Entanglement dynamics of bipartite system in squeezed vacuum reservoirs

Smail Bougouffa[1] **and Awatif Hindi**[2]

[1]Department of Physics, Faculty of Science, Taibah University, P.O.Box 30002, Madinah, Saudi Arabia

[2]Physics Department, College of Science, PO Box 22452, King Saud University, Riyadh - 11495, Saudi Arabia

E-mail: sbougouffa@taibahu.edu.sa
sbougouffa@hotmail.com

Abstract. Entanglement plays a crucial role in quantum information protocols, thus the dynamical behavior of entangled states is of a great importance. In this paper we suggest a useful scheme that permits a direct measure of entanglement in a two-qubit cavity system. It is realized in the cavity-QED technology utilizing atoms as flying qubits. To quantify entanglement we use the concurrence. We derive the conditions, which assure that the state remains entangled in spite of the interaction with the reservoir. The phenomenon of sudden death entanglement (ESD) in a bipartite system subjected to squeezed vacuum reservoir is examined. We show that the sudden death time of the entangled states depends on the initial preparation of the entangled state and the parameters of the squeezed vacuum reservoir.

PACS numbers: 03.67.Mn, 03.65.Yz, 03.65.Ud, 42.50.Lc

Submitted to: *Phys. Scr.*

1. Introduction

In contrast to classical theories, quantum mechanics assures the existence of nonlocal correlations between two systems spatially separated and without any direct interactions, which Schrodinger named as entanglement [1]. Entanglement is a key feature of various quantum information processes such as quantum teleportation [2], quantum dense coding [3], quantum cryptography [4] and quantum computing [5]. Due to the crucial role of entanglement in quantum information processes, the study of entanglement has attracted a lot of interest in recent years. With various studies on entanglement, the mean question which may be posed is how to know that a quantum state is entangled. For a pure bipartite state, the Schmidt decomposition [6] can be used to decide whether the state is entangled and the degree of entanglement can be quantified by the partial von Neumann entropy [7]. Hence, in principle, the problem of entanglement for pure states of a bipartite system has been completely solved. On the other hand, quantum systems predictably undergo decoherence processes and quantum systems are mostly in mixed states. For density matrix of a quantum system consisting of two subsystems, some criteria on entanglement have been established [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Moreover, the generation of entangled states has been investigated in various systems from atoms or ions, photons and quadrature-phase amplitudes of the electromagnetic field [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]. It is clear that the experimental and theoretical studies of bipartite systems have made a great growth in recent years.

Real quantum systems are necessarily subjected to their environments, and these reciprocal interactions often result in the dissipative evolution of quantum coherence and loss of useful entanglement. Decoherence may be investigated in both local and global dynamics, which may lead to the eventual deterioration of entanglement [37]. Yu and Eberly have investigated the time evolution of entanglement of a bipartite qubit system undergoing various modes of decoherence. Particularly, they found that, even when there is no interaction, there are certain states whose entanglement decays exponentially with time, while for other closely related states, the entanglement vanishes abruptly in a finite time which depends upon the initial preparation of the qubits, a phenomenon termed entanglement sudden death (ESD)[38] and was recently observed in two sophistically designed experiments with photonic qubits [39]and atomic band [40]. Furthermore, it has also been observed in cavity QED and trapped ion systems [41]. On the other hand, the phenomenon ESD has motivated many theoretical investigations in other bipartite systems involving pairs of atomic, photonic, and spin qubits [42, 43, 44, 45], multipartite systems [46, 47] and spin chains [48, 49, 50]. In addition, ESD has also been investigated for different environments [37, 38, 55, 52, 53]. However, numerous investigations on ESD in a variety of systems have been done so far, the question of ESD in interacting qubits remains open [54]. On the other hand, from the quantum technological point of view, states that show exponential decay of entanglement, and therefore maintain some trace of this all considerable correlation for an infinite time, are of importance. Although the vanishingly small entanglement present in the exponential tail will be of limited practical importance, however it is of interest to identify exactly in what situations ESD will occur [55, 56].

