An equation for the Ramsey number $R(p_1, p_2, \cdots p_t; r)$

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Abstract:The Ramsey number $R(p_1, p_2, \cdots p_t; r)$ *is a valve value such that as long as the cardinality n of the n-set V is no less than R,however all the* $\binom{n}{r}$ *r-subsets of V are distributed into t boxes, V will always have a property W.Thus, by calculating the number of ways of distribution of r-subsets that makes W true,one can get an equation for* $R(p_1, p_2, \cdots p_t; r)$ *.The evaluation of the general term in this eq. and the counting of the frequencies of occurrence of the various values the general term takes are merely a matter of elementary counting.*

Contents:

0 Notation

The event "All the $\binom{p_i}{r}$ *r-subsets of the* j^{th} p_i*-subset are in the* i^{th} *box." is denoted by* A_{ij}*.* A_{ij} *sometimes also means the* j^{th} p_i*-subset itself.*$(1 \leqslant i \leqslant t, 1 \leqslant j \leqslant \binom{n}{p_i}; p_i \geqslant r)$

The number of distributions of $\binom{n}{r}$ *r-subsets into t boxes that makes event X true is denoted by N（X）.*

Tr(X) represents the family consisting of all the r-subsets of the set X.

|X| means the cardinality of the set X.

Because f will usually be a very complicated expression,I shall write t^f *as txp(f),just like sometimes* e^x *being written exp(x).However,* t^f *will still appear if convenient.*

Unless otherwise stated,all the letters appear in this paper is understood to be non-negative integers.

1 Description of the problem

The most general (finite) Ramsey number $R(p_1, p_2, \dots p_t; r)$ *is defined by the following existence theorem:*

$R(p_1, p_2, \dots p_t; r)$ *is the smallest integer n that has the following property : Set V has n elements.However all the* $\binom{n}{r}$ *r-subsets of V is distributed into t boxes,the following proposition (event) W will always be true: There always exists a* p_1*-subset, all* $\binom{p_1}{r}$ *r-subsets of which are in box 1;or there always exists a* p_2 *-subset, all* $\binom{p_2}{r}$ *r-subsets of which are in box 2;or...or there always exists a* p_t*-subset, all* $\binom{p_t}{r}$ *r-subsets of which are in box t.*

2 The basic eq. for $R(p_1, p_2, \dots p_t; r)$

Denoting by N(X) the number of ways of distribution of r-subsets that makes event X true,we see that the preceding definition amounts to the statement that all the $t\mathrm{xp}(\binom{n}{r})$ *ways of distribution of* $\binom{n}{r}$ *r-subsets into t boxes will make W true,i.e.*

$N(W)=t\mathrm{xp}(\binom{n}{r})$*.Further denoting by* A_{ij} *the event "All the* $\binom{p_i}{r}$ *r-subsets of the* j^{th} p_i*-subset are in the* i^{th} *box." (*A_{ij} *sometimes also means the* j^{th} p_i*-subset itself.* $1\leqslant i\leqslant t, 1\leqslant j\leqslant \binom{n}{p_i}$ *;*$p_i \geqslant r$*),we can,from the definition of W,express W as*

$W=\bigcup_{i=1}^{t}\bigcup_{j=1}^{\binom{n}{p_i}} A_{ij}$ *,so from the inclusion/exclusion principle we get the basic eq:*

$$N(W)=N\left(\bigcup_{i=1}^{t}\bigcup_{j=1}^{\binom{n}{p_i}} A_{ij}\right)$$

$$= \sum_{k=1}^{\binom{n}{p_1}+\binom{n}{p_2}+\cdots+\binom{n}{p_t}} (-1)^{k-1} \sum_{(1,1)\le(i_1,j_1)<(i_2,j_2)<\cdots<(i_k,j_k)\le(t,\binom{n}{p_t})} N(A_{i_1j_1}\cap A_{i_2j_2}\cap\cdots\cap A_{i_kj_k})$$

$$= t^{\binom{n}{r}} \qquad (1)$$ [1]

Next we are essentially doing the following two things:(i)Calculate what values can the general term $N(A_{i_1j_1}\cap A_{i_2j_2}\cap\cdots\cap A_{i_kj_k})$ *in eq.(1) take and (ii) Calculate the frequencies of occurrence of the various values.If these can be worked out,then* $R(p_1,p_2,\cdots p_t;r)$ *is just the smallest positive integer that satisfies eq.(1).*

