Determinism and Quantum Mechanics : some proposed tests

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Can the ancient, philosophical concept of determinism be formally translated into a scientific hypothesis, and finally be confronted with experiments? Are deterministic predictions, as expected, in contradiction with Quantum Mechanics, so that they can be quantified and eventually ruled out? After remarking that determinism and realism correspond to different concepts, we discuss these ideas in the present paper, and show an example where such contradiction appears. We consider a simple quantum system and its environment, including the measurement device, and make the assumption that the time evolution of the total system, the quantum system plus this environment, is deterministic, i. e., its time evolution is given by a dynamical trajectory characterized by some initial conditions. From this, we prove a type of Bell inequalities which are violated by Quantum Mechanics. We discuss other types of experiments, now in the macroscopic domain, where determinism can be tested.

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I. INTRODUCTION

Let us consider the time evolution of an isolated physical system, either classical or quantum. Does it always exist, as a matter of principle, a trajectory which, from some initial conditions, gives the values of the different quantities of the system at any time, as it is, for example, the case in Newtonian mechanics? This constitutes the concept of determinism in nature. As a matter of fact, determinism can be present under some circumstances, but the problem we want to address is whether determinism is *always* present *regardless of our capacity of prediction in practice*. Therefore, to discard determinism it suffices to find at least one example where it is contradicted by the experimental data. This is what we are going to discuss along the present paper. Our claim is that determinism fails because it enters in contradiction with Quantum Mechanics (QM).

Obviously QM, as stated in standard textbooks, is an extremely successful non-deterministic theory, since the result of measurements can only be predicted in an statistical way. The best we can do is to consider an isolated quantum system, which will follow an unitary evolution, and ask ourselves about the result of a given measurement. Then, the rules of QM apply and only probabilities can be predicted (of course, if the quantum system is open, the situation is more complicated). This is the end of the story for QM: it is non-deterministic. To allow for some kind of deterministic behavior in quantum systems, one should depart from the standard formulation and introduce some sort of *hidden variables* that lie behind the quantum description, as in many alternatives to QM, and explore its experimental consequences, as made in the literature.

There are two important points that must be discussed. The first one is the requirement that the system has to be isolated. As mentioned above, the evolution of open systems adds an extra difficulty to the study of QM, since the system becomes correlated with its environment, and its information is therefore degraded [1, 2]. Since we are going to assume determinism, we need to restrict ourselves to isolated systems. We will come back to this point later.

The second point relates determinism with realism, and the confrontation with experiments, in particular those experiments that evidence a violation [3] [4] of Bell inequalities [5] [6]. As it is very well known, this violation entails, modulus some loopholes [7], the failure of local realism, i.e. the NON existence of local hidden variables that claim for an objective reality beyond the quantum description. Of course, one could still assume the existence of some kind of non-local realism by giving up special relativity, an unrealistic assumption that would raise tremendous experimental and theoretical problems. An even weirder idea is the suggestion by Bell [8] that we could rely on a vast *conspiracy* which would arrange the causally unconnected hidden variable values in order to produce the correlations, between the two particle measurement outcomes, which are responsible for the observed violation of Bell inequalities.

In this paper, we explore a different idea, that can be eventually discarded by experiments, and is based on a strict determinism. Differently to the above alternatives, we can find some scenarios, as shown in the next Section, in which determinism enters in contradiction with quantum mechanics. The reason why the violation of Bell inequalities can be reconciled with **non local** realism, but not with determinism, is that the deterministic assumption is a stronger condition than realism. More precisely, realism postulates the existence of some hidden variables values behind the outcome of a performed measurement, *without asking that these hidden values remain the same after performing the measurement*. On the contrary, the deterministic assumption postulates the existence of some *standing* hidden variable values (the unknown initial conditions) behind *all* the successive self-responses of the isolated system along a certain time.

