

A realist interpretation of quantum mechanics based on undecidability due to gravity

Rodolfo Gambini¹, Luis Pedro García-Pintos¹, Jorge Pullin²

¹ Instituto de Física, Facultad de Ciencias, Universidad de la República, Iguá 4225, CP 11400 Montevideo, Uruguay

² Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001

Abstract. We summarize several recent developments suggesting that solving the problem of time in quantum gravity leads to a solution of the measurement problem in quantum mechanics. This approach has been informally called “the Montevideo interpretation”. In particular we discuss why definitions in this approach are not *for all practical purposes* (fapp) and how the problem of outcomes is resolved.

1. Introduction

The problem of measurement in quantum mechanics arises in standard treatments as the requirement of a reduction process when a measurement takes place. Such process is not contained within the unitary evolution of the quantum theory but has to be postulated externally and is not unitary. It is usually justified through the interaction with a large, classical measuring device.

Two main objections have been levied onto two aspects of the solution of the problem of measurement through decoherence.

1) Since the evolution of the system plus environment is unitary, the coherence could potentially be regained.

2) The second criticism has to do with the fact that in a picture where evolution is unitary “nothing ever occurs”. This is Bell’s “and/or” problem. The final reduced density matrix of the system plus the measurement device will describe a set of coexistent options and not alternative options with definite probabilities.

We shall argue that a solution to the problem of time in quantum gravity leads to a modification of quantum mechanics such that the two above objections can be overcome. The result is an objective description of quantum mechanics that is compatible with unitarity and does not require a reduction postulate.

The plan of the article is as follows. In the next section we briefly describe the proposed solution to the problem of time in quantum gravity. In section III we discuss how such solution leads to a modification of Schrödinger’s equation. In section IV we discuss the solution to the objections to environmental decoherence. In section V we argue that fundamental undecidability leads to the solution of the problem of outcomes and macro-objectification. We end with a discussion.

2. The problem of time in quantum gravity

Quantum gravity is expected to be most relevant in situations where one cannot assume that one has a clock external to the system under study (for instance when one considers the universe as a whole as is done in quantum cosmology). As a consequence of that one has to choose as clock one of the variables of the system under study. Since the variables of the system under study are all represented by quantum operators, time is not going to be a classical parameter as in ordinary quantum mechanics.

In fact one can choose to describe ordinary quantum mechanics in terms of a “real clock” i.e. a variable represented by a quantum operator subject to quantum fluctuations, etc. There usually is an objection that no monotonous variable can be conjugate to the Hamiltonian and this is seen as an obstruction to time being represented by a physical variable. But this is easily solved: one picks variables that work well as clocks for a while. If one needs a longer lasting clock one adds another variable when the first stopped being monotonous pretty much like the hour hand on a clock does when the minute hand goes around a complete circle. The way to describe quantum mechanics with a real clock is relationally. You pick an observable, let us call it $T(t)$ and then pick a set of observable quantities we want to study that commute with $T(t)$, i.e. $O^1(t), \dots, O^N(t)$. Here we denote by t the ideal classical time that appears in the ordinary Schrödinger equation. One then computes the conditional probability,

$$P\left(O^i = O_0^i | T = T_0\right) = \lim_{\tau \rightarrow \infty} \frac{\int_{-\tau}^{\tau} dt \text{Tr}\left(P_{O_0^i}(t) P_{T_0}(t) \rho P_{T_0}(t)\right)}{\int_{-\tau}^{\tau} dt \text{Tr}\left(P_{T_0}(t) \rho\right)}, \quad (1)$$

where $P_{O_0^i}(t)$ is the projector on the eigenspace associated with the eigenvalue Q_0^i at time t and similarly for $P_{T_0}(t)$. These conditional probabilities are positive and add to one. If one chooses operators with continuum spectrum such probability of giving a definite value vanishes, one has to ask about the probability of being in an interval around the eigenvalue and the above expression is unchanged.