In the past few years, numerous methods by which the entanglement of quantum systems can be detected and described have been suggested. Possibly the mainly influence to date has been the simple procedure derived by Wooters [57] for measuring entanglement for an arbitrary mixed state of pair two-level systems. Furthermore, for two qubits, concurrence [57] offers a convenient measure of the entanglement of formation. This has provided a very useful tool for measurement of experimental quantum states and is to day commonly used in evaluating the abilities of emerging quantum information technologies.

The purpose of this paper is to propose an efficient scheme for quantum teleportation to generate entangled number states of bipartite system under the influence of squeezed vacuum reservoir. Thus we investigate the time evolution of these entangled states. We examine the problem of ESD for this proposed scheme for different initial entangled state and the parameters of the squeezed vacuum reservoir.

2. Bipartite model system

Recently, Zubairy et all [58] have suggested a new scheme in their examination of the quantum disentanglement eraser. In this simple scheme, the concurrence can be directly measured from the visibility for an explicit class of entangled states. We propose here the same scheme but with some adjustment. A two-level atom with the upper level $|e\rangle$ and the lower level $|g\rangle$ passes consecutively through cavity A, a squeezed vacuum reservoir and a cavity B as shown in figure 1. The incident atom is initially prepared in the excited state $|e\rangle$ and the decay of the radiation field inside a cavity may be described by a model in which the mode of the field of interest is coupled to a whole set of reservoir modes. We assume that initially the two cavities are in vacuum state $|0\rangle$ and the atom always leaves the setup in the ground state $|g\rangle$.

In the interaction picture and the rotating-wave approximation, the Hamiltonian is simply

$$H(t) = \hbar \sum_{j=A,B} \sum_{\mathbf{k}} \left[g_{\mathbf{k}}^{(j)} b_{\mathbf{k}}^{(j)\dagger} a_j e^{-i(\nu-\nu_{\mathbf{k}})t} + g_{\mathbf{k}}^{(j)*} a_j^{\dagger} b_{\mathbf{k}}^{(j)} e^{i(\nu-\nu_{\mathbf{k}})t} \right] \qquad (1)$$

where $a_j (j = A, B)$ and $a_j^{\dagger}$ are the destruction and creation operators of the mode of the electromagnetic field of frequency ν. $b_{\mathbf{k}}^{j}$ and $b_{\mathbf{k}}^{\dagger j}$ are the modes of cavity j of frequency $\nu_{\mathbf{k}}$ which damp the field and $g_{\mathbf{k}}^{(j)}$ is the coupling constant of the interaction between the electromagnetic field and the cavity.

3. Entanglement dynamics in squeezed reservoirs

Here we are concerned with the case in which cavity fields are exposed in broadband squeezed vacuum reservoirs. From the general analysis of system-reservoir interactions, when the modes $b_{\mathbf{k}}^{j}$ are initially in a squeezed vacuum, with the Hamiltonian (1) and the squeezing bandwidths of the squeezed reservoirs are much larger than the atomic line-widths, we can get directly the master equation for the reduced density matrix for the field in the cavities as [59]

$$\begin{aligned}\dot{\rho}(t) = \sum_{j=A,B} \Big[&- \frac{\kappa^{(j)}}{2}(N_j+1)\left(a_j^\dagger a_j \rho(t) - 2a_j\rho(t)a_j^\dagger + \rho(t)a_j^\dagger a_j\right) \\ &- \frac{\kappa^{(j)}}{2}N_j\left(a_j a_j^\dagger \rho(t) - 2a_j^\dagger \rho(t)a_j + \rho(t)a_j a_j^\dagger\right) \\ &+ \frac{\kappa^{(j)}}{2}M_j\left(a_j a_j \rho(t) - 2a_j\rho(t)a_j + \rho(t)a_j a_j\right) \\ &+ \frac{\kappa^{(j)}}{2}M_j^*\left(a_j^\dagger a_j^\dagger \rho(t) - 2a_j^\dagger \rho(t)a_j^\dagger + \rho(t)a_j^\dagger a_j^\dagger\right)\Big] \end{aligned} \tag{2}$$