Let us also discuss the above ideas in connection with those of Bohm [9]. It has been claimed that Bohm's ideas provide a non-local realistic theory which reproduces all quantum predictions: any quantum particle would have its own well defined trajectory, at least in the absence of measurements, and therefore constitutes a deterministic theory to a certain extent. Then, perhaps this determinism could still be preserved in Bohm's theory framework, when successive measurements on the particle are made. However, in Section 3, we will see that, whatever the virtues of the Bohm theory are, it cannot produce this determinism in the presence of measurements and, at the same time, avoid entering in contradiction with QM.

Similar results to the ones reported above, although without mentioning the relationship with Bohmian mechanics, have been previously obtained by De Zela [10] by adding to the deterministic postulate a *non-invasive* measurement condition or, alternatively, a *non contextuality* condition. In the present paper, we do not need to impose any supplementary condition to the determinism postulate. On the other hand, we stress the fact that our time Bell-like inequalities apply to any system, microscopic o macroscopic, provided it has dichotomic responses and determinism is assumed. Using this, we will propose some experiments in the macroscopic domain to test determinism, and we will also refer to other experiments [11] which have already been performed with a negative result for determinism in the macroscopic world.

II. DETERMINISM AND VIOLATION OF BELL INEQUALITIES

We start with an ensemble of free spin 1/2 particles, all of them prepared on the same quantum state, and consider the following ideal experiment: we first fix a set of three space directions given by the unit 3-vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. Then, on each particle we perform two successive spin measurements, each time along one randomly selected direction out of this set: We prepare one particle and make the above two consecutive measurements, then prepare the next particle and proceed in the same way, etc ...

Given a particle, which is considered as free evolving, it may interact to some extent with its environment in an uncontrollable way, and certainly with the experimental apparatus. The latter includes the device that randomly selects the measurement direction among the three initially fixed directions. Consider now the physical system that includes the particle to be measured, the interacting environment and the experimental facility. We will refer to this system as the *enlarged system*, hereafter represented by E, and will assume that such system can be considered as an isolated system during the whole process. This is the usual starting point in the study of open quantum systems [1, 2] (where 'open' refers here to the measured particles).

We now define our notion of determinism following what happens, for example, with the Newtonian determinism (where the initial position and velocity, i. e., the initial conditions, allow us to know the entire trajectory of a particle). We will make the corresponding hypothesis for the enlarged system. By this we mean that, not only the successive spin measurement outcomes, $\pm\hbar/2$, are determined from the initial conditions, but also the successively selected measurement directions are determined too, independently of the selection mechanism. Notice that we do not put any restrictions on the assumed initial conditions: In particular, they could range over non causally connected space-time regions.

Let us be more precise about our enlarged system and the measurement process. Let us denote by λ the initial conditions that we postulate to exist, in addition to the quantum description of the prepared state. Imagine that, each time we prepare our spin 1/2 particles, system E starts from different initial conditions, i. e., λ values. We will perform two consecutive spin measurements on each prepared particle. These two consecutive spin measurements will be performed at two randomly selected times out of three fixed values t_1, t_2, and t_3. To each selected time, t_1, t_2 and t_3, we associate a constant measurement direction, $\vec{a}$, $\vec{b}$ and $\vec{c}$, respectively. In other words, we establish a correspondence $\{t_1 \to \vec{a}, t_2 \to \vec{b}, t_3 \to \vec{c}\}$ and keep it unchanged during the measurement process for the entire set of particles. Thus, the randomness in the direction selection arises only as a consequence of the randomness in the selected pair of times out of the set $\{t_1, t_2, t_3\}$. Denote by S the values of the measurement outcomes, which are conveniently normalized to ± 1 . According to the determinism postulate, there exists a function (unknown to us) which provides those outcomes for each value of time t_i, starting from the initial conditions, i. e., the parameter values λ. Let us represent this function by $S = S(\lambda, t_i, \vec{x}(\lambda, t_i))$, $i = 1, 2, 3$, with $\vec{x}(\lambda, t_i) \in \{\vec{a}, \vec{b}, \vec{c}\}$. Notice that this notation is actually redundant: according to the above discussion, once we have fixed λ and t_i the value of S becomes determined, therefore we could remove the argument $\vec{x}$ in S , although we will keep it for convenience for the following discussion.

