# Quantum computation over the butterfly network

Yoshiyuki Kinjo,[1] Mio Murao,[1,2] Akihito Soeda,[1] and Peter S. Turner[1]

[1] *Department of Physics, Graduate School of Science, the University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo, Japan*

[2] *Institute for Nano Quantum Information Electronics, the University of Tokyo, 4-6-1, Komaba, Meguro-ku, Tokyo, Japan*

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In order to investigate distributed quantum computation under restricted network resources, we introduce a quantum computation task over the butterfly network where both quantum and classical communications are limited. We consider performing a two qubit global unitary operation on two unknown inputs given at different nodes, with outputs at two distinct nodes. By using a particular resource scenario introduced by Hayashi [1], which is capable of performing a swap operation by adding two maximally entangled qubits (ebits) between the two input nodes, we show that any controlled unitary operation can be performed without adding any entanglement resource. We also construct protocols for performing controlled traceless unitary operations with a 1-ebit resource and for performing global Clifford operations with a 2-ebit resource.

Keywords: butterfly network, distributed quantum computation, controlled unitary

## I. INTRODUCTION

Distributed quantum computation aims to perform a large scale quantum computation using a collection of smaller scale quantum computers connected by communication channels. There are several distributed quantum computation architectures proposed for different purposes [2]. In general, distributed computation can be modelled by a combination of computation at each node and communication between the nodes, for both quantum and classical cases. For distributed quantum computation, initially shared entanglement among the nodes can be used as a resource, as well as quantum and classical communication channels. The amount of communication between the nodes required to perform quantum computation tasks has been analyzed by quantum communication complexity theory [3].

As the 'distributedness' of a quantum computation increases, the scale (*i.e.* the number of qubits) of each quantum computer at a node decreases and the number of nodes increases. The communication resources (quantum and classical channels and shared entanglement) form an increasingly large network and the amount of communication required grows. In any such large network, one will inevitably be faced with a bottleneck problem, where communication capacities in some region are lower than that required by a straightforward implementation of the protocol. This bottleneck restricts the total performance of communication. In network information theory, this problem has been extensively studied for the last decade or so under the name network source coding [4]. Although solving general network problems are difficult, a solution of the 2-pair communication (communications of two disjoint sender-receiver pairs) bottleneck problem is known for a simple directed network called the *butterfly network* [5] in the classical case.

In the quantum case, where the no-cloning theorem holds, the method used in the classical case cannot be applied directly, since it involves cloning inputs. Nevertheless, in [6], it is shown that efficient network source coding on the quantum butterfly network, where edges represent 1-qubit quantum channels, is possible for transmitting approximated states. Asymptotic rates of high fidelity quantum communication have been obtained for various networks including the butterfly network with and without additional entanglement [7]. In [1], it is shown that perfect quantum 2-pair communication over the butterfly network is possible if we add two maximally entangled qubits (ebits) between the inputs and allow each channel (edge) to use either 1 qubit of communication or 2 (classical) bits of communication. Recently it has been shown that if we allow free classical communication between all nodes, perfect 2-pair communication over the butterfly network is possible without additional resources [8].

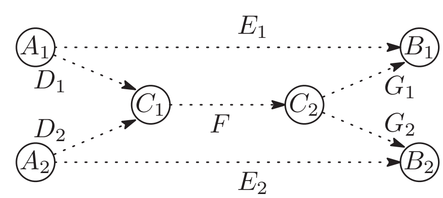


FIG. 1. The (horizontally placed) butterfly network. The 2-pair communication problem aims to transmit information (bit or qubits) from $A_1$ to $B_2$ and from $A_2$ to $B_1$ concurrently via nodes $C_1$ and $C_2$. The directed edges $D_1$, $D_2$, $E_1$, $E_2$, $F$, $G_1$ and $G_2$ denotes communication channels. The channel $F$ exhibits the bottleneck.