where $\kappa^{(j)}(j = A, B)$ is the decay rate in the cavity, $N_j = sinh^2(r_j)$ and $M_j = cosh(r_j)sinh(r_j)exp(-i\theta_j)$, with r_j being the squeeze parameter and θ_j being the reference phase for the squeezed fields which surrounds the cavities A and B. If $N_j = M_j = 0$, the remaining terms are due to vacuum fluctuations.

To investigate the effect of interaction among the bipartite on decoherence we have to investigate the dynamics of bipartite entanglement. Furthermore, the concept of concurrence initiates from the original work of Hill and Wootters [57] where the closed expression of the entanglement of formation of a system of two qubits was derived. They established that the entanglement of formation is a convex monotonic increasing function of the concurrence. Here we use concurrence, to illustrate the degree of entanglement for any bipartite system. This measure satisfies necessary and sufficient condition for being good measure of entanglement for 2X2 system. The concurrence varies from $\mathcal{C} = 0$ for a separable state to $\mathcal{C} = 1$ for a maximally entangled state. The explicit expression for concurrence can be written as

$$C(t) = max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}) \tag{3}$$

where λ's are the eigenvalues of the non-hermitian matrix $\rho(t)\widetilde{\rho}(t)$ arranged in decreasing order of the magnitude. The matrix $\rho(t)$ is the density matrix for the bipartite and the matrix $\widetilde{\rho}(t)$ is given by

$$\widetilde{\rho}(t) = (\sigma_y^A \otimes \sigma_y^B)\rho^*(t)(\sigma_y^A \otimes \sigma_y^B) \tag{4}$$

where $\rho(t)^*$ is the complex conjugation of $\rho(t)$ and σ_y is the Pauli matrix given in quantum mechanics. In the general case, we consider the field states in Fock basis in two identical high-Q cavities A and B that represent a bipartite system surrounding the entangled field as

$$|\Psi\rangle_{AB}(0) = \alpha_1|0_A0_B\rangle + \alpha_2|0_A1_B\rangle + \alpha_3|1_A0_B\rangle + \alpha_4|1_A1_B\rangle \tag{5}$$

where $\alpha_i(i = 1, 2, 3, 4)$ are the probability amplitudes with $\sum_{i=1}^{4} |\alpha_i|^2 = 1$. We use the basis ($|1\rangle = |0_A0_B\rangle, |2\rangle = |0_A1_B\rangle, |3\rangle = |1_A0_B\rangle, |4\rangle = |1_A1_B\rangle$) to define the density matrix of the two qubit system. The equations of motion in terms density matrix elements can be obtained using the master equation 2.

4. Results and conclusion

Here we will consider some interesting initial entangled states for the bipartite which can be prepared and have potential applications in the quantum information processing tasks [56]. We will begin by the examination of the EPR-states which are perceptions in quantum information science, a vital part of quantum teleportation and characterize the simplest possible examples of entanglement.

(i) Assume that the initially entangled state of the field in two cavities to be in a NOON state given by

$$|\Psi\rangle_{AB}(0) = \alpha|0_A 1_B\rangle + \sqrt{1-\alpha^2}|1_A 0_B\rangle \tag{6}$$

This kind of state can be generated as it is mentioned in [56] and having its potential application in Heisenberg-limited metrology and quantum lithography [60]. The solutions of the master equation for this initial NOON state case are given in the Appendix A.

(ii) Consider now the initially entangled bipartite to be in a another EPR-state given by

$$|\Psi\rangle_{AB}(0) = \alpha|0_A 0_B\rangle + \sqrt{1-\alpha^2}|1_A 1_B\rangle \tag{7}$$

This kind of state can be prepared as we have mentioned in [56]. States like these have been realized in experiments with trapped ions [61].