We now follow the original proof of Bell inequalities [5] in order to arrive to similar inequalities for our measurement outcomes. Let us consider the following three expectation values

$$P(a,b) = \int d\lambda \rho(\lambda) S(\lambda, t_1, \vec{a}) S(\lambda, t_2, \vec{b}), \tag{1}$$

$$P(a,c) = \int d\lambda \rho(\lambda) S(\lambda, t_1, \vec{a}) S(\lambda, t_3, \vec{c}), \tag{2}$$

$$P(b,c) = \int d\lambda \rho(\lambda) S(\lambda, t_2, \vec{b}) S(\lambda, t_3, \vec{c}), \qquad (3)$$

where $\rho(\lambda)$ stands for a common probability density of the λ values, which satisfies $\int \rho(\lambda) = 1$. Taking here a common probability density $\rho(\lambda)$ instead of three different probabilities $\rho_{ab}(\lambda)$, $\rho_{ac}(\lambda)$ and $\rho_{bc}(\lambda)$ needs some explanation that we provide below.

Before this, let us make an observation. People familiarized with the original proof of Bell will have noticed that, once we have admitted this common probability and we have considered in (1) that $S(\lambda, t_2, \vec{b})$ does not depend on $\vec{a}$ (and similarly for $S(\lambda, t_3, \vec{c})$ in (2) and (3)), that is, once we have assumed this factorization between the two consecutive measurement directions, the proof of the corresponding inequalities comes in an inevitable way, as we will show. It has to be remarked that, in the present case, this factorization is not an *ad hoc* assumption: as we will see, it is a consequence of our central assumption of determinism. Let us discuss this in more detail.

One could have some doubts if we can take, as we have done, the same output value, $S(\lambda, t_2, \vec{b})$ in (1) as in (3), since in the first case this value is paired up with a precedent $S(\lambda, t_1, \vec{a})$, whereas in the second case is paired up with a following different S value, $S(\lambda, t_3, \vec{c})$ (we could raise a similar question about the two other outcome values $S(\lambda, t_1, \vec{a})$ and $S(\lambda, t_3, \vec{c})$). Nevertheless, as we have remarked, our postulate of determinism makes sure that, once λ and t_i have been given, the corresponding outcome S is fixed irrespective of the other outcome to which is paired up in a given order. This allows us to take the same value $S(\lambda, t_2, \vec{b})$ in Eqs. (1) and (3), and the same value value $S(\lambda, t_3, \vec{c})$ in Eqs. (2) and (3).

Let us come back to the use of the same probability function $\rho(\lambda)$ in the three equations (1)-(3). One might raise the following objection. Since we are selecting, in each case, a different subset of pairs in time, this would translate into a dependence of $\rho(\lambda)$ on the particular subset, which would imply the use of three different functions, say ρ_{ab}, ρ_{ac} and ρ_{bc} in those equations. Again, the response to this objection comes from our hypothesis of deterministic evolution for the enlarged system. This assumption implies, in particular, that the selection of the measurement directions, $\vec{a}$, $\vec{b}$, $\vec{c}$, responds to this kind of evolution. In other words, these directions only depend on the initial conditions λ and the corresponding times t_i. But, then, why does ρ not depend on these values of time, besides depending on λ? The answer is obvious, since these times are always posterior to the corresponding initial conditions λ. Therefore, they cannot affect the probability distribution of λ.

