

# The Dynamical Nonlocality of Neutral Kaons and the Kaonic Quantum Eraser

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Testing quantum foundations for systems in high energy physics gets currently more and more attention e.g. witnessed for entangled neutral K-mesons by the approved programme of the KLOE collaboration at the accelerator facility DAPHNE (Frascati, Italy). We focus on this quantum system in high energy physics and discuss two topics, Bell inequalities and the kaonic quantum eraser, and show how the neutral kaon system differs from systems of ordinary matter and light. In detail, we show a relation of the imbalance of matter and antimatter to the violation of a Bell inequality and discuss another Bell inequality which is maximally violated for a non-maximally entangled state though neutral kaons can be considered as two state systems. We compare in general this system in high energy physics with bipartite qudits. Last but not least we review the quantum marking and eraser procedure and explain why neutral kaons offer more eraser possibilities than usual quantum systems.

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## I. INTRODUCTION

Particle physics has become an interesting testing ground for questions of foundations of quantum mechanics as the accelerator experiments reach higher and higher precisions. This allows not only for deep tests of the standard model and possibly beyond but as well on foundations of quantum mechanics. E.g. massive meson–antimeson systems entangled in their flavour are specially suitable to test foundations. In this paper we focus at the neutral kaon system entangled in strangeness and discuss Bell inequalities and the quantum marking and eraser procedures which are one of the three issues of the recently approved programme of the KLOE collaboration (see Ref. [1]) at the DAPHNE accelerator in Frascati (Italy). In detail we

- show a Bell inequality which is related to a symmetry violation in particle physics, i.e. connecting two different concepts in physics [2, 3]
- show a Bell inequality which can be tested directly in experiments, but is not violated for the maximally entangled spin singlet state [3], but surprisingly for a non-maximally entangled state [4]. This opens — for the first time — the possibility of a direct experimental test whether local realistic theories can be refuted
- discuss the generalized Bell inequality for any pure entangled state
- discuss quantum marking and erasure experiments with neutral kaons [5, 6] which offers two more erasure options not available by other quantum systems

Of course also other features can be/have been discussed, e.g. one can discuss decoherence models for entangled K-mesons and B-mesons (e.g. Refs. [7, 9]), its connection to entanglement measures and show the recent bounds obtained by various accelerator experiments (for kaons see e.g. Refs. [10–12] or for B-mesons see e.g. Ref. [13]). In a quantum gravity picture space–time might be subjected to inherent fluctuations at the Planck scale. This would cause pure states to evolve to mixed states, i.e. decoherence of apparently isolated matter systems would occur suggested by Hawking [14]. This decoherence necessarily implies that the quantum mechanical operator generating CPT transformations cannot be consistently defined [15]. Such a model was formulated in Ref. [16, 17] and estimates of the parameter by the KLOE experiment can be found in Ref. [18]. An information theoretic analyzes of the neutral kaon system [19, 22] is also available offering a comparison to usual quantum systems.

In summary, understanding the nature of entanglement and its manifestation needs also studies of entanglement at different energy scales and as recent works (e.g. about high-dimensional genuine multipartite entanglement [23–25]) show there are still many open questions concerning entanglement.

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## II. BELL INEQUALITIES FOR NEUTRAL KAONS

The neutral K-mesons or simply kaons are bound states of quarks and anti-quarks or more precis the strangeness state  $+1$ ,  $K^0$ , is composed of an anti-strange quark and a down quark and the strangeness state,  $\bar{K}^0$ , is composed of a strange and anti-down quark. In accelerator experiments one can produce a spin singlet state, i.e. one has the same scenario as Einstein, Podolsky and Rosen considered in 1935 which in the following is written down for different quantum systems (spin- $\frac{1}{2}$ , ground/excited state, polarisation entangled photons, single neutrons in an interferometer, molecules, K-meson, B-mesons):

