Note on "A note on Phys. Rev. Lett. 105, 170404 (2010)"

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Cavalcanti and Scarani (CS) raised some noteworthy points [1] regarding our recent work on Bell test for continuous variables [2]. In particular, they argue that our proposed test cannot exclude all possible hidden-variable models by giving a counter-example. Here, we provide a brief response to their comments.

(i) First, we have indeed used a constraint, $X_j^2 + Y_j^2 = N_j$, where X_j and Y_j are the real and the imaginary part of a complex amplitude and N_j is its intensity for local systems $j = 1, 2$, in addressing our proposed test, which was already mentioned in the footnote [22] of our paper [2]. CS constructed a counter-model, i.e. $X_1 = X_2 = Y_1 = Y_2 = 1$ and $N_1 = N_2 = 0$, to explain the violation of our inequalities, e.g.,

$$(\langle X_1X_2\rangle + \langle Y_1Y_2\rangle)^2 + (\langle X_1Y_2\rangle - \langle Y_1X_2\rangle)^2 \leq \langle N_1N_2\rangle.$$

However, their model is beyond physical domain; if a single realistic object possesses a nonzero amplitude ($X_jY_j \neq 0$), it must possess a nonzero intensity as well ($N_j \neq 0$). This is a rather device-independent reasoning, expressed as $X_j^2 + Y_j^2 = N_j$, although an actual experiment (photon-counting) to measure intensity may entail device-dependent assumptions. In fact, possible hidden-variable descriptions to account for the violation will be all *unphysical* in this sense.

(ii) Nevertheless, as CS suggested, it is worthwhile to relax the constraint to some extent in order to rule out a broader class of hidden-variable models, though unphysical, which may warrant further investigation. This can be particularly important for the security test of quantum cryptography, where an adversary may intervene with devices for which certain assumptions could have been made to establish the cryptographic protocol under use. On the other hand, if one wants to confirm whether there exists a correlation that no classical, physical, models can explain, the assumption on the adversary is not necessary. Instead, a "good-will" scenario will be sufficiently meaningful in the current stage; a third party prepares an ensemble of correlated state and distribute them to Alice and Bob who perform local measurements with measurement settings randomly chosen each time by an independent random-number generator. All events, detected and undetected, must be included in analyzing correlation data to address unbiased statistics. It seems that no experiment has ever been done even under this good-will scenario to rule out all physical hidden-variable models with both issues on the locality assumption and the detection efficiency appropriately addressed, and we have suggested a possible test along this line in [2].

(iii) CS raised another point (comment 3 in [1]), which puts forward a possibility that the ensembles (horizontally-polarized light) for measuring X_j and Y_j could be different from those (vertically-polarized light) for measuring N_j by an adversary's intervention. This again seems to be an issue more relevant to, e.g., cryptography. If the argument is carried to a pure Bell test, we point out that no experiment can avoid this type of "conspiracy" even under ideal situations (perfect detector, etc.); note that each party measures only one observable at a time in every Bell test, therefore, an extremely harsh objection can be made that each pair of measurement settings has addressed a completely different ensemble. This is also related to the issue on "measurement independence" discussed, e.g., by Hall [3].

(iv) Finally, one can readily show that the violation of our inequalities is attributed to the quantum commutation rule ($[\hat{a}, \hat{a}^\dagger] = 1$), which may be the only component absent in classical probabilistic descriptions; Once all observables are commutable with each other, those inequalities will be readily satisfied. Moreover, when the constraint $X^2+Y^2 = N$ is discussed at the level of single-mode states, a quantitative test of wave-particle "duality" may arise; the discrepancy between wave-like intensity $\langle X^2 + Y^2\rangle$ (homodyne detection) and the particle-like intensity $\langle N\rangle$ (photon counting) turns out to be $\langle X^2 + Y^2\rangle - \langle N\rangle = \frac{1}{2}$ for each quantum state, which may also prove the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$, or equivalently $[\hat{X}, \hat{Y}] = \frac{i}{2}$. Interestingly, CS also suggested a very similar idea as a positive ramification of our approach. Further investigations with more details will be given elsewhere.

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