Nevertheless, perhaps ρ could still depend on the running time, t, during the successive preparations of our $1/2$ spin particles. Actually, this objection could also be raised for all the proofs of the original Bell-like inequalities, where the independence of ρ on t is implicitly assumed without any comment. Some experimental "reproducibility" principle operating in the natural world is behind this assumed time independence, here and in these previous cases. Obviously without this principle, it would be difficult to compare the theoretical predictions with experiments.

Let us go on with the derivation of our inequalities. We take the difference

$$P(a,b) - P(a,c) = \int d\lambda \rho(\lambda) S(\lambda, t_1, \vec{a}) \qquad (4)$$

$$\times [S(\lambda, t_2, \vec{b}) - S(\lambda, t_3, \vec{c})]. \qquad (5)$$

Henceforth, the proof of the inequalities goes along the same lines as the proof of the original Bell inequalities in the Bell's seminal paper [5]. First, since $S^2(\lambda, t_2, \vec{b}) = 1$, the above difference can be written as

$$P(a,b) - P(a,c) \;=\; \int d\lambda\, \rho(\lambda) S(\lambda, t_1, \vec{a}) S(\lambda, t_2, \vec{b}) \, [1 - S(\lambda, t_2, \vec{b}) S(\lambda, t_3, \vec{c})]. \qquad (6)$$

Then, taking absolute values, we are led to

$$|P(a,b) - P(a,c)| \leq \int d\lambda \rho(\lambda) [1 - S(\lambda, t_2, \vec{b}) S(\lambda, t_3, \vec{c})], \qquad (7)$$

that is, to the well known Bell inequality

$$|P(a,b) - P(a,c)| \leq 1 - P(b,c). \qquad (8)$$

In quantum mechanics, leaving aside the experimental difficulties to perform the kind of experiment we are considering (see [10] for some sound proposals), the three mean values in (8) can be theoretically calculated as the corresponding expected values. These values become $P(a,b) = \vec{a}.\vec{b}$ (see for instance [12]) and similarly for $P(b,c)$ and $P(c,a)$. Thus, inequality (8) becomes

$$|\vec{a}.\vec{b} - \vec{a}.\vec{c}| + \vec{b}.\vec{c} \leq 1, \qquad (9)$$

which is violated for $\vec{b}.\vec{c} = 0$ and $\vec{a} = (\vec{b}+\vec{c})/\sqrt{2}$, in which case the left hand side of inequality (9) reaches the value $\sqrt{2}$.

Thus, for the enlarged system consisting on a $1/2$-spin particle and the measurement apparatus, including the device selecting the time pairs and the corresponding measurement directions, plus the affecting environment if any, the assumed determinism enters in contradiction with quantum mechanics. If such an experiment is performed, and the above inequality is found to be violated, as we expect from QM, one can conclude that there is no trajectory for this quantum particle independently of the kind of realism, local or non local, associated to the initial conditions.

At this point, it is interesting to compare our result with a similar one stated in [13]: there, under the following three postulates, *macroscopic realism per se*, *noninvasive measurability* and *induction*, Leggett claims for some Clauser-Horn-Shimony-Holt (CHSH) inequalities [6], relating the successive outcomes of a system with random dichotomic responses against four types of measurements. Our determinism postulate entails macroscopic realism per se and the induction postulates, and also noninvasive measurability *for the enlarged system* (the spin $1/2$ particle plus the environment including the measurement device). Thus, the only difference between Leggett approach and ours is that we replace the dubious non-invasive measurability by a more clear postulate as determinism, for the isolated enlarged system.

Finally, notice that inequality (8) could be applied to any system, macroscopic or not, with a random dichotomic response to three possible kinds of consecutive measurements (only a consecutive pair of these three measurements being randomly selected each time), provided that determinism is assumed for the time evolution of the system. Then, looking for the possible violation of such inequality could lead to test determinism in the macroscopic world (see [11] for an example of it), beyond the microscopic case of a half one spin particle we have just tested.

