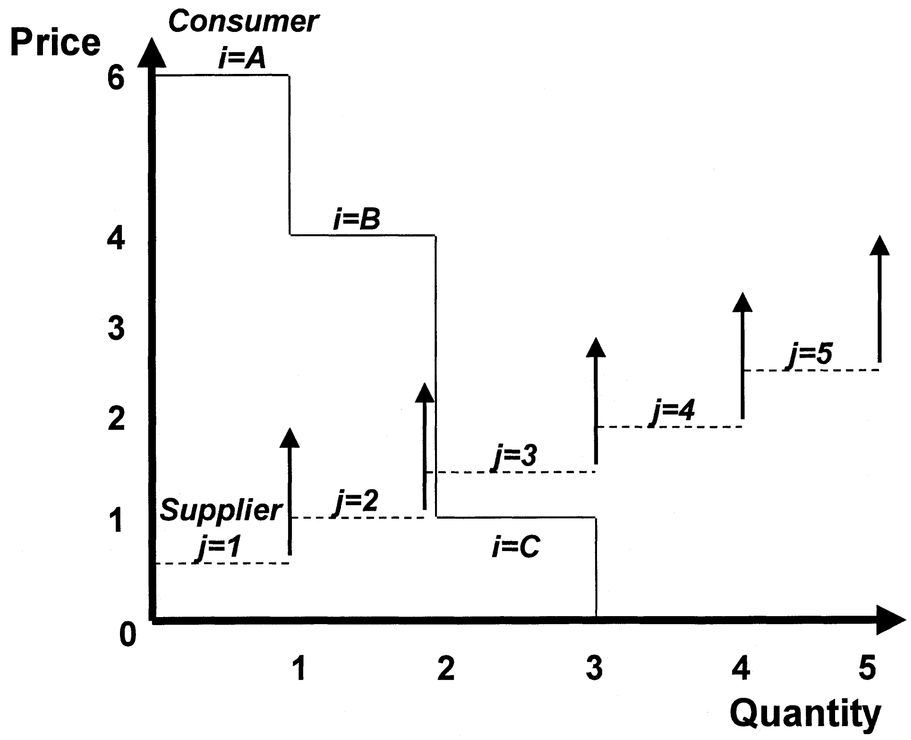

Figure 2. Supply and demand schedules if true costs/benefits are bid.

A declares its value to be 6, so it pays just 2 units, which equals its value of 6 minus the difference (= 4) between the optimal welfare (= 7) and the welfare without A (= 3). Consequently, A earns a net surplus of 6 – 2, or 4.

Now if A bids any amount other than 6 then either (but not both) of two cases occurs: (a)

Table 2. Results of Simultaneous Supply-Demand Bidding: Separate Consumers, Suppliers Case

Consumer i (Generator j)	SW Without Player	Direct Benefit to i (if Negative, Cost to j)	Payment by i (if Negative, Payment to j)	Profit
$i = A$	4 (B's benefit)- 1 (j = 1's cost) = 3	6	6 – (7 – 3) = 2	6 – 2 = 4
$i = B$	5	4	2	2
$i = C$	7	0	0	0
$j = 1$	5	– 1	– 3	2
$j = 2$	6	– 2	– 3	1
$j = 3, 4, \ldots$	7	0	0	0
Total	—	7 (= social welfare)	– 2 (net revenue to auctioneer)	9

the actual SW is unchanged, because the same solution results, or (b) A's profit falls. Thus, any profit maximizing bid also maximizes social surplus. This can be shown by considering all of A's possible strategies (assuming the other bidders are truthful):

- For any bid by A higher than 2, the solution is unchanged from the SW maximizing solution (A and B receive one unit apiece, and these units are supplied by producers 1 and 2). So the total welfare remains at 7. Further, A's profit remains 4 for any of these bids. (E.g., at a bid of 2.1, apparent welfare falls by 0.1 if A drops out, so A's payment is $2.1 - 0.1 = 2$; subtracted from A's value of 6, a profit of 4 results, as before.) This is case (a).
- For any bid lower than 2, the solution changes by excluding A. SW falls to 3 as only one unit is supplied (from producer 1 to B). A's profit falls to zero (since it consumes nothing). This is case (b).
- For a bid of exactly 2, the auctioneer's calculation of apparent social surplus finds a tie between the (true) SW maximizing solution (market supply of 2) and the suboptimal solution (market supply of 1). If the auctioneer breaks the tie by picking the first of the two options, we have case (a). If the tie is instead broken by choosing the second option, case (b) results.

As another example of the calculations, consider $j = 2$ on the supply side. If it is excluded, 2 units are still exchanged in the market, but producer $j = 3$ supplies the second unit rather than $j = 2$. As a result, costs go up by 1 unit, and the total surplus shrinks by 1. The payment to $j = 2$ is then calculated as 3, equal to its cost ($= 2$) plus the loss in surplus if it is excluded ($= 1$). Supplier 2's profit is therefore 1 ($= 3 - 2$). Bidding its true fixed cost of 1 and variable cost of 1 is profit-maximizing for 2; so too is bidding any amount that satisfies the following inequality (given truthful bidding by others and a constraint that bids be nonnegative):

$$\text{Fixed cost bid} + \text{Variable cost bid} < 3$$

This is because any such bid will induce the auctioneer to choose just suppliers 1 and 2 and dispatch them up to their capacity, which is socially optimal. On the other hand, bidding any combination of bids that satisfies the following inequality will yield a suboptimal solution:

$$\text{Fixed cost bid} + \text{Variable cost bid} > 3$$

In this case, the auctioneer will instead choose supplier $j = 3$, inflating social costs and lowering SW. Producer 2 then earns zero profit, which is suboptimal from its perspective.