$$\begin{aligned}
|\psi^-\rangle &= \frac{1}{\sqrt{2}}\{|\uparrow\rangle_l \otimes |\downarrow\rangle_r - |\downarrow\rangle_l \otimes |\uparrow\rangle_r\} \\
&= \frac{1}{\sqrt{2}}\{|0\rangle_l \otimes |1\rangle_r - |1\rangle_l \otimes |0\rangle_r\} \\
&= \frac{1}{\sqrt{2}}\{|H\rangle_l \otimes |V\rangle_r - |V\rangle_l \otimes |H\rangle_r\} \\
&= \frac{1}{\sqrt{2}}\{|I\rangle_l \otimes |\uparrow\rangle_r - |II\rangle_l \otimes |\downarrow\rangle_r\} \\
&= \frac{1}{\sqrt{2}}\{|\text{early}\rangle_l \otimes |\text{late}\rangle_r - |\text{late}\rangle_l \otimes |\text{early}\rangle_r\} \\
&= \frac{1}{\sqrt{2}}\{|K^0\rangle_l \otimes |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l \otimes |K^0\rangle_r\} \\
&= \frac{1}{\sqrt{2}}\{|B^0\rangle_l \otimes |\bar{B}^0\rangle_r - |\bar{B}^0\rangle_l \otimes |B^0\rangle_r\} \\
&= \dots
\end{aligned} \tag{1}$$

Most Bell tests have been performed with polarisation entangled photons, e.g. under Einstein's locality condition [26]. A series of experiments testing the entanglement of molecules has recently been performed, see e.g. Ref. [27]. For single neutrons experiments exist which entangle the outer degree of freedom (path in the interferometer) with inner degrees of freedom (spin), a violation of Bell's inequality can be found, even including the Berry phase, a geometrical phase. The Bell inequality can be violated if the observables are accordingly compensated for the phase [20] which was recently experimentally verified [21]. Different to systems of ordinary matter and light neutral kaons oscillate in time (strangeness oscillation) and are decaying and therefore entanglement manifests itself considerably differently.

Analogous to entangled photon systems Bell inequalities can be derived for neutral kaons, i.e. the most general Bell inequality of the CHSH-type is given by (see Ref. [3])

$$\begin{aligned}
S_{k_n, k_m, k_{n'}, k_{m'}}(t_n, t_m, t_{n'}, t_{m'}) = \\
|E_{k_n, k_m}(t_n, t_m) - E_{k_n, k_{m'}}(t_n, t_{m'})| + |E_{k_{n'}, k_m}(t_{n'}, t_m) + E_{k_{n'}, k_{m'}}(t_{n'}, t_{m'})| \leq 2.
\end{aligned} \tag{2}$$

The upper limit is obtained under the usual assumption for any local realistic theories, i.e. that the most general form of an expectation value is given by

$$E_{k_n, k_m}(t_1, t_2) = \int d\lambda \rho(\lambda) O(k_n, t_n, \lambda) \cdot O(k_m, t_m, \lambda) \quad \text{with} \quad \int d\lambda \rho(\lambda) = 1. \tag{3}$$

Here Alice chooses to measure on the kaon propagating to her left hand side whether it is in the ‘‘quasi-spin’’  $|k_n\rangle$  or not, where  $|k_n\rangle = \alpha|K^0\rangle + \beta|\bar{K}^0\rangle$ , and how long the kaon propagates, i.e. the time  $t_n$  (corresponding to the distance from the source). The same options are given to Bob for the kaon propagating to the right hand side. As in the usual photon setup, Alice and Bob can choose among two settings. We notice already that in the neutral kaon case we have more options than in the photon case, we can vary in the quasi-spin space or vary the detection times or both.

### A. An open quantum formalism describing neutral kaons

In order to discuss fundamental questions of quantum mechanics we have to have an appropriate model to describe the time evolution of the meson system, i.e., oscillation ( $K^0 \leftrightarrow \bar{K}^0$ ) and decay. In Ref. [28] it has been shown that a decaying system can be handled with the open quantum system formulation, i.e. a master equation of the Lindblad type. For that one has to enlarge the original Hilbert space. The particle decay is then incorporated by a certain

Lindblad generator. With this master equation the Hamiltonian of the whole system (“surviving” and “decaying” states) is then Hermitian and trace preserving just like common quantum systems. Moreover, the time evolution is clearly completely positive and Markovian. The effective “environment” causing the decay of the mesons is what in quantum field theory is the quantum–chromo–dynamic vacuum.