In this paper, we investigate the performance of efficient distributed quantum computation over such bottlenecked networks where both quantum and classical communication is restricted. We combine both quantum computation, namely, performing a gate operation on inputs, and network communication, namely, sending outputs, in a *single task*. The task we consider is to deterministically implement a global unitary operation on two inputs at distant nodes and obtain two outputs at distinct nodes connected by the particular butterfly network introduced by Hayashi [1]. We show that any controlled unitary operation can be performed over the butterfly network without adding an entanglement resource. We also present constructions of protocols for performing controlled traceless unitary operations with a 1-ebit resource and for performing global Clifford operations with a 2-ebit resource.

Our construction shows that by taking an appropriate coding, we can perform global unitary operations on spatially separated inputs and distribute the outputs at the same time, even when restricted to a network where the quantum channel connecting inputs and outputs is both directed and bottlenecked. Depending on the cost of resources in a given physical realization of the network, the way of coding varies. In addition, by studying the implementation of Clifford operations on the butterfly network, we also see the different characteristics of quantum and classical information, where the latter can be 'compressed' and sent through the bottleneck whereas the former cannot. In the bigger picture, results like these are the first step in a theory of *network quantum resource inequalities*, which formalizes such tradeoffs, like standard resource inequalities do [9], in the much more complicated network scenario.

The rest of the paper is organized as follows. In Section II, we introduce our task of implementing a global unitary operation over a network, and review Hayashi's protocol [1] in the context of implementing a swap operation. We show the protocols for implementing controlled unitary operations in Section III, controlled traceless operations in Section IV, and Clifford operations in Section V. In Section VI, a summary and discussions are presented.

## II. IMPLEMENTATION OF A SWAP OPERATION

In this section, we introduce our task of quantum computation over a network, and review Hayashi's protocol [1] for 2-pair communication in the context of this task, namely, implementation of a swap operation over the butterfly network.

We consider qubit Hilbert spaces and denote the computational basis of a qubit as $\{|0\rangle, |1\rangle\}$. We say that a two qubit unitary operation $U$ is implementable over a network, if we can obtain a joint output state $U|\psi_1\rangle \otimes |\psi_2\rangle$ of qubits at the node $B_1$ and $B_2$, for any input qubit $|\psi_1\rangle$ and $|\psi_2\rangle$ given at the node $A_1$ and $A_2$, respectively, by performing general operations including measurements at each node and communicating qubit and bit information through channels specified by edges. Trivially, if the unitary operation is a tensor product of the local unitary operations, it is implementable over any network.

In Hayashi's protocol [1] for 2-pair communication, a special butterfly network described by the nodes $A_1$, $A_2$, $B_1$, $B_2$, $C_1$ and $C_2$, and edges $D_1$, $D_2$, $E_1$, $E_2$, $F$, $G_1$ and $G_2$ shown in Fig. 1. An additional entanglement resource of 2 ebits is shared between the node $A_1$ and $A_2$. The defining characteristic of the butterfly network in Hayashi's protocol is that each edge can be chosen to be a single use, one way channel with either one qubit quantum capacity or two bit classical capacity. Although a quantum channel of single-qubit capacity can send a single-bit of classical information, it cannot faithfully send two bits of information. On the other hand, a classical channel cannot faithfully send single qubit information either. Thus, the single-qubit quantum and 2-bit classical channels are mutually inequivalent resources. Note that superdense coding [10] implies that a single qubit quantum channel and shared 1-ebit entanglement together have the capacity of 2-bit classical channel, and teleportation shows 2-bit classical channel and shared 1-ebit entanglement together have the capacity of a single qubit quantum channel, however here those ebit resources are not available.