There is a technical problem in carrying out the above construction in theories like general relativity that are generally covariant and it is the construction of observables to use in the construction. This was for many years a stumbling block for carrying out this program. Recently we proposed to use evolving constants of the motion as the observables and this construction has appeared satisfactory in model systems that could not be treated before. Since we are orienting this article towards readers not primarily interested in quantum gravity we omit the details here and refer to our paper on the subject [1].

3. A modified Schrödinger equation

An important question is how the conditional probabilities we introduced in the previous section evolve. We have been able to show that if one make some judicious assumptions, namely, that the clock does not interact with the system, that the clock is in a highly classical state (a coherent state where the “hand” of the clock is sharply peaked in space and moves in a monotonous way), then one can define a density matrix labeled by the eigenvalues of T ,

$$\rho(T) \equiv \int_{-\infty}^{\infty} U_{\text{sys}}(t) \rho_{\text{sys}} U_{\text{sys}}(t)^\dagger \mathcal{P}_t(T) \quad (2)$$

where the “sys” subscript indicates that it is the density matrix and the evolution operator corresponding to the system under study, it does not include the clock. We are assuming the density matrix of the whole universe is of the form, $\rho = \rho_{\text{sys}} \otimes \rho_{\text{clock}}$. The probability $\mathcal{P}_t(T)$ is the probability that the clock reads T when the ideal Schrödinger time is t . Such probabilities

are not directly observable, that is the reason the variable t always appears integrated. The explicit expression for the probability is,

$$\mathcal{P}_t(T) = \frac{\text{Tr}(P_T(t)\rho)}{\int_{-\infty}^{\infty} dt' \text{Tr}(P_T(t')\rho)}. \quad (3)$$

If one assumes one has a clock that follows the ideal Schrödinger time perfectly, then $\mathcal{P}_t(T) = \delta(t - T)$. In reality there will be departures. If we assume the departures are very small,

$$\mathcal{P}_t(T) = \delta(t - T) = \delta(T - t) + a(T)\delta'(T - t) + b(T)\delta''(T - t) + \dots, \quad (4)$$

one can show that the density matrix satisfies an approximate Schrödinger equation (for a pedagogical discussion of the derivation, see [2]),

$$\frac{\partial \rho(T)}{\partial T} = i[\rho(T), H] + \sigma(T)[H, [H, \rho(T)]]. \quad (5)$$

where $\sigma(T) = \partial b(T)/\partial T$. This type of equation is a particular case of the Lindblad equation considered in decoherence, but in this particular form it has the virtue that it conserves energy. The effect of the extra term is to make the off diagonal elements of the density matrix in the energy basis go to zero exponentially. That is, the evolution in terms of the real clock is not unitary. The underlying evolution in terms of the idealized time t is, but the “real clock” T cannot keep track of it accurately enough to keep a unitary evolution. After some time for a given T one will have a distribution of possible t 's associate with it and even if one started with a pure state one ends with a superposition of pure states for a given value of T .

At this point it is worthwhile asking: can one make the effect arbitrarily small by choosing better clocks? That requires to estimate the minimum uncertainty one can have in a clock. A full discussion would require more understanding of quantum gravity that we have right now. There have been several heuristic estimates in the literature [3] and some controversy surrounding them. We prefer to just consider generically that the uncertainty in the measurement of time has to grow with the observed time and if the effect is due to gravity, Planck's time $t_{\text{Planck}} = 10^{-44}\text{s}$ should be involved. This leads to postulate an uncertainty in the measurement of a period of time T of the form,

$$\delta T = t_{\text{Planck}}^{1-a} T^a. \quad (6)$$

The heuristic arguments we mentioned above yield $a = 1/3$ or $a = 1/2$ but in practice this makes little difference. As long as the clock error increases with the time measured, there will be a fundamental loss of coherence due to the use of real clocks.

