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Nonlocal Two Particle Interferometry & Entanglement Generation from Majorana Bound States

Sougato Bose and Pasquale Sodano
Dept. of Physics and Astronomy, University College London,
Gower Street, WC1E 6BT London, United Kingdom,
Department of Physics and Sezione INFN, University of Perugia, Via A. Pascoli, 06123 Perugia, Italy
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We show that a one dimensional device supporting a pair of Majorana bound states at its ends can produce remarkable Hanbury-Brown Twiss like interference effects between well separated Dirac fermions of pertinent energies. We find that the simultaneous scattering of two incoming electrons or two incoming holes from the Majorana bound states leads exclusively to an electron-hole final state. This "anti-bunching" in electron-hole internal pseudospin space can be detected through current-current correlations. Further, we show that, by scattering appropriate spin polarized electrons from the Majorana bound states, one can engineer a non-local entangler of electronic spins for quantum information applications. Both the above phenomena should be observable in diverse physical systems enabling to detect the presence of low energy Majorana modes.

Quantum Indistinguishability has striking manifestations when two identical particles are brought together at a beam splitter. For example, two bosons in identical states would "bunch" together when exiting a beam-splitter purely due to interference effects [1]. Two fermions, on the other hand, would exit separately or "anti-bunch" [2]. These effects are indeed an instance of the celebrated Hanbury Brown-Twiss effect, which has recently also been tested with Helium atoms [3]. The same quantum indistinguishability is exploited for the production of entangled photons [4], and can also be used to entangle generic massive particles [5]. Of course, all these effects can occur only when the particles are brought together spatially, for instance, at a beam splitter. It is thereby interesting to look for settings where rather well separated identical particles could manifest such phenomena.

Here we report on the possibility of engineering a *non-local* beam splitter enabling the above class of phenomena for distant charged fermions. Here, by "non-local" we mean spatially extended. Going beyond the usual two particle interference in orbital/momentum space, here one finds a *Hanbury Brown-Twiss effect in the electron-hole internal pseudospin space* . This is enabled by Majorana mid-gap low energy modes which transform between electrons and holes [6], effectively making them indistinguishable in a scattering experiment. This Hanbury Brown-Twiss effect is thereby a detector of the Majorana modes.

Recently, low energy Majorana (neutral charge self-conjugated fermion) modes located at the edges of linear devices have been shown to induce non-local phenomena [7–9]. Indeed there are a variety of platforms to realize such devices such as a quantum wire immersed in a p-wave superconductor [7, 10], cold-atomic systems mimicking p-wave superconductors [11], topological insulator-superconductor-magnet structures [8, 12, 13] and potentially also semiconductor systems [14, 15]. The evidences of their non-local nature are distance independent tunneling [7], crossed Andreev reflection [8] and teleportation-like coherent transfer of a fermion [9]. Finally, they may be easily manipulated [6] and are relevant excitations also in conventional superconductors [16]. So far, the primary application envisaged for these fermions has been topological quantum computation [17]. As the second key result of this paper we will show another use of these modes, namely that Majorana bound-states (MBS) could be used to engineer entanglement between the spins of well separated particles, a pivotal resource in quantum information.

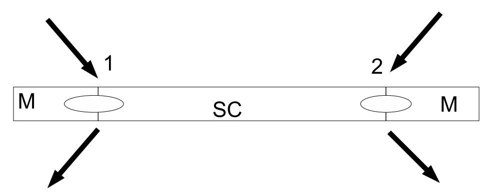

FIG. 1: Non-local beam splitter and electron spin entangler. The MBS are shown as empty ellipses 1 and 2. One specific realization where MBS occur at boundaries between magnets (M) and superconductors (SC) deposited on quantum spin-Hall insulators is depicted, though our results hold more generally. Incoming and outgoing particles are shown by arrows, and may, in practice, be tunneled in/out by STM tips or electron pumps acting as leads.