The solution of the (Eq. 2) depends on the initial state of the two bipartite system. We can show that, for these two classes of the initial states that were be considered above, the solution of (Eq.2) has the matrix shape in the representation spanned by the two-bipartite states

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix} \tag{8}$$

With this form of the density matrix, we can show that the concurrence can be expressed as

$$C(t) = max\left(0, \widetilde{C}_1(t), \widetilde{C}_2(t)\right) \tag{9}$$

where

$$\widetilde{C}_1(t) = 2\left[\sqrt{\rho_{23}(t)\rho_{32}} - \sqrt{\rho_{11}(t)\rho_{44}(t)}\right] \tag{10}$$

$$\widetilde{C}_2(t) = 2\left[\sqrt{\rho_{14}(t)\rho_{41}} - \sqrt{\rho_{22}(t)\rho_{33}(t)}\right] \tag{11}$$

Using this formalism we can investigate the dynamics of entanglement for the two initial states that considered above. However, in the case of squeezed reservoirs, we find that the entanglement sudden death always happens for the two initial entangled states with

$0 < \alpha < 1$. This is shown clearly in numerical results plotted in Fig. 2. In Fig. 3, the time evolution of the concurrence is plotted for different values of the degree of squeezing. We note that, the sudden-death time of entanglement becomes smaller as the degree of squeezed increases. In conclusion, we investigate that, for bipartite entangled states, the entanglement measured by concurrence abruptly disappears during the dynamic evolution in the squeezed vacuum reservoir while for the same class of entangled states, the entanglement decays exponentially for vacuum reservoirs [56]. The results can be extended to the high dimensional bipartite filed states inside the cavities in squeezed vacuum environments(more than one photon in each cavity)where the concurrence can not be used and we have recourse to another measure of entanglement [62], namely, the logarithmic negativity. The results are in Progress and can be reported elsewhere.

Acknowledgments

The research of SB is supported by a grant no (652/431) from the Deanship of Scientific Research, Taibah University.

References

[1] Schrödinger E 1935 *Naturwissenschaften* **23** 807
[2] Bennett C H, BrassardG, Crepeau C, Jozsa R, Peres A, and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895.
[3] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881.
[4] Ekert A K 1991 *Phys. Rev. Lett.* **67** 6961.
[5] Barenco A, Deutsch D, Ekert A, and Jozsa R 1995 *Phys. Rev. Lett.* **74** 4083
[6] Schmidt E 1906 *Math. Annalen* **63** 433.
[7] Bennett C H , Bernstein H J, Popescu S, and Schumacher B 1996 *Phys. Rev. A* **53** 2046.
[8] Peres A 1996 *Phys. Rev. Lett.* **77** 1413.
[9] Horodecki M, Horodecki P, and Horodecki R 1996 *Phys. Lett. A* **223** 1.
[10] Horodecki M, Horodecki P 1996 *Phys. Rev. A* **59** 4206.
[11] Nielsen N and Kempe K 2001 *Phys. Rev. Lett.* **86** 5184.
[12] Terhal B 2000 *Phys. Lett. A* **271** 319.
[13] Simon R 2000 *Phys. Rev. Lett.* **84** 2726 .
[14] Duan L M, Giedke G, Cirac J I, and Zoller P 2000 *Phys. Rev. Lett.* **84**, 2722.
[15] Giedke G, Kraus B, Lewenstein M, and Cirac J I 2001 *Phys. Rev. Lett.* **87** 167904.
[16] Hillery M and Zubairy M S 2006 *Phys. Rev. Lett.* **96**, 050503; 2006 *Phys. Rev. A* **74**, 032333.
[17] Shchukin E and Vogel W 2005 *Phys. Rev. Lett.* **95**, 230502.
[18] Hagley E, Maitre X, Nogues G, Wunderlich C, BruneM, Raimond J M, and Haroche S 1997 *Phys. Rev. Lett.* **79**, 1 .
[19] Turchette Q A, Wood C S, King B E, Myatt C J, Leibfried D, Itano W M , Monroe C, and Wineland D J 1998 *Phys. Rev. Lett.* **81** 3631 .
[20] Molmer K and Sorensen A 1999 *Phys. Rev. Lett.* **82** 1835 .
[21] Sackett C A , Kielpinski D , King B E , Langer C , Meyer V , Myatt C J , Rowe M , Turchette Q A , Itano W M , Wineland D J , and Monroe C 2000 *Nature* **404**, 256
[22] Duan L M, Cirac J I, Zoller P , and Polzik E S 2000 *Phys. Rev. Lett.* **85** 5643.
[23] Julsgaard B, Kozhekln A, and Polzlk E S 2001 *Nature* **413**, 400.
[24] Rauschenbeutel A, P. Bertet P, S. Osnaghi S, G. Nogues G, Brune M , Raimond J M, and Haroche S 2001 *Phys. Rev. A* **64**, 050301.
[25] Freedman S J and J. F. Clauser J F 1972 *Phys. Rev. Lett.* **28**, 938.