Notice two outcomes of the above auction. First, each consumer pays the same price ($= 2$), while each producer is paid the same average price ($= 3$). This occurs because all consumers and producers are the same size (1 unit) and, in the optimal solution, each entity is producing or consuming at either its upper or lower bound. Second, the auctioneer operates at a loss, paying 6 to producers, but receiving only 4 from consumers. This results from the market power of the bidders.

5.2. Example of Supply-Demand Auction, Continued: The Case of a Firm that Bids Both Supply and Demand

Now imagine that one firm owns both consumer A and producer 1. It could meet its own demand internally without participating in the market (assuming that its energy consuming and producing facilities are connected in a way that doesn't require use of the auctioneer's transmission facilities), which would give it a profit of 5 (value = 6 minus cost = 1). This is the same profit it would earn by entering the market and bidding truthfully. In the latter case, it would earn a profit equal to its actual value minus cost $(6 - 1)$, minus its payment, calculated as its bid $(6 - 1)$ minus the (apparent) loss of welfare if it doesn't participate. This loss of welfare is 7 (SW if it participates) minus 2 (SW if only B and firms 2,3, . . . participate), or 5. Thus, its profit is $(6 - 1) - [(6 - 1) + 5] = 5$.

In this case, there is no extra payment resulting from the firm's market power. The reason is that the firm's supply equals its quantity demanded in the optimal solution. This outcome is not generally the case.

The auction is still truth revealing. The proof in section 2 shows that if the combined firm makes any bid that would result in a (socially) suboptimal solution, then its profit would also be less.[13] For instance, if the firm bid to buy at a price of 1.5 while bidding its true supply costs, the auctioneer would accept only the supply bid, and only consumer B would buy. The combined firm's not consuming a unit is socially suboptimal, and its profit from its sale would only be 1 unit.[14] Its total profit will therefore have fallen from 5 to 1.

On the other hand, consider the situation in which the combined firm disguises its bids in such a way that the auctioneer treats its demand bid as being from one entity and the supply bid as being from another, and calculates payments to the demand side separately from payments to the supply side. Then the firm's profits are increased by participating in the auction. This can be shown by totalling the separate profits from A and 1 (table 2). The resulting total profit is 6, rather than the 5 that results if the combined firm self-supplies and submits no bids at all. However, the overall solution remains socially optimal, as long as bids remain truthful.

But if the firm is divided into two entities in this manner, then the auction is no longer truth revealing. An example showing that lying yields even greater profit is sufficient to prove this. Assume that the auctioneer does not separately meter supply and demand, but only checks to see if the net amount bid is correct. Let that combined firm submit a demand-side bid for 2 units worth 6 units apiece and, simultaneously, a bid to supply 2 units costing 1 unit apiece. This is a lie, because it is not capable of demanding or supplying so much. But if both bids are entirely accepted, the net amount (zero) is feasible;

13 This result implicitly assumes that the demand bidder is not a reseller of power in a market in which it has market power. A reviewer pointed out that vertically integrated utilities would have such market power, if the utility is a monopoly in the retail market and regulators allow the utility to pass on its costs to its customers (via, e.g., a purchased power adjustment clause). In that case, there might be an incentive to distort bids so that demand bidders pay higher prices to supply bidders, who could then keep the extra profits earned at the expense of the utility's ultimate customers.

14 Equal to $-1 - [-1 - \{3 - 2\}]$, where 3 is the (apparent) welfare from having B consume the one unit supplied by producer 1, while 2 is the (apparent) welfare from having supplier 1 drop out so that B instead consumes one unit supplied by producer 2.

Table 3. Simultaneous Auction Results, Consumer A/Supplier 1 Collusion

Consumer i (Generator j)	Apparent SW Without Player	Actual Direct Benefit to i (if Negative, Cost to j)	Payment by i (if Negative, Payment to j)	Profit
$i = A$	$4 - 1 = 3$	6	$6 * 2 - (12 - 3) = 3$	$6 - 3 = 3$
$i = B$	$6 + 6 - (1 + 1) = 10$	4	$4 - (12 - 10) = 2$	$4 - 2 = 2$
$i = C$	12	0	0	0
$j = 1$	$6 + 6 + 4 - (2 + 3 + 4) = 7$	-1	$-1 * 2 - (12 - 7) = -6$	$-1 - (-6) = 5$
$j = 2$	$6 + 6 + 4 - (1 + 1 + 3) = 11$	-2	$-2 - (12 - 11) = -3$	$-2 - (-3) = 1$
$j = 3, 4, \ldots$	12	0	0	0
Total	—	7 (= net actual social welfare)	-4 (= net revenue to auctioneer)	11

so the trick is for the firm to design the bids to ensure this outcome. Here, the auctioneer completely accepts both bids, and calculates an (apparent) social welfare of $6 + 6 + 4$ (value to consumers) minus $1 + 1 + 2$ (cost to suppliers), or 12. Table 3 shows the profit calculations.

Despite the infeasible individual bids, the auctioneer's solution is still feasible, as the *net* power provided by the combined firm is still zero. But now the combined firm now earns a total profit of 3 (from $i = A$) plus 5 (from $j = 1$), or 8—which is 2 units higher than in table 2 (the no-collusion case). The result is still social welfare maximizing, but the firm has extracted more money from the auctioneer. Indeed, the firm could do so without limit by simply submitting additional demand bids of 6 accompanied by an equal number of supply bids of 1.

This situation also shows that even if there aren't firms who are in a position to submit (and disguise) both supply and demand bids, there is an incentive for suppliers to collude with consumers to extract more money from the auctioneer.