We consider a one dimensional device supporting two weakly coupled MBS at its ends as shown in Fig.1. The MBS are labeled as 1 and 2 and schematically shown as empty ellipses in the figure. For the sake of clarity, we will first show how this device produces Handbury-Brown Twiss like interference effects between spatially separated Dirac fermions in the *spinless* models investigated in [7, 10]; later, we show how all the results are valid for more realistic spinfull physical settings [12, 14, 15]. The Hamiltonian describing the weak coupling between the MBS γ_1 and γ_2 is given by

$$H_M = iE_M\gamma_1\gamma_2, \tag{1}$$

where γ_j are Majorana operators defined by $\gamma_j = \gamma_j^\dagger$ and satisfying $\gamma_j\gamma_k + \gamma_k\gamma_j = 2\delta_{ij}$. Leads, also labeled as 1 and 2, are connected to the system as shown in Fig.1, allowing for the scattering of Dirac fermions (electrons or holes) from each of the MBS. The unitary scattering matrix S in a model independent form is given by [8] $S(E) = 1 + 2\pi iW^\dagger(H_M - E - i\pi WW^\dagger)^{-1}W$ where W describes the coupling of the scatterer (H_M) to the leads and E is the energy of the incident electrons/holes. The entries w_j of the W matrix are related to the couplings to the leads $\Gamma_j = 2\pi w_j^2$ [8].

For our purposes, it is convenient to assume that $E >> \Gamma_j$, as well as $E \approx E_M$ (i.e., the energies of the incoming Dirac fermions are tuned to be nearly resonant with the Majorana coupling energy). Under these circumstances, the S matrix simplifies to

$$S = \frac{1}{2}\begin{pmatrix} 1 & -i & -1 & -i \\ i & 1 & i & -1 \\ -1 & -i & 1 & -i \\ i & -1 & i & 1 \end{pmatrix} \tag{2}$$

where the basis is $\{|e_1\rangle, |e_2\rangle, |h_1\rangle, |h_2\rangle\}$, with $|e_j\rangle$ and $|h_j\rangle$ representing an electron and a hole in the lead j. Note that this regime is *different* from the one considered by Akhmerov et. al. [8], where only the terms corresponding to crossed Andreev reflection (i.e $\langle h_2|S|e_1\rangle$ and $\langle h_1|S|e_2\rangle$) are maximized. Here we work in a regime where all the entries of S have the same magnitude.

If, at time t, a single electron tunnels into the Majorana mode located at site 1, i.e., the incoming state is $c_1^\dagger|0\rangle$, it transforms, under S, to

$$c_1^\dagger|0\rangle \xrightarrow{\text{MBS}} \frac{1}{2}(c_1^\dagger + ic_2^\dagger - d_1^\dagger + id_2^\dagger)|0\rangle, \tag{3}$$

where $c_j^\dagger$ ($d_j^\dagger$) creates an electron (hole) at site j. In Eq.(3), we have used MBS above the arrow to indicate that Majorana bound states are responsible for the process. Since the transformation (3) is equivalent to a four port beam splitter, with MBS inducing the beam splitting process, one can equally well take MBS to stand for "Majorana Beam-Splitter". Eq.(3) implies that an incoming electron has $\frac{1}{4}$th probability of coming out of each site as an electron or a hole. If another electron scatters at a different time t' on the Majorana mode located at position 2, it will also scatter with exactly the same probabilities for the four possible outcomes. The joint probability for two incoming electrons to exit as two electrons or two holes (whichever the output port) would thus be $\frac{1}{2}$. Next, we will show that when $t = t'$, i.e., simultaneous scattering, two particle interference can take place so that the probability of two electrons or two holes exiting is completely suppressed. By $t = t'$ we mean that the wavepackets of the two incoming electrons (holes) are large enough so that their time of arrival cannot be distinguished when one observes them after the scattering.

When two electrons scatter *simultaneously*, one at site 1 and the other at site 2, one has

$$\begin{aligned} c_1^\dagger c_2^\dagger|0\rangle \xrightarrow{MBS} & \frac{1}{2}(c_1^\dagger + ic_2^\dagger - d_1^\dagger \\ & +id_2^\dagger)\ \frac{1}{2}(-ic_1^\dagger + c_2^\dagger - id_1^\dagger - d_2^\dagger)|0\rangle \\ = & \frac{1}{2}(ic_1^\dagger d_1^\dagger - c_1^\dagger d_2^\dagger + c_2^\dagger d_1^\dagger + ic_2^\dagger d_2^\dagger)|0\rangle. \end{aligned} \tag{4}$$

In the last step of Eq.(4), we have used $d_j^\dagger(E) = c_j(-E)$ (which effectively embodies the indistinguishability between an electron and a hole), where E is energy. From Eq.(4) one sees that the probability for two outgoing electrons (holes) after the scattering is zero. Exactly the *same* holds when two holes scatter simultaneously at leads 1 and 2. This is an interference effect in the same sense as the anti-bunching of fermions at a normal two port beam splitter, where fermions cannot exit through the same port. Instead of being in the spatial channels, here the anti-bunching is in the internal pseudospin space which has particle and hole as its two states. The unitary conversion of an electron to a hole, is, *per se*, not surprising in view of Refs.[6].