As an example of a system whose determinism could be experimentally tested, we suggest a proposal based on the experiment performed in [14], where the authors study a physical system including a two-level system which is continuously and weakly monitored by a detector. In this work, $V(t)$ is the output signal of the detector at time t . By introducing an appropriate (nonlinear) transformation, it is possible to define a new function $U(t)$, which depends on $V(t)$ and runs from -1 to 1 as V increases within the interval $]-\infty,\infty[$. Equations (1)-(3) can be modified using $U(t)$ instead of our variable S , and new Bell-like inequalities (similar to (8)) can be derived. Since the transformation from $V(t)$ to the new variable $U(t)$ is nonlinear, the comparison with the experiment cannot be done using the data for the correlator $K(\tau)$ obtained in [14]. Such comparison needs storing all the individual data for the potential, not just the correlator, and thus requires a modified setup and a new run. However, it can be done in principle starting from the existing setup. Such an experiment would provide an interesting alternative to the one using spin $1/2$ particles for the purpose of ruling out the deterministic hypothesis.

In order to test determinism in the macroscopic world, we could also use any source producing random successions of numbers, as for example the ones originated from atmospheric noise [15]. It is generally stated, according to standard criteria, that these numbers are "true random numbers". Beyond this statement, it would be interesting to test it by investigating the possible violation of the corresponding Bell inequalities deduced under the deterministic assumption, since a hypothetical violation of them would entail the true randomness of the corresponding succession. The unexpected violation of similar inequalities found in the case of an electrocardiogram [11], with outputs obviously generated by an ordinary macroscopic system, might encourage any one to perform such a test.

The Bell inequalities that we propose to test are still inequalities (8) applied to our generator of random numbers. That these inequalities are still valid in the case of such a generator, can be proved by mimicking the procedure that we have followed above in the case of a $1/2$ spin particle. In this case, however, since we are analyzing a set of data after they have been produced, we need to adopt the extra assumption of statistical separability between the data to be analyzed and the random numbers used to select the different times t_i, as described in [11]. To get an idea about the new proof one needs to establish imagine, for the sake of simplicity, that our generator produces (assumed) random number values -1 and 1, at a constant rate: an output every time interval δt. Let us write the successive output times as $n\delta t$, with n standing for the natural numbers from 1 to a given maximum value N. For each value of n, the times $n\delta t$, $(n+1)\delta t$ and $(n+2)\delta t$, will play the role of the three times, t_i, or equivalently, the three directions a, b, c, respectively, in the proof of inequalities (8). Now, for each n value, we randomly select two of these three times. Furthermore, consider the three expectation values $P(a,b)$, $P(a,c)$ and $P(b,c)$, constructed by keeping only the outcomes -1 and 1 corresponding to the pairs of times selected among the three times $n\delta t$, $(n+1)\delta t$ and $(n+3)\delta t$. Then, by invoking the determinism assumption and following an argument similar to the one used in the above proof of inequalities (8), we can conclude the validity of these inequalities for this case.

Finally, in order to stress the point of making the distinction, made in the present paper, between determinism and realism, we consider the case presented in [16], where it is claimed that 42 random numbers have been generated from the measurement outputs of a system of two entangled atoms. We think that the claim is correct and very interesting but, in our opinion, not correctly established. Its pretended proof relies on the observed violation of the corresponding (CHSH) inequalities [6]. As it is well known, these inequalities are proven under the assumption of local realism. This means that the claim about randomness depends on the supplementary assumption of locality, which means that the observed outputs could still be predetermined, if non local realism is allowed. To consistently prove that these 42 numbers are actually random, we should previously prove the CHSH inequalities under the only assumption of determinism, whose initial conditions, as seen in the present paper, are completely general, i.e., local or non local. This new proof of CHSH inequalities come easily by applying to the present case the same method previously used to prove inequalities (8). The role played then by the two successive measurements on the same particle is to be replaced by the two successive measurements performed, respectively, on the two atoms. In this way, we can properly conclude that the reported violation of the CHSH inequalities in [16] really proves the true random character of the 42 numbers obtained. A detailed analysis concerning this topic will be published elsewhere.