A similar effect is introduced by the use of “real rods” to measure space. Such a discussion requires quantum field theory and had not been developed fully so we refer the reader to reference [4].

4. Solution to the objections to environmental decoherence

The presence of a fundamental source of loss of coherence helps deal with the objections levied against the solution of the problem of measurement through environmental decoherence. The two main objections can be characterized as “the information is still there” and “the and/or” problem or “problem of outcomes”. We discuss them in the following two subsections.

4.1. Impossibility of recovering the information

This objection is as follows: although a quantum system interacting with an environment with many degrees of freedom will very likely give the appearance that the initial quantum coherence of the system is lost, the information about the original superposition could be recovered, for

instance, by carrying out a measurement that includes the environment. The fact that such measurements are hard to carry out in practice does not prevent the issue from existing as a conceptual problem.

To analyze this point in some detail, as is customary in discussions of decoherence, let us consider a variation of a model proposed by Zurek [5] in which the quantum system, the environment and the measuring apparatus are under control. The system consists of a cavity with a uniform magnetic field in the z (vertical) direction. Inside the cavity is a spin S that represents the “needle” of the measuring device. A steady flux of spins is pumped into the cavity in a horizontal stream and each spin interacts with S a finite amount of time τ . The stream constitutes “the environment”. We also assume the spins of the environment are separated enough to avoid considering interactions among them. The interaction Hamiltonian is given by,

$$\hat{H}_k^{\text{int}} = f_k \left(\hat{S}_x \hat{S}_x^k + \hat{S}_y \hat{S}_y^k + \hat{S}_z \hat{S}_z^k \right), \quad (7)$$

with f_k the coupling constants. The Hamiltonian due to the presence of the magnetic field when the k -th particle is in the cavity is,

$$\hat{H}_k^B = \gamma_1 B \hat{S}_z \otimes \hat{I}_k + \gamma_2 B \hat{I} \otimes S_z^k, \quad (8)$$

where \hat{I} is the identity matrix acting on the Hilbert space of the needle and I_k is the identity in the Hilbert space of the k -th particle. The introduction of a constant magnetic field pointing in a given direction, is in order to have a definite pointer basis.

We have discussed this model in detail in [6] so we will not repeat all details here. We outline the main result. d’Espagnat [7] suggested that for systems like this one can define a global observable,

$$\hat{M} \equiv \hat{S}_x \otimes \prod_k^N \hat{S}_x^k. \quad (9)$$

This observable has the property that if one starts from a normalized initial state given by,

$$|\Psi(0)\rangle = (a|+\rangle + b|-\rangle) \prod_{k=1}^N \otimes [\alpha_k |+\rangle_k + \beta_k |-\rangle_k], \quad (10)$$

where $|\pm\rangle$ are eigenstates of σ_z , and computes the expectation value of the observable in a state that has evolved unitarily one gets,

$$\langle \psi | M | \psi \rangle = ab^* \prod_k^N [\alpha_k \beta_k^* + \alpha_k^* \beta_k] e^{-2i\Omega_k \tau} + a^* b \prod_k^N [\alpha_k \beta_k^* + \alpha_k^* \beta_k] e^{2i\Omega_k \tau}, \quad (11)$$

where $\Omega_k = \sqrt{4f_k^2 + B^2(\gamma_1 - \gamma_2)^2}$ with γ_1 the magnetic moment of the spins of the environment and γ_2 that of the “needle”. On the other hand, if one assumes a collapse of the wavefunction has occurred one gets that

$$\langle \psi | M | \psi \rangle = 0. \quad (12)$$

If one now studies the same observable but assuming the modified Schrödinger evolution we discussed in the previous section one gets,

$$\langle \hat{M} \rangle = ab^* e^{-i2N\Omega T} e^{-4NB^2(\gamma_1 - \gamma_2)^2 \theta} \prod_k^N \left[\alpha_k \beta_k^* e^{-16B^2 \gamma_1 \gamma_2 \theta} + \alpha_k^* \beta_k \right] \quad (13)$$