We have conducted numerical simulations (see the Appendix) to investigate how much market power such a collusive pair of producer and consumer bidders might exercise. These simulations assume linear demand and supply curves for the bidders and the rest of the market, which is modeled as a competitive fringe. After examining a range of supply and demand elasticities (including ones not reported in the Appendix), we conclude the following.

First, there is indeed an incentive for a supply bidder and demand bidder to secretly collude by submitting and coordinating separate dishonest sets of bids. By doing so, they can extract additional payments from the auctioneer. They do this by having the supplier understate its costs and the demand bidder overstate its willingness to pay, as found in the simple example earlier in this section. As a result, they sell and consume, respectively, too much relative to the social welfare maximizing solution. The total profit accruing to the colluders increases, although the individual profits earned by each may or may not increase.

Second, the welfare loss resulting from such supply-demand collusion is less than \$0.1 per MWh of total market demand if the sizes of the supply bidder and demand bidder are 20% or less of the competitive fringe. This loss is less than 0.2% of the price of power received by the competitive fringe. The highest percentage welfare losses occur when demand and supply are very elastic (elasticities of 1.2 and 1.6, respectively) and are much smaller for more reasonable elasticities. The welfare loss also falls quickly if either of the producer's or consumer's market shares are below that level. The subsidy from the auctioneer (the difference between the auctioneer's revenue and payments) is about double the welfare loss. On the other hand, if the colluding parties each make up approximately one-third of the market, then the welfare loss can climb to 2% of the price of power. In a multi-billion dollar power market (such as California's), this can amount to many tens of millions of dollars annually.

The problem of supplier-consumer collusion becomes much worse if the auctioneer can only measure the net amount demanded by the supplier and consumer together. In that case, the colluders can milk the auctioneer without limit by shifting their supply and demand curves to the right by very large but equal amounts. As a result, when making the payment calculation (1), the auctioneer will conclude that the loss of welfare that occurs when one or the other party is omitted is very large, which will, on net, yield a huge payment to the colluders. Thus, the lack of budget balance makes the VCG scheme vulnerable to collusion between suppliers and consumers. They can collude to implement strategies that are strongly Pareto-dominant for them at the expense of the auctioneer. However, when parties are metered separately, extreme collusion of this kind would be difficult.[15]

6. Problems and Issues with the VCG Auction

There are no free lunches in auction design. The VCG auction has a number of apparently desirable properties: In isolated contexts, it is truth revealing and leads to efficient consumption and production decisions. This sounds attractive, but it will not work as well as one might hope, and it comes at a steep price, especially if the market is uncompetitive.

First, the VCG auction is generally not self-funding. In other words, the auction will generally (but not always) require a separate source of funds to make the extra payments that motivate bidders to reveal their true costs or values.[16] The amount involved is potentially large, especially if large producers and consumers can collude to game the

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15 This potential for supplier-consumer collusion is addition to the possibilities of collusion by bidders on the same side of the market. VCG auctions share the extra vulnerability to cheating by bidders of Vickrey and progressive auctions (see Robinson 1985 and Graham and Marshall 1987) and the Vickrey auction's vulnerability to cheating by the bid-takers (Rothkopf and Harstad 1995).

16 However, the VCG auction is not necessarily revenue deficient. Consider a situation in which consumers have demand curve $P = 1 - Q$, and there are two potential suppliers, each with fixed set-up cost of 0.15 (incurred if anything is produced), zero marginal cost, and capacity 0.5. If all transactions must take place through the VCG auction, the VCG equilibrium results in the auctioneer charging each consumer $P = 0.5$ and paying one supplier 0.15 to supply $Q = 0.5$. Revenue ($PQ = 0.25$) exceeds the payment to the supplier. However, this auction is unsustainable, since in general the supplier could withdraw from the auction and offer a lower price to consumers, making both consumers and supplier better off. In general, a VCG auction that yields positive net revenue for the auctioneer will suffer from this problem (see Note 19, below).

VCG auction, as discussed in section 5.2 and the Appendix. The provision of such funds is problematical (although the use of ''uplifts'' to recover costs not otherwise included in energy prices is common in power markets). Even if it is possible to obtain the funds, there is bound to be some inefficiency associated with obtaining them (Krishna and Perry 1998).[17] For instance, if a tax is imposed, those taxed will tend to make inefficient decisions to avoid the tax.

Second, when it works properly the VCG auction requires revelation of private information by competitors. This may seem desirable or at least non-objectionable, and in a completely isolated single-auction context, it may be so. However, as with the Vickrey auction for single items which the VCG auction generalizes, there are severe disadvantages to such revelation. (See Rothkopf et al. (1990) or Engelbrecht-Wiggans and Kahn (1991) for discussions of the disadvantages of single-item Vickrey auctions in non-isolated contexts.)

We now discuss some of the potential problems with the revelation of private information in the VCG auction. To start with, bidders may not want others they deal with (suppliers, unions, regulators, customers, lenders, etc.) to know their precise costs or values. Private information provides power and profit in negotiations, and bidders may well prefer to not reveal their true costs or values even if it involves some potential loss of expected profit in a particular auction. For example, a fuel supplier with market power can negotiate a better deal with an electric generator if it knows the generator's exact costs. If the generator anticipates such a negotiation, it has an incentive to shade its bid. Thus, in a richer context, the VCG auction is not truth revealing and, therefore, not necessarily efficient.[18]

One particular aspect of this concern is particularly striking. The extra payment to each bidder is a direct and highly relevant measure of that bidder's market power. This would be valuable information for government regulation and anti-trust actions. However, the bidder will be aware of this and may, accordingly, shade its bid (not bid actual value or cost). Shading for this reason would not occur if the government pledged, in a completely believable way, to never use the information to bring an adverse action. But such a pledge is almost impossible to make, as successor administrations and congresses are not bound by their predecessors' policies.