[26] Clauser J F 1975 *Phys. Rev. Lett.* **36**, 1223.
[27] Fry E S and Thompson R C 1976 *Phys. Rev. Lett.* **37** 465 .
[28] Ou Z Y and Mandel L 1988 *Phys. Rev. Lett.* **61**, 50.
[29] Kwiat P G , Mattle K , Weinfurter H , and Zeilinger A 1995 *Phys. Rev. Lett.* **75** 4337.
[30] Bouwmeester D, Pan J -W, Daniell M, Weinfurter H, and Zeilinger A 1998 *Phys. Rev. Lett.* **82**, 1345.
[31] Pan J -W , M. Daniell M, Gasparoni S, Weihs G, and Zeilinger A 2001, *Phys. Rev. Lett.* **86**, 4435.
[32] Zhao Z, Chen Y -A, Zhang A -N, Yang T, Briegel H J, and Pan J -W 2004 *Nature* **430**, 54.
[33] Tsujino K, Hofmann H F, Takeuchi S, and Sasaki K 2004 *Phys. Rev. Lett.* **92**, 153602.
[34] Eisenberg H S, Khoury G, Durkin G A, Simon C, and Bouwmeester D 2004 *Phys. Rev. Lett.* **93**, 193901.
[35] Braunstein S L and van Loock P 2004 quant-ph/0410100
[36] Zhang Y, Wang H, Li X Y, Jing J T, Xie C D , and Peng K C 2000 *Phys. Rev. A* **62** 023813.
[37] Ikram M, Li F L and Zubairy M S 2007 *Phys. Rev. A* **75** 062336.
[38] Eberly J H and Yu T 2007 *Science* **316** 555.
[39] Almeida M P, de Melo F, Hor-Meyll M, Salles A, Walborn S P, Souto Ribeiro P H ,L. Davidovich L 2007 *Science* **316** 579.
[40] Laurat J, Choi K S, Deng H, Chou C W, and Kimble H 2007 *Phys. Rev. Lett.* **99** 180504.
[41] Frana Santos M, Milman P, DavidovichL and Zagury N 2006 *Phys. Rev. A* **73** 040305(R).
[42] Marek P, Lee J and Kim M S 2008 *Phys. Rev. A* **77** 032302.
[43] Gong Y-X ,Zhang Y S, Dong Y L, Niu X L, Huang Y F and Guo G C 2008 *Phys. Rev. A* **78** 042103.
[44] Tolkunov D, Privman V and Aravind P K 2005 *Phys. Rev. A* **71** 060308(R).
[45] Chou C H, Yu T and Hu B L 2008 *Phys. Rev. E* **77** 011112.
[46] Lopez C E, Romero G, Lastra F, Solano E and Retamal J C 2008 *Phys. Rev. Lett.* **101** 080503.
[47] Solomon A 2008 *Eur. Phys. J. Special Topics* **160** 391-397.
[48] Cormick C and Paz J P 2008 *Phys. Rev. A* **78** 012357.
[49] Lai C Y, Hung J T,C. Mou C Y and Chen P 2008 *Phys. Rev. B* **77** 205419.
[50] Abliz A, Gao H J, Xie X C, Wu Y S and Liu W M 2006 *Phys. Rev. A* **74** 052105.
[51] Al-Qasimi A and James D F V 2008 *Phys. Rev. A* **77** 012117.
[52] Ficek Z and Tanas R 2006 *Phys. Rev. A* **74** 024304.
[53] Ficek Z , and Tanas R 2008 *Phys. Rev. A* **77** 054301.
[54] Das S and Agarwal G S 2009 quant-ph/0905.3399 v1.
[55] Al-Qasimi A and James D F V 2008, *Phys. Rev. A* **77** 012117.
[56] Bougouffa S 2010, *Opt. Commun* **283** 2989.
[57] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245.
[58] Zubairy M S , Agarwal G S and Scully M O 2004 *Phys. Rev. A* **70** 012316.
[59] Scully M O and Zubairy M S 1997 *Quantum Optics*, Cambridge University Press.
[60] Bouwmeester D 2004 *Nature* **429**734.
[61] Leibfried D et al 2003 *Nature***422** 412.
[62] Tahira R, Ikram M, Bougouffa S and Zubairy M S 2010 *J. Phys. B: At. Mol. Opt. Phys.* **43** 035502.