The 2-pair communication can be regarded as performing a distributed swap operation over the butterfly network, where two arbitrary quantum inputs $|\psi_1\rangle$ and $|\psi_2\rangle$ at $A_1$ and $A_2$, respectively, are transferred to nodes $B_2$ and $B_1$, respectively. We can write this as a distributed computation $U_{swap}|\psi_1\rangle \otimes |\psi_2\rangle = |\psi_2\rangle \otimes |\psi_1\rangle$, where the tensor product on the LHS is that between nodes $A_1$ and $A_2$ and on the RHS between $B_1$ and $B_2$, in that order. Let us denote the input qubits at the node $A_1$ and $A_2$ by $S_1$ and $S_2$, respectively. The qubits of the shared ebits at the node $A_1$ are denoted $H_{1,1}$ and $H_{1,2}$, while those at the node $A_2$ are $H_{2,1}$ and $H_{2,2}$. The qubits $H_{1,i}$ and $H_{2,i}$ for $i = 1, 2$ are both in the maximally entangled state $|\Phi^+\rangle = (|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)/\sqrt{2}$. For this protocol, channels $E_1$ and $E_2$ are one qubit quantum channels, while all others are two bit classical channels.

The protocol is as follows:

1. At the node $A_1$, perform a Bell measurement on input qubit $S_1$ and $H_{1,1}$ while at the node $A_2$, perform a Bell measurement on the other input qubit $S_2$ and $H_{2,2}$. Let $i, j$ be the two bits of classical information given by the measurement result at $A_1$ and $k, l$ as that at $A_2$. Now $X^i Z^j$ and $X^k Z^l$ correspond to the combination of Pauli $X$ and $Z$ corrections for quantum teleportation [11] associated with each measurement.

2. At $A_1$, apply $X^i Z^j$ to $H_{1,2}$ while at $A_2$, apply $X^k Z^l$ to $H_{2,1}$.

3. Send qubit $H_{1,2}$ from $A_1$ to $B_1$ through the quantum side channel $E_1$ and qubit $H_{2,1}$ from $A_2$ to $B_2$ through the quantum side channel $E_2$. Send $i, j$ from $A_1$ to $C_1$ and $k, l$ from $A_2$ to $C_1$ via the two bit classical channels $D_1$ and $D_2$ respectively.

4. At $C_1$, compute $i + k, j + l \pmod 2$. Then send $i + k, j + l$ to the node $C_2$ via the two bit classical channel $F$.

5. Distribute $i+k, j+l$ from $C_2$ to $B_1$ and $B_2$ via the two bit classical channels $G_1$ and $G_2$, respectively.

6. At the node $B_1$, apply the Pauli corrections $X^{i+k}Z^{j+l}$ on the qubit received from $A_1$, and at $B_2$ apply the same operation on the qubit received from $A_2$.

This protocol can be presented by the quantum circuit and the butterfly network shown in following FIG.2.

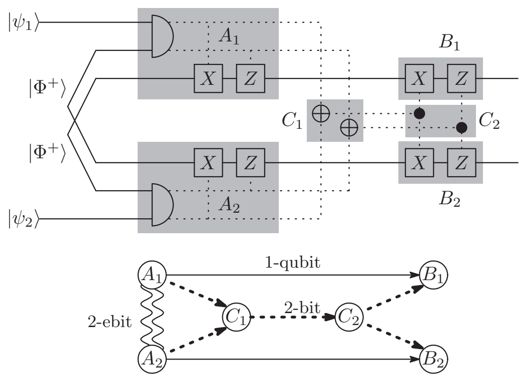


FIG. 2. *The upper figure:* The quantum circuit for implementing a swap operation on the first qubit and the sixth qubit. Each shaded block indicates operations at a node. $H$ denotes a Hadamard operation, and detectors denote Bell measurements in the computational basis. The dotted line represents a controlled operation depending on the measurement outcome. *The lower figure:* The (horizontal) butterfly network corresponding to the quantum circuit above, showing the amount of communication required in the protocol. The solid line denotes a single qubit channel, and the thin dotted line a single bit channel.