Another concern about information revelation is fear of cheating. Bidders who reveal their costs or values are vulnerable to cheating by a bid-taker who uses this information to affect the payment due a bidder by using the information to create insincere (but losing) bids by a confederate that result in a lower payment to the winning bidders (Rothkopf and Harstad, 1995). For instance, in a VCG auction with two bids of \$100 and \$200 to supply a single item, the maker of the lower bid is entitled to a payment of \$200. However, if the bid-taker fakes or solicits an insincere bid of \$120, it can lower its payment to \$120.

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17 They show that among all possible mechanisms that guarantee efficient allocation for one-sided auctions with private values, the VCG auction maximizes the revenue received by the auctioneer. Thus, if the VCG auction is revenue deficient, it will be necessary to impose some inefficiency in order to achieve revenue sufficiency. However, the Krishna and Perry (1998) result was not demonstrated for double auctions (such as that in section 5, above), nor for auctions with non-private values.

18 However, this problem could be at least partially solved with an independent auctioneer that does not reveal all bid information. Such secrecy, however, could cloak corruption.

A third disadvantage of the VCG auction is that there is no market clearing price. This is illustrated by the gas pipeline example in section 3, in which different successful bidders pay different amounts per unit of capacity. Indeed, this can occur even if the bidders have the same valuation per unit, if the quantities they request differ. Such discrimination may raise fairness issues. Further, the lack of a market clearing price may pose practical problems by making it more difficult to settle small deviations from the agreed upon transaction. In many auctions, a small shortfall in the quantity to be delivered by or to a winning bidder is routinely handled by adjusting its payment based upon the price. Such adjustments will be important in electricity auctions, as there are significant errors in demand forecasts (generally 3–5% for day-ahead forecasts). In that case, the VCG auction makes payments to generators to cover several hours of fuel costs together with fixed costs incurred in the first hour; as a result, an unambiguous hourly price might not be identifiable.

7. Conclusions

The benefits of efficient energy system operation can be huge; even a small fraction of 1% of fuel costs can amount to many millions of dollars for a typical utility. In designing auctions for electric power and natural gas markets, it is therefore natural to desire that such auctions provide incentives for truthful bidding. We have studied a modified Vickrey-Clarke-Groves auction that could be used for this purpose, in which an auctioneer solves a mathematical program to determine the winning bids. The VCG auction, in theory, motivates profit-maximizing firms to submit bids reflecting their true supply costs consumption benefits.

However, we have also identified several practical problems that diminish the attractiveness of this auction for energy markets. These problems appear to be generic to the VCG mechanism, and would therefore also be of concern for non-energy markets. Future work should further examine these problems to determine to what extent they might be mitigated. But as these problems appear to stem directly from adjustments to payments that are made to ensure truthful revelation, it appears that costless fixes are impossible.

Perhaps the most important concern is the extra payments the VCG auctioneer must make when both demand and supply are bid and there is market power. Since in that case the VCG auction will often result in a loss for the auctioneer, how and whom could the auctioneer tax in order to make up such losses? This requires a ''second-best'' approach in which the auction cannot be subsidized (i.e., the auctioneer receives at least as much revenue from consumers as it pays to suppliers). An investigation of that question should also quantify the losses of economic surplus result from alternative tax mechanisms.[19]

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19 Relevant to this discussion is the result that there generally exists no mechanism that simultaneously achieves truthful revelation while at the same time satisfying a constraint that no party (including the auctioneer) is made worse off by participating in the auction (Mas-Colell et al. 1995). This result implies that, in general, there exists no truthful revelation mechanism that also guarantees revenue sufficiency and leaves all bidders no worse off than they would be if they withdrew from the auction (Myerson and Satterthwaite 1983).

Those losses should then be put in context: given that efficiency losses can result under the VCG auction due to taxation and possible collusion among bidders, how do they compare to efficiency losses under other auctions? This should be the subject of future research. For instance, there are several alternative auction structures under consideration for power markets. In the California Power Exchange, for example, only linear pricing is allowed (i.e., each bidder submits a single $/unit bid to be applied to all units purchased or sold), even when true cost functions are more complex. Such simplified bidding means that bidders cannot submit fully truthful bids, and inefficient system dispatch or other misallocations may result. In contrast, the UK system allows all bids to include all cost components. Quantifying losses under other auction schemes would not be simple. For instance, in general, equilibria in linear pricing auctions are mixed (probabilistic). Also, methods for calculating price equilibria (using, e.g., Cournot or supply function-based Nash equilibria) in the face of nonconvex costs have yet to be developed. Indeed, equilibrium prices may be undefinable (Johnson and Svoboda 1996).

Appendix. Simulations of Buyer-Seller Collusion

Assumptions

Power providers consist of a single large generator and a competitive fringe. The competitive fringe has a marginal cost (and thus supply) curve of:

$$MC_{CF} = a + mQ_{SCF}, \tag{A.1}$$

where MC_{CF} = the marginal cost [$/MWh], a is the price intercept, m is the slope of the marginal cost curve, and Q_{SCF} is the quantity supplied [MW] by the competitive fringe. If the large generator behaves competitively, its supply at any price would be αQ_{SCF}, implying that its marginal cost curve is:

$$MC_{LG} = a + mQ_{LG}/\alpha, \tag{A.2}$$

where MC_{LG} and Q_{LG} are the marginal cost and supply provided by the large generator, respectively. Because the large bidder may behave strategically, the marginal cost curve is not necessarily its supply curve.