The neutral kaon system is a decaying two–state system due to the strangeness oscillation in time,  $K^0 \leftrightarrow \bar{K}^0$ , and is usually described via an effective Schrödinger equation which we write in the Liouville von Neumann form ( $\hbar \equiv 1$ )

$$\frac{d}{dt}\rho = -i H_{eff} \rho + i \rho H_{eff}^\dagger \quad (4)$$

where  $\rho$  is a  $2 \times 2$  density matrix and  $H_{eff}$  is non-Hermitian. Using the usual Wigner-Weisskopf-approximation the effective Hamiltonian can be decomposed as  $H_{eff} = H - \frac{i}{2}\Gamma$  where the mass matrix  $H$  and the decay matrix  $\Gamma$  are both Hermitian and positive. To obtain this Hamiltonian the weak interaction Hamiltonian is treated as a perturbation and interaction between the final states is neglected, in particular the decay states of the neutral kaons, e.g. pions, are assumed to be stable. The second approximation is to consider only the first pole contribution and this leads to the exponential time evolution of the two diagonal states of  $H_{eff}$

$$|K_{S/L}(t)\rangle = e^{-im_{S/L}t} e^{-\frac{\Gamma_{S/L}}{2}t} |K_{S/L}\rangle, \quad (5)$$

$m_{S/L}$  and  $\Gamma_{S/L}$  are the masses or decay constants of the short or long lived kaons. What makes the neutral kaon systems so attractive for many physical applications is the huge factor between the two decay rates, i.e.  $\Gamma_S \approx 600\Gamma_L$ .

Considering Eq. (5)—which describes the surviving components of the neutral kaon evolving in time—we notice that the state is not normalized for  $t > 0$ . Indeed, we are not including all information on the system under investigation. For  $t > 0$  the neutral kaon is a superposition of the surviving components and decaying components. In Ref. [28] the authors showed that by at least doubling the original two–dimensional Hilbert space the decay can be incorporated into the dissipator of the enlarged space via a Lindblad operator. Let us start with the well known master equation in the Lindblad form

$$\frac{d}{dt}\rho = -i[\mathcal{H}, \rho] - \mathcal{D}[\rho] \quad (6)$$

where the dissipator has the general form

$$\mathcal{D}[\rho] = \frac{1}{2} \sum_{j=0} (\mathcal{A}_j^\dagger \mathcal{A}_j \rho + \rho \mathcal{A}_j^\dagger \mathcal{A}_j - 2\mathcal{A}_j \rho \mathcal{A}_j^\dagger).$$

The density matrix is 4 dimensional and lives on  $\mathbf{H}_{tot} = \mathbf{H}_s + \mathbf{H}_f$  where  $s/f$  denotes “surviving” and “decaying” or “final”, respectively, and has the following components

$$\rho = \begin{pmatrix} \rho_{ss} & \rho_{sf} \\ \rho_{fs} & \rho_{ff} \end{pmatrix}$$

where  $\rho_{ij}$  with  $i, j = s, f$  denote  $2 \times 2$  matrices and  $\rho_{sf} = \rho_{fs}^\dagger$ . The Hamiltonian  $\mathcal{H}$  is the Hamiltonian  $H$  from the effective Hamiltonian  $H_{eff}$  extended to the total Hilbert space

$$\mathcal{H} = \begin{pmatrix} H & 0 \\ 0 & 0 \end{pmatrix}.$$

To incorporate the decay ability we define one Lindblad generator, e.g.  $\mathcal{A}_0$ , to be

$$\mathcal{A}_0 = \begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix} \quad \text{with } B : \mathbf{H}_s \rightarrow \mathbf{H}_f,$$

where  $B^\dagger B = \Gamma$  and  $\Gamma$  is the decay matrix of  $H_{eff}$ . All other Lindblad generators ( $j > 0$ ) can only act on the surviving component of the density matrix, i.e.:

$$\mathcal{A}_j = \begin{pmatrix} A_j & 0 \\ 0 & 0 \end{pmatrix} \quad \text{with } j \neq 0.$$

Rewritten the master equation for the density matrix on the total Hilbert space  $\mathbf{H}_{tot}$  decomposed into the components of  $\rho$  we have

$$\begin{aligned}\dot{\rho}_{ss} &= -i[H, \rho_{ss}] - \frac{1}{2} \underbrace{\{B^\dagger B, \rho_{ss}\}}_{\Gamma} - \tilde{D}[\rho_{ss}], \\ \dot{\rho}_{sf} &= -iH\rho_{sf} - \frac{1}{2} \underbrace{B^\dagger B}_{\Gamma} \rho_{sf} - \frac{1}{2} \sum_j A_j^\dagger A_j \rho_{sf}, \\ \dot{\rho}_{ff} &= B \rho_{ss} B^\dagger,\end{aligned}$$

where  $\tilde{D}[\rho_{ss}] = \frac{1}{2} \sum_{j=1} (A_j^\dagger A_j \rho + \rho A_j^\dagger A_j - 2A_j \rho A_j^\dagger)$  describes any decoherence or dissipation which may occur. Indeed, we immediately see that this master equation describes the original effective Schrödinger equation (4) and thus the decay of neutral kaons (and in addition decoherence):