In [1], it has been shown that this protocol is optimal even for asymptotic cases, and that two ebits of entanglement are necessary and sufficient for implementing the swap operation, (i.e. a 2-pair communication), in this butterfly network scenario using information theoretical arguments. The swap operation is significant since it is the most 'global' operation in terms of entangling power [12] and delocalization power [13], compared to controlled unitary operations. Our work is motivated by the question of whether or not we can reduce the resource requirement by weakening the entangling/delocalization power of the network-implemented unitary operations.

## III. IMPLEMENTATION OF CONTROLLED UNITARY OPERATIONS

We consider the deterministic implementation of controlled unitary operations over the butterfly network in the setting of Hayashi's protocol, where we can choose a single qubit quantum channel or a 2-bit classical channel for each edge of the network. We denote a controlled unitary operation by $C_U = |0\rangle\langle 0| \otimes \mathbb{I} + |1\rangle\langle 1| \otimes U$ where $U$ is a single qubit unitary operation. The controlled unitary operations have at most half of the entangling power of the swap operation, (which is 2-ebits), and accordingly they require only half of the resource ebits in entanglement assisted LOCC, (where, similarly, swap requires 2-ebits). Considering this comparison, it is natural to expect controlled unitary operations to require 1 ebit of entanglement shared between the two input nodes in order to be implemented over the butterfly network. However, we discover a protocol implementing any controlled unitary operation over the butterfly network *without* using any entanglement resource.

This protocol is based on the implementation of a controlled phase operation $C_{U_\theta}$, where a single qubit phase operation $U_\theta$ is given by $U_\theta = |0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|$ using the quantum circuit shown in the upper figure of Fig. 3. In order to perform $C_{U_\theta}$ over the butterfly network, operations shown in each shaded block are performed at each node in the upper figure of of Fig. 3, and quantum/classical information is transmitted between the nodes using the quantum/classical communication specified by the edges shown in the lower figure of Fig. 3.

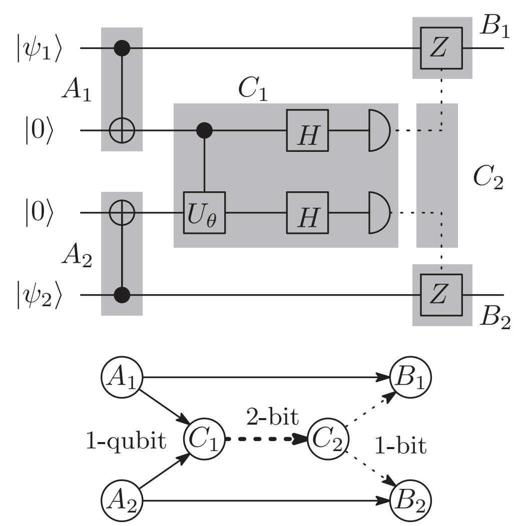


FIG. 3. *The upper figure:* The quantum circuit for implementing a controlled phase operation on the first qubit and the fourth qubit. Each shaded block indicates operations at a node. $H$ denotes a Hadamard operation, and detectors denote projective measurements in the computational basis ($Z$-measurement). The dotted line represents a controlled operation depending on the measurement outcome. *The lower figure:* The (horizontal) butterfly network corresponding to the quantum circuit above, showing the amount of communication required in the protocol. The solid line denotes a single qubit channel, the thick dotted line denotes a two bit channel and the thin dotted line a single bit channel.

Since any controlled unitary operation is locally unitary equivalent to a controlled phase operation, namely, we can write $C_U = (v_1 \otimes v_2)C_{U_\theta}(u_1 \otimes u_2)$ using appropriate single qubit unitary operations $u_1$, $u_2$, $v_1$ and $v_2$, the protocol implementing $C_{U_\theta}$ over the butterfly network allows one to implement any controlled unitary operation by adding the single qubit operations $u_1$, $u_2$, $v_1$ and $v_2$ at the nodes $B_1$ and $B_2$, respectively.