On the demand side, there is also a single large player and a competitive fringe. The competitive fringe has demand curve:

$$P = P_o - (P_o/Q_o)Q_{DCF}, \tag{A.3}$$

whose integral is the total benefit/value received by those consumers. P_o is the price intercept and Q_o the quantity demanded by the fringe when price is zero. Q_{DCF} is the actual quantity demanded by fringe consumers. If the large consumer behaved competitively, its quantity demanded at any price would be βQ_{DCF}:

$$P = P_o - (P_o/Q_o)Q_{LC}/\beta, \tag{A.4}$$

where Q_{LC} is the large consumer's quantity demanded. Its total value is the curve's integral.

We vary α to simulate the effect of different sized supply bidders. The parameters a and m can varied to determine the impact of different supply elasticities. We similarly change β to represent changes in the size of the large consumer; P_o and Q_o are altered to simulate different demand elasticities. Two sets of competitive fringe demand and supply functions are considered: low demand elasticity and high demand elasticity. These are shown in table 4, along with the solution when $\alpha = \beta = 0$ (no large players). Other cases have also been considered, with broadly similar results.

Model

First, consider the large supply bidder. It submits a bid schedule with intercept B and slope m/α. Thus, the bidder can choose a price intercept different from its true intercept a, but it does not alter its slope. Raising B above a would be equivalent to the classic oligopolist strategy of restricting supply. We assume that the bidder chooses m to maximize its profit, which in the VCG process equals:

$$\begin{aligned} \textit{Supplier Profit} = {} & [SW - SW_{-LG}] + [BQ_{LG} + 0.5mQ_{LG}{}^2/\alpha] \\ & - [aQ_{LG} + 0.5mQ_{LG}{}^2/\alpha]. \end{aligned} \tag{A.5}$$

The first two bracketed terms represent the payment from the auctioneer to the large generator (equation (1)), and the last bracketed term is the generator's actual cost. The payment is in two parts: the improvement in social welfare resulting from the bidder's participation (the first bracketed term), and the integral of the bid curve (the second bracketed term). Social welfare is calculated by the auctioneer as the integral of the demand function(s) (perhaps including the bid function submitted by the large consumer) minus the integral of the supply function(s) (including the large generator's bid function).

We call the difference between (a) the auctioneer's payment and (b) the payment made if the auctioneer only pays the market clearing price the *market power payment*:

Table 4. Demand and Supply Assumptions

				Equilibrium When $\alpha = \beta = 0$		Resulting Price Elasticity of:	
a \$/MWh	m \$/(MW)^2	P_o \$/MWh	Q_o MW	P	Q	Demand	Supply
10	0.027	40	1000	22.1	449	1.2	1.6
10	0.027	120	550	22.1	449	0.2	1.6

$$\textit{Market Power Payment} = [SW - SW_{-LG}] + [BQ_{LG} + 0.5mQ_{LG}{}^2/\alpha] - PQ_{LG}, \quad \text{(A.6)}$$

where P is the market clearing price calculated by the auctioneer. In general, the market power payment is nonnegative when costs are convex.[20] It is an index of the bidder's market power, since if the bidder possessed none, the bidder would be paid the market clearing price and no more, and the market power payment would be zero.

Turning to the large demand bidder, it submits a bid schedule with intercept P_{LC} and slope $-(P_o/Q_o)/\beta$. Like the supply bidder, the demand bidder can lie about its price intercept but not about its slope. Dropping the intercept below its true value P_o corresponds to the oligopsonist strategy of withholding demand. The demand bidder picks P_{LC} to maximize its net benefits:

$$\begin{aligned}\textit{Consumer Profit} = {} & [SW - SW_{-LC}] \\ & - [P_{LC}Q_{LC} - 0.5(P_o/Q_o)Q_{LC}{}^2/\beta] \\ & + [P_oQ_{LC} - 0.5(P_o/Q_o)Q_{LC}{}^2/\beta]. \qquad \text{(A.7)}\end{aligned}$$

Analogous to (A.5), the first two bracketed terms are the payment from the auctioneer and the last is the actual value to the large consumer.

The ''payment'' by the auctioneer to the demand bidder is, of course, negative, since the bidder is buying, and paying for, power. However, what it pays is, in general, less than the market clearing price; this discount is exactly analogous to the market power payment to supply bidders, and so we also refer to it by that name:

$$\begin{aligned}\textit{Market Power Payment} = {} & PQ_{LC} - \{[SW - SW_{-LC}] \\ & - [P_{LC}Q_{LC} - 0.5(P_o/Q_o)Q_{LC}{}^2/\beta]\}. \qquad \text{(A.8)}\end{aligned}$$

Finally, consider collusion between the large supply and demand bidders. They choose P_{LC} and B, respectively, as above. If they are recognized and treated as one entity by the auctioneer, then their joint profit is:

$$\begin{aligned}\textit{Profit} = {} & [SW - SW_{-LC,-LG}] - [P_{LC}Q_{LC} - 0.5(P_o/Q_o)Q_{LC}{}^2/\beta] + [BQ_{LG} + 0.5mQ_{LG}{}^2/\alpha] \\ & + [P_oQ_{LC} - 0.5(P_o/Q_o)Q_{LC}{}^2/\beta] - [aQ_{LG} + 0.5mQ_{LG}{}^2/\alpha], \qquad \text{(A.9)}\end{aligned}$$

where SW is calculated including all bidders, and $SW_{-LC,-LG}$ is obtained by considering just the fringe demand and supply. The market power payment is:

20 The difficulty with nonconvex costs is that a market clearing price—and therefore a competitive baseline—often cannot be unambiguously defined.

$$Market\ Power\ Payment = P(Q_{LC} - Q_{LG}) - \{[SW - SW_{-LC,-LG}] - [P_{LC}Q_{LC} - 0.5(P_o/Q_o){Q_{LC}}^2/\beta] + [BQ_{LG} + 0.5m{Q_{LG}}^2/\alpha]\}. \tag{A.10}$$

This is nonnegative and, except for very small bidders, is usually positive.