- (1) By construction the time evolution of  $\rho_{ss}$  is independent of  $\rho_{sf}, \rho_{fs}$  and  $\rho_{ff}$ .
- (2) Furthermore  $\rho_{sf}, \rho_{fs}$  completely decouples from  $\rho_{ss}$ . Consequently, these components  $\rho_{sf}, \rho_{fs}$  can without loss of generality be chosen to be zero, they are not physical and cannot be measured.
- (3) With the initial condition  $\rho_{ff}(0) = 0$  the time evolution is solely determined by  $\rho_{ss}$  and simply given by

$$\rho_{ff}(t) = B \int_0^t dt' \rho_{ss}(t') B^\dagger. \quad (7)$$

And indeed, this are the properties which we expect to describe a particle decay!

The extension for entangled kaons is straight forward given by replacing the Hamiltonian and decay generator, respectively

$$\begin{aligned}\mathcal{H} &\longrightarrow \mathcal{H} \otimes \mathbb{1} + \mathbb{1} \otimes \mathcal{H} \\ A_0 &\longrightarrow A_0 \otimes \mathbb{1} + \mathbb{1} \otimes A_0\end{aligned} \quad (8)$$

Summarizing, we showed that particle decay and decoherence/dissipation are related phenomena and that the Wigner Weisskopf approximation is Markovian and completely positive.

### B. A Bell inequality sensitive to the imbalance of matter and antimatter in our universe

Let us first choose all times equal zero and choose the quasi-spin states  $k_n = K_S, k_m = \bar{K}^0, k_{n'} = k_{m'} = K_1^0$  where  $K_S$  is the short-lived eigenstate, one eigenstate of the time evolution, and  $K_1^0$  is the  $\mathcal{CP}$  plus eigenstate. Here  $\mathcal{C}$  stands for charge conjugation and  $\mathcal{P}$  for parity. The neutral kaon is famous for violating the combined transformation  $\mathcal{CP}$  for which Fitch and Turlay got the Nobel Prize in 1980. In Ref. [2] the authors showed that after optimizing and a further trick the Bell inequality (2) can be turned into

$$\delta \leq 0 \quad (9)$$

where  $\delta$  is the  $\mathcal{CP}$  violating parameter in mixing. Experimentally,  $\delta$  corresponds to the leptonic asymmetry of kaon decays which is measured to be  $\delta = (3.27 \pm 0.12) \cdot 10^{-3}$ . This value is in clear contradiction to the value required by the BI above, i.e. by the premises of local realistic theories! In this sense the violation of a symmetry in high energy physics is connected to the violation of a Bell inequality, i.e. to nonlocality. This is clearly not available for photons, they do not violate the  $\mathcal{CP}$  symmetry. Although the BI (9) is as loophole free as possible, the probabilities or expectations values involved are not directly measurable, because experimentally there is no way to distinguish the short-lived state  $K_S$  from the  $\mathcal{CP}$  plus state  $K_1^0$  directly.

### C. A Bell inequality sensitive to the strangeness oscillation

Another choice for the generalized Bell inequality (2) is obtained by setting all quasispins equal  $\bar{K}^0$  and varying all times

$$\begin{aligned}S_{\bar{K}^0, \bar{K}^0, \bar{K}^0, \bar{K}^0}(t_1, t_2, t_3, t_4) = \\ |E_{\bar{K}^0, \bar{K}^0}(t_1, t_2) - E_{\bar{K}^0, \bar{K}^0}(t_1, t_3)| + |E_{\bar{K}^0, \bar{K}^0}(t_4, t_2) + E_{\bar{K}^0, \bar{K}^0}(t_4, t_3)| \leq 2.\end{aligned} \quad (10)$$