Note that this protocol does not use the full capacity of the butterfly network at the edges $G_1$ and $G_2$, they are only used for transmitting 1-bit, instead of the 2-bit capacity allowed in Hayashi's setting. This extra 1-bit capacity could be used for another task, *e.g.* distributing a shared random bit. It is also remarkable that the operations required at nodes $A_1$, $A_2$, $B_1$ and $B_2$ do not depend on the angle $\theta$ of the controlled phase operation $C_{U_\theta}$, thus, the distributed quantum computation $C_{U_\theta}$ can be implemented without revealing the identity of the operation to the parties at the input and output nodes, as well as the fact that the party at node $C_1$ who is actually performing the unitary operation only transmits classical information.

## IV. IMPLEMENTATION OF CONTROLLED TRACELESS UNITARY OPERATIONS

In this section, we consider a situation where one of the inner channels, say $D_2$, is restricted to a *single* bit classical channel. We find a protocol that implements a slightly weaker class of controlled unitary operations, controlled *traceless* unitaries, over such a restricted butterfly network by adding 1-ebit of entanglement shared between the input nodes $A_1$ and $A_2$. At first sight this protocol consumes more resources than the protocol presented in the previous section for implementing a weaker class of controlled unitary operations, but as it only requires classical communication of 1-bit for the channel $D_2$, comparison of the resource requirements between these two protocols is not trivial.

This protocol is inspired by the entanglement assisted LOCC implementation of controlled unitary operations [14] shown in Fig. 4. This LOCC implementation requires a 1-ebit entanglement resource and two-way classical communication (1-bit each way) between the two distant parties.

However, this LOCC implementation is not directly implementable over the butterfly network, because in the latter the classical communication is also restricted. This incompatibility is shown in the following way, where a similar argument holds for any node at which the controlled unitary operation $C_U$ appearing in the quantum circuit is performed; here we will assume that $C_U$ is performed at the node $A_2$. Since no incoming communication from other nodes is allowed at node $A_2$, the classically controlled $X$ operation on the third qubit should also be performed by $A_2$. Then, the first controlled-NOT operation must also be performed at $A_2$ from the same reason. But for implementation over the butterfly network, the first qubit should be given at node $A_1$ by definition, therefore it is not possible.

Our idea is that to find an alternative quantum circuit

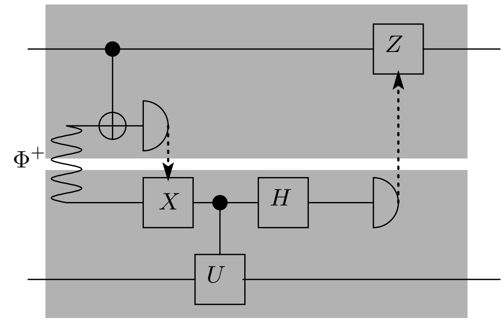


FIG. 4. The quantum circuit for entanglement assisted LOCC implementation of a controlled unitary operation presented in [14]. There are only two nodes; the first two qubits are at the first node (upper shaded area) while the third and forth qubits are at the second (lower shaded area).

to implement $C_U$ on the first and the fourth qubits, where $C_U$ on the third and fourth qubits is performed at the node $A_2$ before performing any other controlled operation required on the third qubit, by restricting the class of unitary operations $U$. If the order of $C_U$ on the third and forth qubit and the (classically controlled) $X$ operation on the third qubit are changed such that

$$C_U(X \otimes \mathbb{I}) = (A \otimes B)C_U, \qquad (1)$$

where $A$ and $B$ are some single qubit unitary operations to compensate, then we arrive at a quantum circuit implementing $C_U$ with the desired property. This quantum circuit is shown in the upper figure of Fig.5. By performing the operations given in each shaded block at each node, and transmission of quantum/classical information between the nodes specified by the edges shown in the lower figure of Fig.5, such a (restricted) $C_U$ operation is implementable over the butterfly network.