On the other hand, if the auctioneer considers the large bidders as separate entities, then the large supplier's profit is given by (A.5) and the demand bidder's profit by (A.7), totaling:

$$Profit = [2SW - SW_{-LG} - SW_{-LC}] - [P_{LC}Q_{LC} - 0.5(P_o/Q_o){Q_{LC}}^2/\beta] + [BQ_{LG} + 0.5m{Q_{LG}}^2/\alpha] + [P_oQ_{LC} - 0.5(P_o/Q_o){Q_{LC}}^2/\beta] - [aQ_{LG} + 0.5m{Q_{LG}}^2/\alpha], \tag{A.11}$$

where SW_{-LG} is calculated including the large consumer but excluding the large generator, and SW_{-LC} incorporates the generator but omits the consumer. (A.9) and (A.11) differ if the first bracketed terms in each differ—that is, if the effect of simultaneously omitting both the large consumer and generator differs from the sum of the effects of omitting the large consumer and omitting the large generator separately.

The market power payment in this case is usually larger than in (A.10):

$$Market\ Power\ Payment = P(Q_{LC} - Q_{LG}) - [2SW - SW_{-LG} - SW_{-LC}] - [P_{LC}Q_{LC} - 0.5(P_o/Q_o){Q_{LC}}^2/\beta] + [BQ_{LG} + 0.5m{Q_{LG}}^2/\alpha]\}. \tag{A.12}$$

Results

Two sets of results are discussed: when there is just a large demand or supply bidder, but not both; and when both are present.

The simulations of a lone supply or demand bidder confirm the theoretical result of truthful revelation: the large supply bidder (demand bidder) should submit its true cost (true benefit) as its bid in order to maximize profit.

But a price is paid for this efficiency: a payment by the auctioneer to the bidder over and above the market clearing price. This is the market power payment (A.6, A.8). Figures 3a and 3b show its magnitude for various sizes of the large supplier and consumer. The market power payment received by the large bidder is expressed as a percentage of the price received by the competitive fringe. It is relatively unimportant if the bidder's size is 10% of the competitive fringe, amounting to 2% or less of the competitive price. This is no more than 0.2% of the total amount paid to suppliers, and under the assumptions in the appendix, this amount also equals the auctioneer's revenue shortfall. These and other simulations show that larger market power payments occur under low supply elasticity or demand elasticity. That payment, however, rises to as much as 10% of the fringe's price (5% of total auction payments) if the bidder's size rises to 50% of the competitive fringe (i.e., one-third of the entire market). As the PJM simulations of section 4 show, the market power payments can be considerably greater than that under some circumstances. Of course, the auctioneer has to get this money from somewhere, and the resulting tax will generally cause distortions and diminish welfare.

Turning now to the large supply and demand bidders case, if they bid as one entity, and are so treated by the auctioneer, then again we have truthful revelation—at the expense of extra payments by the auctioneer to the bidders (a premium for the bidder's supply, and a discount for the bidder's purchases, (A.10)). The premiums per MWh bought or sold by the joint supply-demand bidders are almost exactly those paid in the generator or demand alone cases.

However, if instead the bidders disguise their cooperation and are treated as separate entities by the auctioneer, the bidders can increase their profits by lying—truthful revelation is lost. To maximize joint profit, the supply bidder shaves its bid, while the demand bidder raises its bid. For instance, figure 4 shows that if the size of each of the large bidders is 20% of competitive fringe, the supply bidder drops its price by $1.5/MWh, while the demand bidder raises its bid by a like amount. This strategy is the opposite of the behavior resulting from

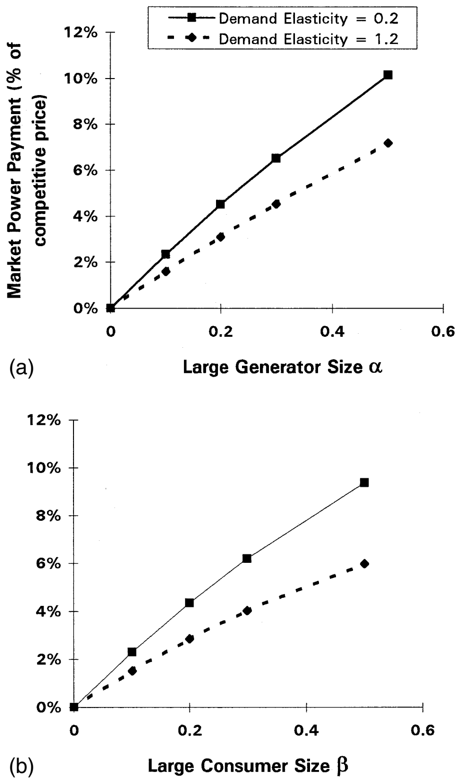

Figure 3. Market power payments. (a) Large supplier alone case. (b) Large consumer alone case.

classic oligopoly or oligopsony models—supplies and demands are expanded here, not contracted. As long as the supplier and consumer are equal sized, the amount of distortion in their bids is the same, and is directly proportional to the size of the bidders (figure 4). Larger distortions occur if demand or supply elasticity are decreased.