$$+ ba^* e^{i2N\Omega T} e^{-4NB^2(\gamma_1 - \gamma_2)^2 \theta} \prod_k^N \left[\alpha_k \beta_k^* + \alpha_k^* \beta_k e^{-16B^2 \gamma_1 \gamma_2 \theta} \right] \quad (14)$$

where $\Omega = B(\gamma_1 - \gamma_2)$. We see the expectation value is exponentially damped with an exponent that is large. Actually putting in realistic numbers one concludes that if one has an “environment” with more than 10^7 particles the expectation value decays too fast to be measured within the accuracies of the experiment, as we will discuss in the next subsection.

Due to the fundamental decoherence induced by quantum clocks the expectation value of the observables exponentially decreases and is more and more difficult to distinguish from the vanishing value resulting from collapse. A similar analysis allows to show that “revivals” [8] of coherence of the wavefunction of the system plus the measurement device (but excluding the environment) are also prevented by the modified evolution. When the multi-periodic functions in the coherences tend to take again the original value after a Poincaré time of recurrence the exponential decay for sufficiently large systems completely hides the revival under the noise amplitude.

Thus, the difficulties found in testing macroscopic superpositions in a measurement process are enhanced by the corrections resulting of the use of physical clocks.

4.2. Fundamental undecidability and the problem of outcomes

To address the second objection to the decoherence solution to the measurement problem, the “and-or” problem or as others put it “nothing ever happens” we will introduce a criterion to determine when an event takes place in a theory where evolution is always unitary.

When one takes into account the way that time enters in general covariant systems including the quantum fluctuations of the clock, the unitary evolution of the total system (system plus apparatus plus environment) becomes indistinguishable from the collapse. This is also true for revivals and the observation of the coherences of the reduced density matrix of the system plus the measuring device. We call such situation “undecidability”. We are going to show that undecidability is not only for all practical purposes (FAPP) but fundamental (see [9] for details). From the previous discussion one can gather that as one considers environments with a larger number of degrees of freedom and as longer time measurements are considered, distinguishing between collapse and unitary evolution becomes harder. But is this enough to be a fundamental claim?

Given the expectation value we discussed

$$\langle \hat{M} \rangle \sim \exp\left(-6NB^2(\gamma_1 - \gamma_2)^2 T_{\text{Planck}}^{4/3} \tau^{2/3}\right) \quad (15)$$

is it possible for a big enough ensemble to distinguish it from zero?

Brukner and Kofler [10] have recently proved that from a very general quantum mechanical analysis together with bounds from special and general relativity there is a fundamental uncertainty in the measurements of angles associated with quantum mechanical spin, even if one uses a measuring device of the size of the observable Universe. The bound is given by $\Delta\theta \geq \ell_{\text{Planck}}/R$ with ℓ_{Planck} the Planck length and R the radius of the universe. This uncertainty in the measurement of angles in our example implies there is an uncertainty in the measurement of $\langle M \rangle$. If that uncertainty is bigger than the value of $\langle M \rangle$ one can measure there is no way of distinguishing a collapse from a unitary evolution even in principle. For the model in question this occurs for $N \sim 10^7$ spins. So one sees that even if one considers ensembles of growing size, this does not help in distinguishing the expectation value in the case of collapse or unitary evolution since there are fundamental limitations to measurement. It is like attempting to measure a distance with an accuracy greater than the smallest spacing in a ruler. No matter how many measurements one makes, the error will be at least as large as the smallest spacing on the ruler.