A sufficient condition for $U$ to satisfy Eq.(1) is that $\mathrm{tr}\, U = 0$. To see this, we rearrange Eq.(1) as

$$A \otimes B = C_U(X \otimes \mathbb{I})C_U^\dagger = |1\rangle\langle 0| \otimes U + |0\rangle\langle 1| \otimes U^\dagger. \qquad (2)$$

By taking partial traces of Eq.(2), we obtain

$$(\mathrm{tr} A)B = 0 \text{ and } (\mathrm{tr} B)A = (\Re\, \mathrm{tr} U)X + (\Im\, \mathrm{tr} U)Y. \qquad (3)$$

Since $B = 0$ is uninteresting, we therefore have $\mathrm{tr} A = 0$ and denote $A$'s eigenvalues by $\pm\alpha$. Then $B$'s eigenvalues are $\pm 1/\alpha$ or both $1/\alpha$ since the eigenvalues of $A \otimes B$ are equal to those of $X \otimes \mathbb{I}$, which are $\pm 1$. The case when $B$'s eigenvalues are degenerate is trivial: $B$ is equal to the identity up to some factor. Otherwise $\mathrm{tr}\, B = 0$ and from Eq.(3) we can conclude $\mathrm{tr}\, U = 0$. Thus, a controlled traceless unitary operation is implementable over this butterfly network.

## V. IMPLEMENTATION OF CLIFFORD OPERATIONS

In this section we construct a protocol for implementing Clifford operations on the butterfly network by

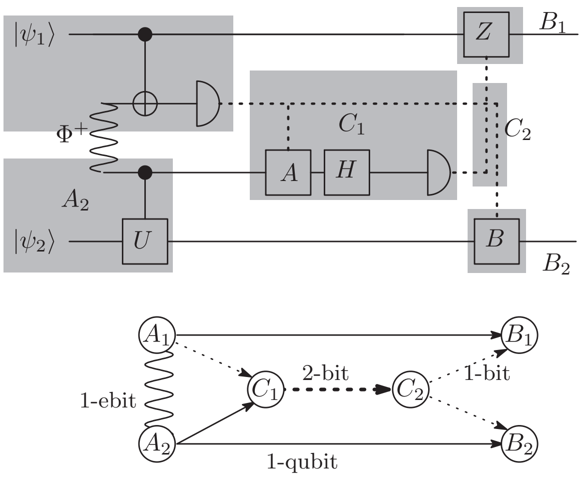


FIG. 5. *The upper figure:* The quantum circuit for implementing a controlled traceless unitary operation on the first qubit and the fourth qubit. *The lower figure:* The (horizontal) butterfly network corresponding to the quantum circuit above, showing the amount of communication required in the protocol.

slightly modifying the protocol for the swap operation $U_{swap}$ of section II. Here, a Clifford operation $U_{Cl}$ is defined as any operation that maps the Pauli group to itself, the group of which is known to be generated by a controlled-NOT operation, a Hadamard operation $H$, a phase operation $S = |0\rangle\langle 0| + i|1\rangle\langle 1|$, and Pauli operations. Any two qubit Clifford operation can be written in the form of $U_{Cl} \cdot U_{swap}$ by an appropriate choice of $U_{Cl}$, since $U_{swap}$ also belongs to the Clifford group. Her we construct a protocol for implementing $U_{Cl} \cdot U_{swap}$ over the butterfly network.

Suppose that a given Clifford operation $U_{Cl}$ satisfies $U_{Cl}(X_1 \otimes \mathbb{I}) = (P_1 \otimes P_2)U_{Cl}$ and $U_{Cl}(Z_1 \otimes \mathbb{I}) = (Q_1 \otimes Q_2)U_{Cl}$, where $P_1$, $P_2$, $Q_1$, $Q_2$ represent Pauli operators. The initial state of the protocol is given by $|\psi_1\rangle_{S_1}|\Phi^+\rangle_{H_{1,1}H_{2,1}}|\psi_2\rangle_{S_2}|\Phi^+\rangle_{H_{2,2}H_{1,2}}$, using the notation introduced in section II. First, perform a Bell measurement on $S_1$ and $H_{1,1}$ at the node $A_1$ and then perform $U_{Cl}$ at the node $A_2$ on $H_{2,1}$ and $S_2$. By denoting the measurement outcomes at the node $A_1$ to be $i, j$, the resulting state can be written as