It is worthwhile for the bidders to produce power that costs more than the price and to buy power that is worth less than the price because of the additional payments they receive. The reason for these additional payments is that the distorted bids result in an inflated estimate by the auctioneer of the increase in SW that results from each party's participation. If each bidder is 30% of the size of the fringe competitors, the resulting market power payment (A.12) is between 25% and 50% higher than the market power payment (A.10) that would occur if the bidders were treated as one entity. This increase in the payment grows as the bidder sizes get larger—in some cases doubling (e.g., when bidders are the same size as the competitive fringe).

These distorted bids impose a loss in allocative efficiency—even ignoring the additional efficiency losses that would result if the auctioneer taxes someone to come up with the market power payments. Figure 5 quantifies these welfare losses as a percentage of the price

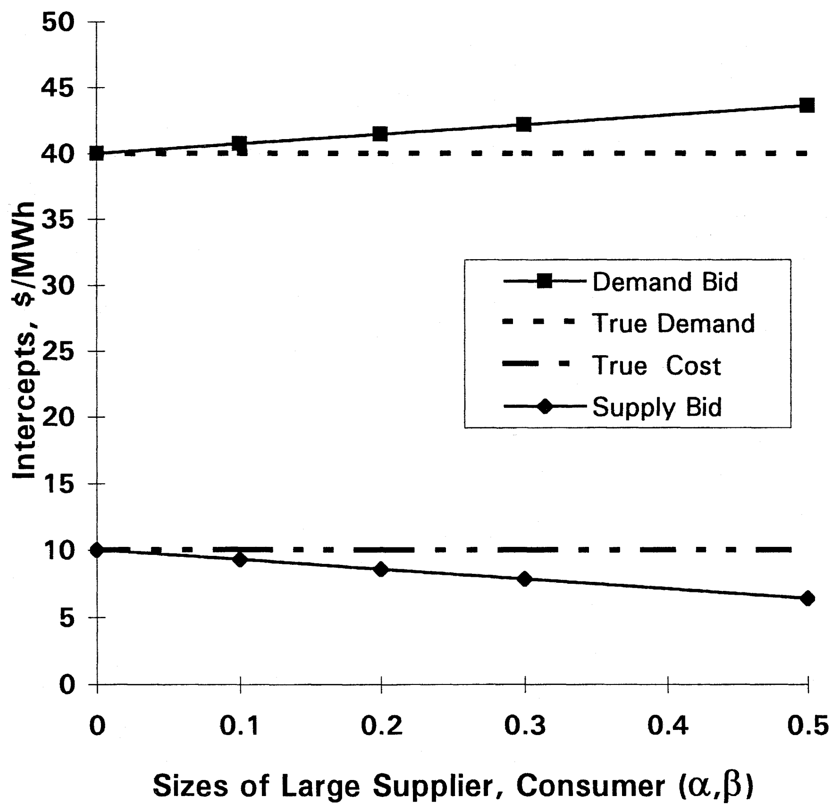

Figure 4. Bid distortions when supplier and consumer collude and coordinate separate bids.

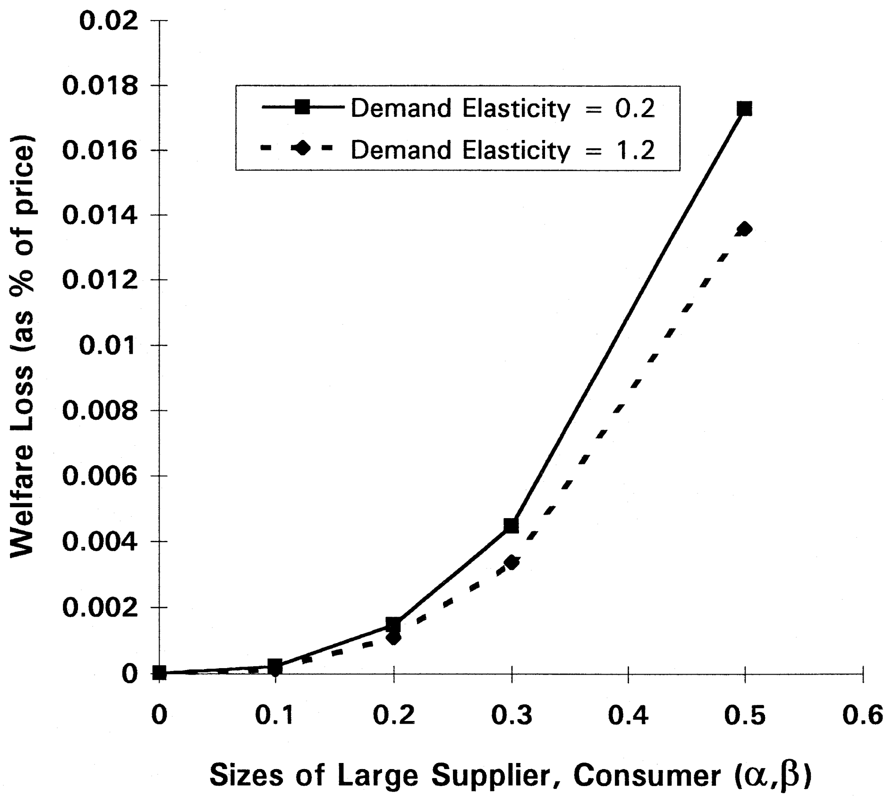

Figure 5. Loss of welfare due to bid distortions when supplier and consumer collude.

received by the competitive fringe. Sometimes these losses are negligible—for instance, if the bidders are only 10% of the size of the competitive fringe. However, these losses can grow to 2% of the price if the bidders are as large as 50% of the competitive fringe.