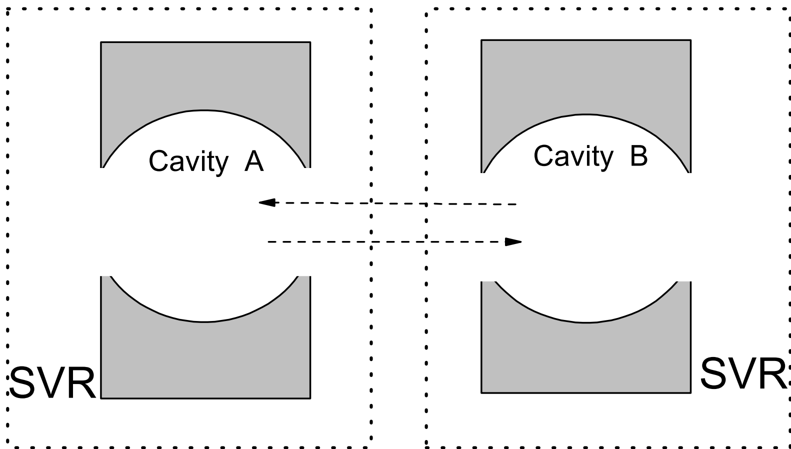

Figure 1. Two independent systems of identical cavities containing initial entangled fields. The entangled fields do not have directional interaction with each other but independently interact with their local environment in each cavity

Appendix A. Solutions of equations of motion of the density matrix elements for squeezed vacuum reservoirs

The equation of motion of density matrix elements for the state (Eq. 6) can be obtained and for the sake of simplicity, we assume that the cavities are identical $\kappa^{(A)} = \kappa^{(B)} = \kappa$, $N_A = N_B = N$ and $M_A = M_B = M$. On solving these equations of motion we get the time evolution of the density elements matrix

$$\rho_{11}(t) = -\frac{a+3}{8b^2}\left[1+a+2\sinh(b\kappa t) - \frac{1+a}{b}\cosh(b\kappa t)\right]e^{-a\kappa t}$$
$$\rho_{22}(t) = -\frac{1}{16}\left[(16\alpha^2+\frac{4}{b^2}-4)+(1-\frac{1}{b^2})\cosh(b\kappa t)\right]e^{-a\kappa t}$$
$$\rho_{33}(t) = \frac{1}{16}\left[(-16\alpha^2+\frac{4}{b^2}+12)+(1-\frac{1}{b^2})\cosh(b\kappa t)\right]e^{-a\kappa t}$$
$$\rho_{44}(t) = \frac{1}{8b^2}\left[a^2-1+\frac{1}{4}(a-1)\Big(2b\sinh(b\kappa t)-(a+1)\cosh(b\kappa t)\Big)\right]e^{-a\kappa t}$$
$$\rho_{14}(t) = -\frac{M}{|M|}\alpha\sqrt{1-\alpha^2}\sinh(|M|\kappa t)e^{-a\kappa t}$$
$$\rho_{32}(t) = \alpha\sqrt{1-\alpha^2}\cosh(|M|\kappa t)e^{-a\kappa t}$$