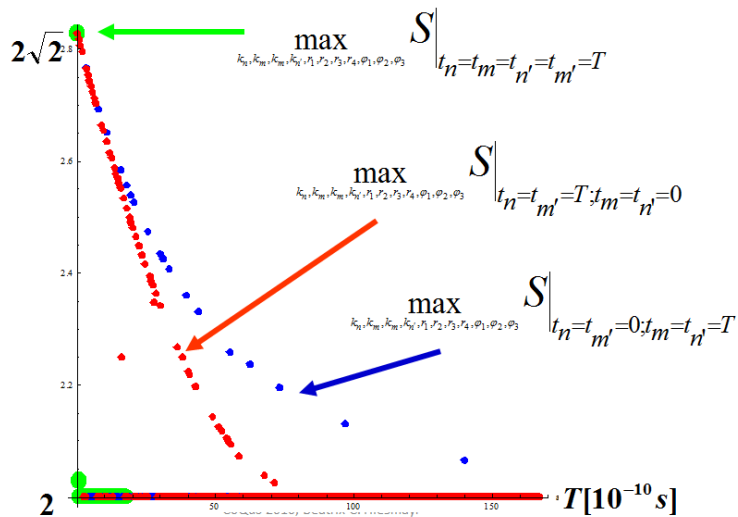


FIG. 1: Here the numerically optimized values of the CHSH-Bell type inequality (2) over all possible quasi-spin choices and any pure state is shown for different choices of times.

This has the advantage that it can in principle be tested in experiments. Alice and Bob insert at a certain distance from the source (corresponding to the detection times) a piece of matter forcing the incoming neutral to react. Because the strong interaction is strangeness conserving one knows from the reaction products if it is an antikaon or not. Note that different to photons a  $NO$  event does not mean that the incoming kaon is a  $K^0$  but also includes that it could have decayed before. In principle the strangeness content can also be obtained via decay modes, but Alice and Bob have no way to force their kaon to decay at a certain time, the decay mechanism is a spontaneous event. However, a necessary condition to refute any local realistic theory are *active* measurements, i.e. exerting the free will of the experimenter (for more details consult Ref. [29]).

Thus the question is: Can we violate the Bell–CHSH inequality sensitive to strangeness (10) for a certain initial state and what is the maximum value?

The first naive guess would be yes. In Refs. [2, 29] the authors studied the problem for an initial maximally entangled Bell state, i.e.,  $\psi^- \simeq K^0 \bar{K}^0 - \bar{K}^0 K^0$ , and found that a value greater than 2 **cannot** be reached, i.e. one cannot refute any local realistic theory. The reason is that the particle–antiparticle oscillation is too slow compared to the decay or vice versa, i.e., the ratio of oscillation to decay  $x = \frac{\Delta m}{\Gamma}$  is about 1 for kaons and not 2 necessary for a violation. On the other hand we have seen that the decay property acts as a kind of “decoherence”. From decoherence studies we know that some states are more “robust” against a certain kind of decoherence than others.

This leads to the question: Exists there another maximally entangled Bell state or another initial non-maximally entangled state which leads to a violation of the above Bell inequality because it is more robust against the decay property?

Indeed this is the case! The violation is not high, it is about 2.1 (see Ref. [29]) and is obtained for a non-maximally entangled state. This opens for the first time the possibility of a direct experimental test, the remaining question is how to produce any state that violates the Bell inequality.

#### D. What kind of states do violate the generalized Bell inequality?

In principle it is interesting to ask which states are violating the generalized CHSH-Bell type inequality (2). I.e. we optimize over all four quasispins Alice and Bob can choose in principle and any initial pure state where the four times are given. Thus we have to optimize over  $2 \times 4 + 6 + 1 = 15$  parameters. For that we used a special parametrization and an adapted Nelder-Mead optimization which was also used in optimization of a Bell operator for bipartite qudits (see Refs. [30, 31]). Each point in Fig. 1 is the maximum over three optimizations, but certainly as it often fails, it has further to be improved. If we choose  $t_n = t_m = t_{n'} = t_{m'} = T$  then after a rather short time  $T$  no violation of the generalized Bell inequality (2) can be obtained (big green dots). On the other hand if we choose two times to be equal to zero,  $t_n = t_{m'} = T; t_m = t_{n'} = 0$ , then we find that the violation of the Bell inequality can be obtained for times  $T$  which certainly can be realized in experiments (red dots). If we choose  $t_n = t_{m'} = 0; t_m = t_{n'} = T$ , the violation is even higher, this reflects the non-symmetry of the measurement choices for the Bell inequality (blue dots).