Of course, in a practical realization, the condition $E \sim E_M$ required for obtaining the scattering matrix S of Eq.(2) may not be exactly met. To see the effect of an energy mismatch, we denote by δE the amount by which E deviates (either positively or negatively) from E_M; this deviation is, however, assumed to be much lower than E_M itself (i.e., $\delta E << E_M$). Without assuming $\delta E << E_M$, one may end up in qualitatively different regimes: e.g., for δE comparable to $-E_M$, one reaches the regime of Ref.[8] of only crossed transmission. For $\delta E << E_M$, the scattering matrix as a function of δE is given by

$$S_{\delta E} = \frac{i\Gamma}{\delta E + i\Gamma}S + \frac{\delta E}{\delta E + i\Gamma}I \tag{5}$$

where $\Gamma = \Gamma_1 \sim \Gamma_2$ and I is the 4×4 identity matrix. In deriving Eq.(5), one ignores the second and higher powers of both $\delta E/E_M$ and Γ/E_M as $E_M >> \Delta E, \Gamma$. It is easy to check that, despite the above approximation, $S_{\delta E}$ is unitary; furthermore, Eq.(5) holds for any value of the ratio $\delta E/\Gamma$ as long as $E_M >> \Delta E, \Gamma$. Using $S_{\delta E}$, one readily obtains that the probability of observing an electron-electron output state becomes finite and equal to $\frac{(\delta E)^2}{(\delta E)^2+\Gamma^2}$, which, of course, vanishes when $E \sim E_M$.

So far, we have described the scattering as a process where one sends particles one by one through the leads at specific times. However, in practice, rather than controlling times, one could control the energies ϵ_1 and ϵ_2 of the particles in their respective leads, so as to make them behave indistinguishably when $\epsilon_1 \sim \epsilon_2$. Then, the standard way to observe the predicted fermion anti-bunching is through a measure of the correlations between the currents in leads 1 and 2. We assume the leads to support a discrete set of electronic energy states of density ν, so that the current in lead j is given by [18, 19]

$$I_j(t) = \frac{e}{h\nu}\sum_{\epsilon,\epsilon'} e^{i(\epsilon-\epsilon')t}\{a_j^\dagger(\epsilon)a_j(\epsilon') - b_j^\dagger(\epsilon)b_j(\epsilon')\} \quad (6)$$

where a_j and b_j denote the incoming and outgoing particles and ϵ and ϵ' are the energies of the particles. The spectral density of the current fluctuations $\delta I_j = I_j - \langle I_j\rangle$ between the leads at zero frequency is [19]

$$P_{ij} = \lim{}_{T\to\infty}\frac{h\nu}{T}\int_0^T dt\ \mathrm{Re}\langle\delta I_1(t)\delta I_2(0)\rangle. \quad (7)$$

Using $S_{\delta E}$ of Eq.(5) and considering an incoming two electron state $c_1^\dagger(\epsilon_1)c_2^\dagger(\epsilon_2)|0\rangle$, where $c_j^\dagger(\epsilon_j)$ denotes an electron of energy ϵ_j in lead j, one finds

$$P_{ij} = \frac{e^2}{h\nu}\frac{\Gamma^2}{\{(\delta E)^2+\Gamma^2\}^2}\{(\delta E)^2-\Gamma^2\}\delta_{\epsilon_1,\epsilon_2}, \quad (8)$$