and $\rho_{21}(t) = \rho_{12}^*(t) = 0, \rho_{31}(t) = \rho_{13}^*(t) = 0, \rho_{32}(t) = \rho_{23}^*(t), \rho_{41}(t) = \rho_{14}^*(t), \rho_{42}(t) = \rho_{24}^*(t) = 0, \rho_{43}(t) = \rho_{34}^*(t) = 0$, where $a = 4N+1$ and $b^2 = 8N^2+8N+1$.

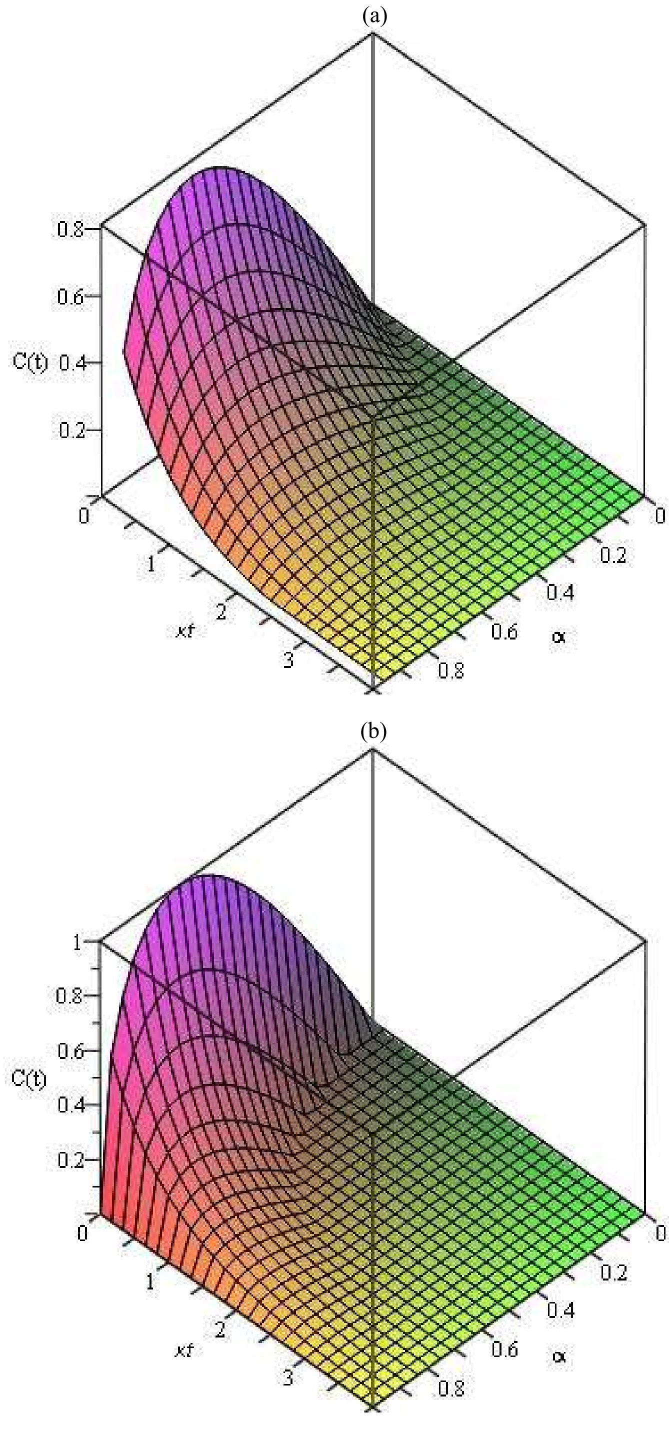

Figure 2. (Color online)Entanglement dynamics of the two initial states of the bipartite system in squeezed vacuum reservoirs for $r = 0.2$. (a) For the First initial NOON state. (b) For the second EPR initial state.