### III. THE KAONIC QUANTUM ERASER

Two hundred years ago Thomas Young taught us that photons interfere. Nowadays also experiments with very massive particles, like the fullerenes, have impressively demonstrated that fundamental feature of quantum mechanics. It seems that there is no physical reason why not even heavier particles should interfere except for technical ones. It is well known that the knowledge on the path through the double slit is the reason why interference is lost. The gedanken experiment of Scully and Drühl in 1982 [32] surprised the physics community, if the knowledge on the path of the particle is erased, interference is brought back again. Since that work many different types of quantum erasures have been analyzed and experiments were performed with atom interferometers and entangled photons where the quantum erasure in the so-called “delayed choice” mode captures best the essence and the most subtle aspects of the eraser phenomenon. In this case the meter, the quantum system which carries the mark on the path taken, is a system spatially separated from the interfering system which is generally called the object system. The decision *to erase or not* the mark of the meter system—and therefore *to observe or not* interference—can be taken long after the measurement on the object system has been completed. This was nicely phrased by Aharonov and Zurek in their review article [33] as “erasing the past and impacting the future”.

Some of these experiments use a kind of double slit device and entanglement. Both are available for neutral kaons, i.e. their time evolution can be viewed as a double slit varying in time [22]. In Refs. [5, 6, 34] it was shown that there exist four different types of quantum erasure concepts for neutral kaons:

- (a) Active eraser with *active* measurements
- (b) Partially passive quantum eraser with *active* measurements
- (c) Passive eraser with “*passive*” measurements on the meter
- (d) Passive eraser with “*passive*” measurements

Two of them (a), (b) can be considered from the theoretical point of view to be analogous to performed erasure experiments with entangled photons. In option (a) the erasure operation is carried out “actively”, i.e., by exerting the free will of the experimenter, whereas in the option (b) the erasure operation is carried out “partially actively”, i.e., the mark of the meter system is erased or not by a well known probabilistic law, e.g., by a beam splitter or in case of kaons by the decay property. However, different to photons the kaons can be measured by an *active* or a *passive* procedure (= decay modes). This offers new quantum erasure possibilities (c), (d) and proves the very concept of a quantum eraser, namely sorting events.

### IV. SUMMARY AND DISCUSSION OF THE “DYNAMICAL NONLOCALITY” IN THE NEUTRAL KAONS SYSTEM

We have shown a generalized Bell inequality (Ref. [3]) where both the quasi-spins, i.e. a certain superposition of the strangeness eigenstates (kaon, antikaon), and the times can be varied. Our first choice was to set all times zero and vary in the quasi-spins, by a further trick (Ref. [2]), this Bell inequality can be turned into an experimentally testable conditions. In detail we found that no local realistic theory can be constructed which includes the  $\mathcal{CP}$ -violating property of this system. This is certainly a different feature only available in meson-antimesons systems and nicely relates foundations of quantum mechanics with the very powerful concept of symmetries in particle physics. The disadvantage is that the expectation values involved cannot directly be tested in experiments.

Another choice of measurement settings (quasispins equal the strangeness eigenstates), not lacking this disadvantage, was shown to be non-violating if the initial state is the maximally entangled antisymmetric Bell state. Finding that the decay property (in general of any decaying system) can be modeled by an open quantum formalism gave the idea that there may exist states which are more robust against this “decoherence” caused by the decay property. Indeed, such states can be found, in particular a non-maximally entangled state violates the Bell inequality maximally. This opens for the first time the possibility of an experimental test, however, it is still open how to produce such a state and certainly some more subtle problems are awaiting their solution before we will have a conclusive experimental test.

From the theoretical point of view it is interesting to compare the neutral kaon system with other systems. For bipartite qubit systems any pure entangled state violates the Bell inequality and the maximal entanglement is obtained for the maximally entangled states, in contrast to the kaonic two-state system where a non-maximally entangled state maximally violates the Bell inequality. If the entanglement of two spin-1/2 particles is considered by an initial observer moving at different speeds, one finds that firstly the entanglement is only Lorentz invariant for certain partitions of

the Hilbertspace and secondly that the antisymmetric Bell state behaves differently than the symmetric one in general (see Ref. [37]).

If one considers a generalization of the CHSH-Bell inequality for bipartite entangled qudits, the CGLMP-Bell inequality (Ref. [36]), one finds for bipartite qutrits that a non-maximally entangled state violates the Bell inequality by a higher amount. Interestingly, no monotonic relation between the amount of entanglement and the violation of the Bell inequality can be found for bipartite qudits, which was demonstrated for the magic simplex states [35] in Ref. [30].

Last but not least, we showed that if Alice and Bob could choose any measurement setups and one could produce any initial state for the neutral kaon system, then for rather long times a violation of the Bell inequality can be found.

In summary, the neutral kaon system is a unique laboratory to test fundamental questions of particle physics but as well to test fundamental questions of quantum mechanics.

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