where $\delta_{\epsilon_1,\epsilon_2}$ is the Kronecker delta function. Note that, when the incident electrons are distinguishable i.e., $\epsilon_1 \neq \epsilon_2$, then, as expected, $P_{ij} = 0$ since for an electron exiting one lead there could equally well be an electron or a hole exiting the other lead. When, instead, $\epsilon_1 = \epsilon_2$ (i.e., the particles are indistinguishable), then for $|\delta E| < |\Gamma|$, the domination of the electron-hole final state (as in Eq.(4)) makes $P_{ij} < 0$, which allows to the detect the predicted "anti-bunching" in pseudospin space. For $|\delta E| > |\Gamma|$ a process of amplitude $\Gamma\delta E$ in which only one of the electrons scatter, while the other remains in its lead, dominates; Fermi statistics now makes the electrons anti-bunch spatially (the more conventional antibunching [2, 19]), contributing to a positive P_{ij}. As in Ref.[8], our results are not inconsistent with those of Bolech and Demler [20], since their results apply when the energy of the incoming electrons is much higher than E_M.

So far our discussion has been confined to a spinless model, while for the promising implementations [12–15], the Majorana modes should involve superpositions of operators of different spins. For example, for a realization in a ferromagnet-s-wave superconductor-ferromagnet structure on a quantum spin-Hall edge [13], one has

$$\gamma_1 = \frac{1}{\sqrt{2}}(c_{1,\uparrow} - ic_{1,\downarrow} + ic_{1,\downarrow}^\dagger + c_{1,\downarrow}^\dagger)$$
$$\gamma_2 = \frac{1}{\sqrt{2}}(c_{2,\uparrow} + ic_{2,\downarrow} - ic_{2,\downarrow}^\dagger + c_{2,\downarrow}^\dagger), \quad (9)$$

where $c_{j,\sigma}$ creates an electron with spin σ in lead j. Defining the spin states $|\pm y\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm i|\downarrow\rangle)$, and using the basis $\{|e_{1,+y}\rangle, |e_{2,-y}\rangle, |h_{1,+y}\rangle, |h_{2,-y}\rangle, |e_{1,-y}\rangle, |e_{2,+y}\rangle, |h_{1,-y}\rangle, |h_{2,+y}\rangle\}$, the scattering matrix is found to be

$$S_{\mathrm{spinfull}} = \begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & S \end{pmatrix}, \quad (10)$$

where in (10), I and $\mathbf{0}$ are the 4×4 Identity and null matrices, while S is the scattering matrix given by Eq.(2).

When one uses S_{spinfull} to study the scattering of the incident state $c_{1,-y}^\dagger c_{2,+y}^\dagger|0\rangle$, one only needs the lower-right 4×4 block of S_{spinfull}. Thus, precisely the same electron-hole output state as in Eq.(4) is obtained, apart from the fact that, now, the spin indices $-y$ and $+y$ are pinned to the sites 1 and 2 respectively. Thus, by choosing the spin polarizations of the incoming electrons pertinently, one can observe *all* the effects described till now. This should be possible in a variety of systems as Majorana modes of the form given by Eq.(9) are quite generic, e.g., also realizable in semiconductor-superconductor-magnet structures [15].

As a brief aside, we point out that, when one electron and one hole scatter at sites 1 and 2 respectively, for $\delta E << \Gamma$, the incoming state $c_1^\dagger d_2^\dagger|0\rangle$ evolves to $\frac{1}{2}(-c_1^\dagger c_2^\dagger - ic_1^\dagger d_1^\dagger + ic_2^\dagger d_2^\dagger - d_1^\dagger d_2^\dagger)|0\rangle$, implying the interferometric vanishing of the probability of one outgoing electron and one outgoing hole in separate leads.

Next, we propose a protocol for the generation of entanglement between spins of well separated particles incoming at site 1 and at site 2. For this purpose, we choose the realization of Majorana fermions given by Eq.(9) and make two electrons with parallel spins in the $\uparrow$ direction come in *simultaneously* i.e., choose the initial state $c_{1,\uparrow}^\dagger c_{2,\uparrow}^\dagger|0\rangle$. Then, using S_{spinfull}, one gets

$$c_{1,\uparrow}^\dagger c_{2,\uparrow}^\dagger|0\rangle \xrightarrow{MBS} \frac{1}{4}(c_{1,\uparrow}^\dagger c_{2,\uparrow}^\dagger - c_{1,\downarrow}^\dagger c_{2,\downarrow}^\dagger + 2c_{1,\uparrow}^\dagger c_{2,\downarrow}^\dagger$$
$$+ \quad ...)|0\rangle, \quad (11)$$