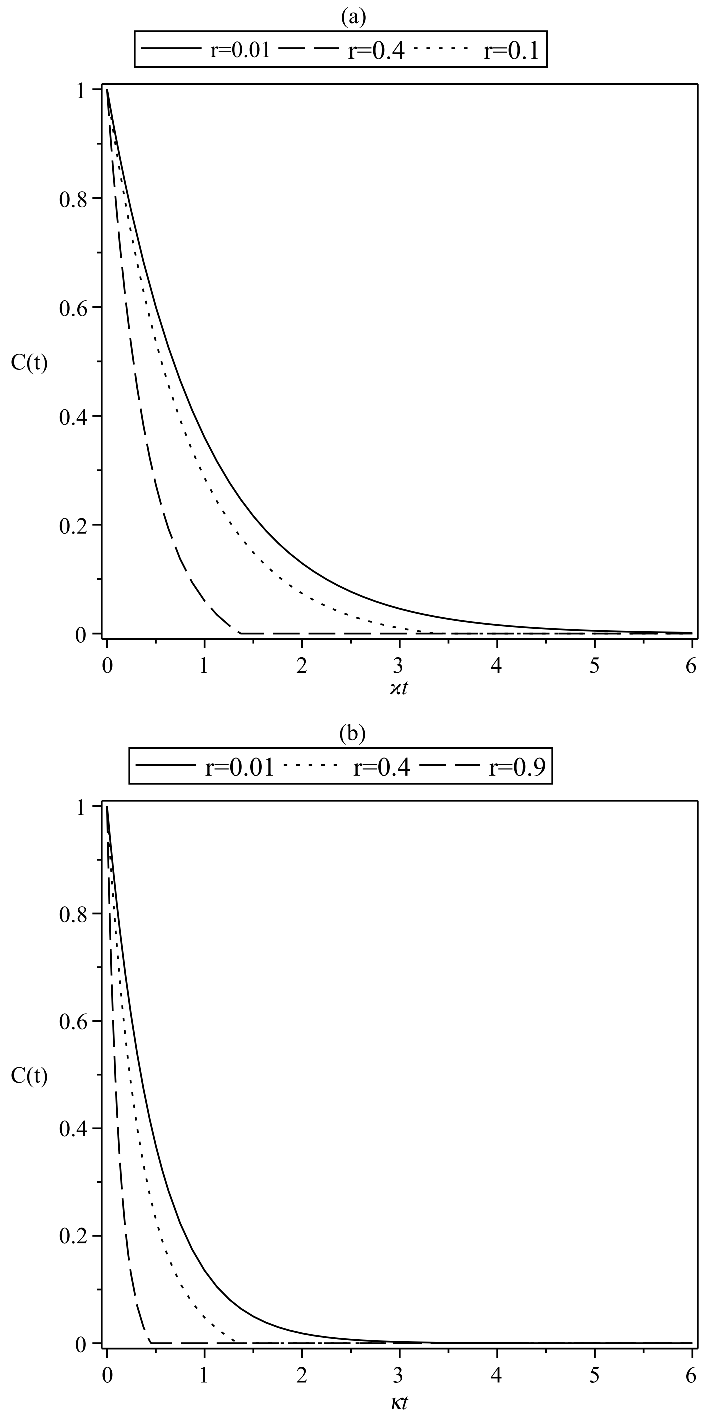

Figure 3. Entanglement dynamics of the two initial states of the bipartite system in squeezed vacuum reservoirs for different values of the degree of squeezing and $\alpha = \frac{1}{\sqrt{2}}$. (a) For the First initial NOON state. (b) For the second EPR initial state.