A crucial assumption in the above analysis is that the auctioneer can verify the power sales and purchases by each bidder. It turns out that if only the *net* power sold or purchased by a colluding buyer and seller can be monitored but the buyer and seller are treated as separate entities, then an even more extreme distortion can result. The buyer/seller will then have an incentive to greatly exaggerate the quantities that they would be willing to buy/sell at a given price, and they can increase their net surplus without limit at the expense of the auctioneer.

References

Ausubel, L. M., P. Cramton, R. P. McAfee, and J. McMillan. 1997. "Synergies in Wireless Telephony: Evidence from the Broadband PCS Auctions." *Journal of Economics and Management Strategy* 6: 497–527.

Bushnell, J. B., and S. S. Oren. 1994. "Bidder Cost Revelation in Electric Power Auctions." *Journal of Regulatory Economics* 6(1): 5–26.

Clarke, E. 1971. ''Multipart Pricing of Public Goods.'' *Public Choice* 2(1): 19–33.

Elmaghraby, W., and S. S. Oren, 1999. ''The Efficiency of Multi-unit Electricity Auctions.'' *Energy Journal* 20(4): 89–116.

Engelbrecht-Wiggans, R. 1980. ''Auctions and Bidding Models.'' *Management Science* 26(1): 119–142.

Engelbrecht-Wiggans, R., and C. M. Kahn. 1991. ''Protecting the Winner: Second-Price versus Oral Auctions.'' *Economics Letters* 35: 243–248.

Federal Energy Regulatory Commission. 1998. Technical Conference, ''In the Matter of Natural Gas Pipeline Company of America,'' Docket No. RP 97-431, May 19–20.

Graham, D., and R. Marshall. 1987. ''Collusive Bidder Behavior at Single-Object Second-Price and English Auctions,'' *Journal of Political Economy* 95: 1217–1239.

Groves, T. 1973. ''Incentives in Teams.'' *Econometrica* 41: 617–631.

Harstad, R. M., M. H. Rothkopf, and K. Wahrer. 1996. ''Efficiency in Auctions When Bidders Have Private Information About Competitors.'' In *Advances in Applied Micro-Economics*, Vol. 6, edited by M. Baye. Greenwich, CT: JAI Press.

Johnson, R. B., S. S. Oren, and A. J. Svoboda. 1997. ''Equity and Efficiency of Unit Commitment in Competitive Electricity Markets.'' *Utilities Policy* 6(1): 9–20.

Johnson, R. B., and A. J. Svoboda. 1996. ''Incentive Effects of Power Exchange Auction Proposals.'' Mimeo, Pacific Gas & Electric Co., San Francisco, CA.

Johnson, R. B., A. J. Svoboda, C. Greif, A. Vojdani, and F. Zhuang. 1998. ''Positioning for a Competitive Electric Industry with PG&E's Hydro-Thermal Optimization.'' *Interfaces* 28(1): 53–74.

Krishna, V., and M. Perry. 1998. *Efficient Mechanism Design*. Mimeo, Department of Economics, Pennsylvania State University, State College, PA.

Mackie-Mason, J. K. 1994. *A Spatial 'Smart Market' for Electric Power and Transmission*. Mimeo, Dept. of Economics, University of Michigan, Ann Arbor, MI.

Mas-Colell, A., M. Whinston, and J. R. Green. 1995. *Microeconomic Theory*. Oxford University Press, New York, Ch. 25.

McAfee, R. P., and J. McMillan. 1987. ''Auctions and Bidding.'' *Journal of Economic Literature* 25: 699–738.

McGuire, B. 1997. *Power Auctions and Intertemporal Produciton Costs: Dealing with Unit Commitment*. Mimeo, University of California Energy Institute, Berkeley, CA.

Myerson, R. B., and M. A. Satterthwaite. 1983. ''Efficient Mechanisms for Bilateral Trading.'' *Journal of Economic Theory* 29: 265–281.

Robinson, M. S. 1985. ''Collusion and the Choice of Auction.'' *Rand Journal of Economics* 16: 141–145.

Rothkopf, M. H. 1994. ''Models of Auctions and Competitive Bidding.'' In *Handbooks in Operations Research, Vol. 6: Operations Research and the Public Sector*, edited by S. M Pollack, M. H. Rothkopf and A. Barnett. New York: Elsevier Science.

Rothkopf, M. H. 1999. ''Daily Repetition: A Neglected Factor in the Analysis of Electricity Auctions.'' *The Electricity Journal* 12(3): 60–70.

Rothkopf, M. H., and R. M. Harstad. 1994. ''Modeling Competitive Bidding: A Critical Essay.'' *Management Science* 40: 364–384.

Rothkopf, M. H., and R. M. Harstad. 1995. ''Two Models of Bid-Taker Cheating in Vickrey Auctions.'' *Journal of Business* 68: 257–267.

Rothkopf, M. H., A. Pekec, and R. M. Harstad. 1998. ''Computationally Manageable Combinational Auctions.'' *Management Science* 44(8): 1131–1147.

Rothkopf, M. H., T. J. Teisberg, and E. P. Kahn. 1990. ''Why are Vickrey Auctions Rare?'' *Journal of Political Economy* 98(1): 94–109.

Vickrey, W. 1961. ''Counterspeculation, Auctions, and Competitive Sealed Tenders.'' *Journal of Finance* 16: 8–37.

Wilson, R. B. 1993. ''Strategic Analysis of Auctions.'' In *The Handbook of Game Theory*, edited by R. Auman and S. Hart. Amsterdam: North Holland.