where ... denotes terms such as $c_{1,\sigma}^\dagger c_{1,\sigma'}^\dagger, c_{2,\sigma}^\dagger c_{2,\sigma'}^\dagger, c_{j,\sigma}^\dagger d_{k,\sigma'}^\dagger$ and $d_{j,\sigma}^\dagger d_{k,\sigma'}^\dagger$, which are not relevant to our discussion. Eq.(11) implies that, when two outgoing electrons are obtained in leads 1 and 2 separately, their state is $|\xi\rangle_{12} = \frac{1}{\sqrt{6}}(|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2 + 2|\uparrow\rangle_1|\downarrow\rangle_2)$ where, as it is usually done [5, 19], one uses the lead label to label the electron. $|\xi\rangle_{12}$ is an entangled state of the spins of electrons 1 and 2, with the amount of entanglement (as quantified by the von Neumann entropy of one of the particles [22]) being 0.19 ebits. Though the entanglement is not very high, $|\xi\rangle_{12}$ is a pure state, and hence of value in quantum information, as its entanglement can be concentrated without loss by local means [22]. Moreover, the probability of obtaining two outgoing electrons in separate leads (i.e., $|\xi\rangle_{12}$) is rather high, namely 3/8. At the expense of decreasing this probability, one may improve the degree of entanglement of the generated state by tuning the polarizations of the incoming electrons. For instance, if the incoming state is $(\frac{1}{\sqrt{10}}c^{\dagger}_{1,+y} + \frac{3}{\sqrt{10}}c^{\dagger}_{1,-y})(\frac{3}{\sqrt{10}}c^{\dagger}_{2,+y} + \frac{1}{\sqrt{10}}c^{\dagger}_{2,-y})|0\rangle$, one obtains an output state of entanglement 0.75 ebits, while the probability of the generation this state becomes 0.055. The spin entanglement of the outgoing electrons could be measured by passing them through separate spin filters as in Ref. [21].

Unlike the entanglement generation scheme of Ref. [5], here particles polarized parallel to each other suffice to generate entanglement. Importantly, in our protocol, particles at a distance from each other can be made entangled; this may avoid the decoherence arising necessarily from the transport needed to separate the particles after a local entangling mechanism. In addition, the distance between the entangled particles can be enhanced by putting n copies of our setup in series with leads connecting the end of one copy to the beginning of another. The probability of obtaining the state $|\xi\rangle_{12}$ in the leftmost and rightmost leads will then be $(3/8)^n$.

One simple setting where the non-local two particle interferometry and the entanglement generation between distant electrons from MBS may be observed can be engineered with magnet-superconductor-magnet junctions deposited on the edge of a 2D quantum spin Hall insulator [8, 13]. Just as in Ref.[8], one can observe these effects when the Majorana modes are separated by several micrometers at temperatures of the order of 10 mK. For this setting, the explicit form of the Majorana operators is exactly the same as in Eq.(9) [13]. Interestingly, strong spin-orbit coupled quantum wires in proximity with ferromagnets and superconductors also support the realization of MBS [14, 15] given in Eq.(9) [15]. As in previous proposals [13, 14, 20], also in our settings, two STM tips could act as the leads 1 and 2 to observe the non-local two particle Hanbury Brown-Twiss interferometry. For the entanglement generation, instead, it will be more useful to have synchronized electron pumps [23] feeding in the incoming electrons. In addition, the filtering of the desired state $|\xi\rangle_{12}$ can be achieved by pumps capturing exactly one outgoing electron from each Majorana bound state.

In this paper, we showed that a one dimensional device with two Majorana bound states at its ends yields a Hanbury-Brown Twiss effect in the internal electron-hole pseudospin space which may be detected in realistic condensed matter settings through current-current correlations. This is a departure from *all* the known multi-particle interference effects which have manifested themselves in spatial bunching and antibunching or spin-spin correlations. Fundamentally, it can be regarded as a manifestation of the quantum indistinguishability between electronic annihilation and hole creation evidencing the presence of Majorana bound states. The same settings may also be used to engineer a non-local entangler of distant electronic spins, which may enable circumventing the decoherence arising from the transport needed to separate entangled particles.

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