The problem of macro-objectification of properties may be described according with Ghirardi [11] as follows: “how, when, and under what conditions do definite macroscopic properties

emerge (in accordance with our daily experience) for systems that, when all is said and done, we have no good reasons for thinking they are fundamentally different from the micro-systems of which they are composed?” We think that undecidability provides an answer to this problem. If a system suffers an interaction with an environment such that the density matrix is such that all the physical predictions of such a state cannot be distinguished from those of a collapsed density matrix, we will claim that an event took place. A fundamental ingredient is that in order to distinguish the density matrices one will be limited by fundamental limitations on the measurements of physical quantities and preparations of physical states. A detailed implementation will depend on the system under study. An example is the limitation on the measurements of spins of Brukner and Kofler we discussed in this section. Notice that for a quantum micro-system in isolation, events would not occur. However for a quantum system interacting with an environment, events will be plentiful. The same goes for a system being measured by a macroscopic measuring device.

5. Discussion

Gravity fundamentally limits how accurate our measurements of space and time can be. This requires reformulating quantum mechanics in terms of real clocks and rods, that have errors in their measurements. The resulting picture of quantum theory is one where there is a fundamental loss of quantum coherence: pure states evolve into mixed states as our clocks and rods cannot keep track of the unitary evolution. This eliminates the problem of revivals and global observables in the solution to the measurement problem through environmental decoherence: just waiting longer does not improve things as more quantum coherence is lost. One also has a definition for when an event takes place: when the fundamental loss of coherence is such that one cannot distinguish the unitary evolution from a reduction, an event has taken place. We call this situation “undecidability” between reduction and unitary evolution. One therefore has a complete, objective formulation of quantum mechanics with a unitary evolution and a notion of event that does not imply the collapse of the wavefunction. We call it the Montevideo interpretation of quantum mechanics. There even exists an axiomatic formulation of the theory [12]. We refer the reader to the references for further details.

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References:

- [1] R. Gambini, R. A. Porto, J. Pullin and S. Torterolo, *Phys. Rev. D* **79**, 041501 (2009) [arXiv:0809.4235 [gr-qc]].
- [2] R. Gambini, R. Porto and J. Pullin, *Gen. Rel. Grav.* **39**, 1143 (2007) [arXiv:gr-qc/0603090].
- [3] F. Károlyházy, A. Frenkel, B. Lukács in “Quantum concepts in space and time” R. Penrose and C. Isham, editors, Oxford University Press, Oxford (1986); Y. J. Ng and H. van Dam, *Annals N. Y. Acad. Sci.* **755**, 579 (1995) [arXiv:hep-th/9406110]; *Mod. Phys. Lett. A* **9**, 335 (1994); G. Amelino-Camelia, *Measurability Of Space-Time Distances In The Semiclassical* 3415 (1994) [arXiv:gr-qc/9603014]; S. Lloyd, J. Ng, *Scientific American*, November (2004).
- [4] R. Gambini, R. A. Porto, J. Pullin, *Phys. Lett.* **A372**, 1213-1218 (2008). [arXiv:0708.2935 [quant-ph]]
- [5] W. Zurek, *Phys. Rev.* **D26**, 1862 (1982).
- [6] R. Gambini, L. P. G. Pintos, J. Pullin, *Found. Phys.* **40**, 93-115 (2010). [arXiv:0905.4222 [quant-ph]].
- [7] B. d’Espagnat “Veiled reality”, Addison Wesley, New York (1995).
- [8] R. Gambini, J. Pullin, *Found. Phys.* **37**, 1074-1092 (2007). [quant-ph/0608243].
- [9] R. Gambini, L. P. Garcia-Pintos, J. Pullin, [arXiv:1009.3817 [quant-ph]].
- [10] C. Brukner, J. Kofler, “Are there fundamental limits for observing quantum phenomena from within quantum theory?”, arXiv:1009.2654 [quan-ph]
- [11] G. Ghirardi, “Sneaking a look at God’s cards” Princeton University Press, Princeton, NJ (2007).

- [12] R. Gambini, L. P. Garcia-Pintos, J. Pullin, “Complete quantum mechanics: An Axiomatic formulation of the Montevideo interpretation,” [arXiv:1002.4209 [quant-ph]].