Now, as mentioned at the end of the Introduction, the non existence of trajectories should also be considered from the point of view of the Bohm hidden variable theory. As we discuss in the next Section, there is no contradiction between our claim of non determinism and that theory.

III. THE LIMITATIONS OF BOHM THEORY OF HIDDEN VARIABLES

Let us examine the common claim that Bohm's hidden variable theories (HVT) can predict the existence of dynamical trajectories and, at the same time, be consistent with QM. It is true that Bohm [9] proves that his theory gives the same probability of finding a particle in a given position, that QM does. From this, he concludes that his "interpretation is capable of leading in all possible experiments to identical predictions to those obtained from the usual interpretation" that is, to those obtained from QM. Then, when considering an entangled extended system, as in Einstein-Podolsky-Rosen experiments (similar to the one considered by Bell in his seminal paper), Bohm assumes that his realism is non-local. In this way, his non-local HVT can explain the observed violation of the ordinary Bell inequalities, in agreement with QM, without having to give up realism (see [5] for example).

Is it always like this? Is it true that we can devise a non-local HVT that lead to the same predictions that QM, *for all conceivable experiments*? As we discuss below, it is doubtful that HVT, even if allowing for non-local realism, could always agree with QM to the extent of assigning a dynamical trajectory to each quantum particle when measurements are present. That is, to the extent of assuming determinism.

First of all, in these theories, each time one performs a measurement on the particle position, if one wants to complete, beyond the obtained outcome, the precedent particle trajectory with a new trajectory piece, one must provide the probability density of the particle position just after this outcome. The provided probability becomes the new initial probability. Then, this new initial probability must be taken the same as the one dictated by standard QM, if we want the HVT to agree henceforth with QM. After this, in the HVT framework, one does not need to worry about how this initial probability evolves in time, until one performs a subsequent measurement, since, in the absence of any measurement, HVT are just designed to predict the same probability evolution as the one predicted by the Schrödinger equation. But, as we will see in a moment, the real point is that when consecutive different measurements are performed on the same particle [12], one expects to find some well definite correlations among the corresponding outcomes: the correlations dictated by QM that lead, for example, to the violation of inequalities (8).

More precisely: both in QM or in HVT, the probabilities of each measurement outcome is given by the corresponding initial quantum state of the particle, just the state previous to the measurement. In HVT, these initial quantum states are supplemented with the assumed initial values of some non-local hidden variables, λ. Nevertheless, the point here is that these λ values, which mimic so perfectly well the quantum evolution of the above probabilities in the absence of measurement, have nothing to do with the explanation of the quantum correlations which are behind the reported quantum violation of inequalities (8). The reason is that *these correlations have only to do with the quantum fact that the initial λ values need to be different before and after a given measurement on the particle*, while they have to be the same if one assumes uncritically that we always can have deterministic dynamical trajectories for quantum particles in the framework of HVT theories.

Thus, if we mean by "trajectory" something more than a simple continuous path, even a zigzagging one, to require the existence of determining initial conditions, and we further accept QM, it seems that there is no room left "for models that force Nature to mimic the concept of trajectory" as it is still expected in [17]. In other words, it seems that henceforth we would have to renounce to a deterministic Bohmian mechanics and also to any deterministic explanation of the quantum wave collapse, for example one in the terms of [18].

To summarize: according to the above discussions, either Quantum Mechanics, or determinism as such, must be false. So, if we accept QM, in view of its great success, we must conclude that determinism as such would contradict experiments. Then the answer to the Leggett question [13] whether "it is indeed realism rather than locality which has to be sacrificed?" would be 'yes'. All in all: against Einstein's old dream, it seems that QM cannot be completed to the extent to allow for the existence of trajectories for quantum particles even when accounting for all its affecting environment.

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