$$|\tilde{\Phi}^{ij}\rangle_{S_1 H_{1,1}}|\tilde{\psi}_{12}^{ij}\rangle_{H_{2,1}S_2}|\Phi^+\rangle_{H_{2,2}H_{1,2}}, \tag{4}$$

where the states

$$|\tilde{\Phi}^{ij}\rangle = X^i Z^j |\Phi^+\rangle \tag{5}$$

and

$$|\tilde{\psi}_{12}^{ij}\rangle = (P_1 \otimes P_2)^i (Q_1 \otimes Q_2)^j U_{Cl}|\psi_1\rangle|\psi_2\rangle \tag{6}$$

denote the post measurement states corresponding to the outcome $i, j$. Next, perform another Bell measurement on $S_2$ and $H_{2,2}$ at the node $A_2$ and denote the measurement outcomes by $k, l$. This effects a teleportation of $|\psi_2\rangle$. The the state is now transformed to

$$|\tilde{\Phi}^{ij}\rangle_{S_1 H_{1,1}}|\tilde{\psi}_{12}^{ijkl}\rangle_{H_{1,2}H_{2,1}}|\tilde{\Phi}^{kl}\rangle_{S_2 H_{2,2}} \tag{7}$$

where

$$|\tilde{\psi}_{12}^{ijkl}\rangle = (\mathbb{I} \otimes X_2^k Z_2^l)(P_1 \otimes P_2)^i (Q_1 \otimes Q_2)^j U_{Cl} U_{swap}|\psi_1\rangle|\psi_2\rangle \tag{8}$$

denote the post measurement state after the second Bell measurement, corresponding to the outcome $i, j, k, l$. The parties at the nodes $A_1$ and $A_2$ now hold the uncorrected outputs $H_{1,2}$ and $H_{2,1}$, respectively. Next $A_1$ and $A_2$ perform $X_2^i Z_2^j P_2^i Q_2^j$ and $P_1^k Q_1^l$ on their qubit $H_{1,2}$ and $H_{2,1}$, respectively, while sending their measurement outcomes to the node $C_2$, just as in the protocol in [1]. The parties at the $B$ nodes receive the classical outcomes $i+k$ and $j+l$ from the corresponding $A$ nodes. Node $B_1$ can correct the quantum information by performing $X_1^{i+k} Z_1^{j+l}$ on her received qubit, whereas node $B_2$ performs $P_1^{i+k} Q_1^{j+l}$. This completes the protocol.

## VI. SUMMARY AND DISCUSSIONS

In this paper, in order to investigate distributed quantum computation under restricted network resources, we introduce a quantum computation task over the butterfly network where both quantum and classical communications are limited. We have studied protocols implementing two qubit unitary operations over a particular butterfly network introduced in [1] by showing several constructions. We have shown that any controlled unitary operation is implementable without an additional entanglement resource. We have shown another construction of a protocol for the case where one of the inner channels of the butterfly network is severely restricted in that it only allows one bit of classical information to be sent. We also presented a modification of the Hayashi protocol that implements global Clifford operations over the butterfly network.

We did not, however, consider optimality of the protocols in this work – proving the impossiblity of certain global operations would be very interesting. One of the reasons for this is because the circuit model is incompatible with the network model for evaluating upper bounds, which is useful for constructing and verifying protocols. This incompatibility may also result in the difficulty of analyzing protocol for gate arrays. For example, in general we cannot say that $U_1 U_2$ is implementable even if we know $U_1$ and $U_2$ are implementable on a network. For evaluating outer upper bounds, the min-cut max-flow theorem [7] or the resource inequality [9] approach can be useful for analyzing the situation where concurrency is properly taken into account.

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