3 Evaluation of the general term in the basic eq.

Let's concentrate on (i) first.We remember that $N(A_{i_1j_1}\cap A_{i_2j_2}\cap\cdots\cap A_{i_kj_k})$ *represents the number of ways of distribution of* $\binom{n}{r}$ *r-subsets into t boxes that makes k events* $A_{i_1j_1}\cdots A_{i_kj_k}$ *simultaneously true.In order that these k events hold simultaneously,we have to and only have to do the following:Put accordingly the* $\left|\bigcup_{s=1}^{k}Tr(A_{i_sj_s})\right|$ *r-subsets contained by these k sets*[2]*into the boxes specified by the k events respectively.After doing this,we achieve the objective that the k events hold simultaneously. The remaining* $\binom{n}{r}-\left|\bigcup_{s=1}^{k}Tr(A_{i_sj_s})\right|$ *r-subsets can then be arbitrarily put into the t boxes.The resulting* $t\,xp\left[\binom{n}{r}-\left|\bigcup_{s=1}^{k}Tr(A_{i_sj_s})\right|\right]$

ways of distribution all make the k events simultaneously true.In other words,we have obtained by the preceding argument that(the second equality in eq.(2)comes from the principle of inclusion and exclusion)

$$N(A_{i_1j_1}\cap A_{i_2j_2}\cap\cdots\cap A_{i_kj_k})=txp\left[\binom{n}{r}-\left|\bigcup_{s=1}^{k}Tr(A_{i_sj_s})\right|\right]$$

$$=txp\left[\binom{n}{r}+\sum_{s=1}^{k}(-1)^s\sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k}\left|\bigcap_{m=1}^{s}Tr(A_{i_{\lambda_m}j_{\lambda_m}})\right|\right]$$

$$=txp\left[\binom{n}{r}+\sum_{s=1}^{k}(-1)^s\sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k}\binom{p_{i_{\lambda_1}i_{\lambda_2}\cdots i_{\lambda_s}}}{r}\right](2)$$

The 3th equality in eq.(2) uses the fact: $\bigcap_{m=1}^{s}Tr(A_{i_{\lambda_m}j_{\lambda_m}})=Tr\ (\bigcap_{m=1}^{s}A_{i_{\lambda_m}j_{\lambda_m}})$.

This means that the common r-subsets of the s sets $A_{i_{\lambda_m}j_{\lambda_m}}\quad(1\le m\le s)$ *can only*

be the r-subsets of their intersection.Since the elements of the common r-subsets must be contained in all the s sets,therefore belong to their intersection.From this fact, it is then easy to see that[3]

$$\left|\bigcap_{m=1}^{s}Tr(A_{i_{\lambda_m}j_{\lambda_m}})\right|=\left|Tr\ (\bigcap_{m=1}^{s}A_{i_{\lambda_m}j_{\lambda_m}})\right|=\binom{\left|\bigcap_{m=1}^{s}A_{i_{\lambda_m}j_{\lambda_m}}\right|}{r}\triangleq\binom{p_{i_{\lambda_1}i_{\lambda_2}\cdots i_{\lambda_s}}}{r}$$

We shall not be too happy after obtaining eq.(2),because there is still one case that shall not be overlooked.Only after this case is considered do we really finish our task (i).The case is that eq.(2) is valid only for those "compatible" k-event (unordered) tuples,namely the k-event tuples whose k events can be simultaneously satisfied.There do exist k-event tuples whose k events cannot be made true simultaneously ,because two (or more) of the k events(sets) may have common r-subsets,and they demand that *the common r-subsets be put into two(or more) different boxes.Obviously, no distribution of r-subsets can satisfy this requirement,since the common r-subsets can only go to one box.(or,the partition of r-subsets into classes require that any two of the classes be disjoint.)So, for this kind of k-set tuples, their corresponding N's are simply zero,not given by eq.(2).Thus,only those k-event tuples are "compatible",any two events(sets) of which have no common r-subsets,unless these two events require their common r-subsets be put into the same box.Having no common r-subsets,the two sets must and only have to have no more than r-1 common elements.Therefore,we get the necessary and sufficient condition for k-event tuple compatibility,which reads:*

For any two sets $A_{i_\mu j_\mu}$ *and* $A_{i_\nu j_\nu}$ *chosen arbitrarily from the k sets,the nonnegative integer* $p_{i_\mu i_\nu} \equiv \left| A_{i_\mu j_\mu} \cap A_{i_\nu j_\nu} \right|$ *must*

$$\le r\text{-}1+\delta_{i_\mu i_\nu}(p_{i_\mu} - r+1)=r\text{-}1 \quad \text{if } i_\mu \neq i_\nu, \textit{i.e. if these two events demand their common r-subsets be put into two different boxes.}$$

$$\text{OR } = p_{i_\mu} \quad \text{if } i_\mu = i_\nu, \textit{i.e. if these two events demand their common r-subsets be put into the same box.}$$

This compatibility condition will be used in p10 of this paper where we shall determine the range of summation in the basic eq.(1).

4 Counting the frequency of the various values (non-zero)of the general term

Now we handle task (ii) mentioned in p3.For expository convenience,let's call the $p_{i_{\lambda_1} i_{\lambda_2} \cdots i_{\lambda_s}}$ *'s appearing in eq.(2) the intersection spectrum of the k sets*

$$A_{i_1 j_1} \cdots A_{i_k j_k} \ (1 \le \lambda_1 < \lambda_2 < \cdots < \lambda_s \le k, 1 \le s \le k, \binom{k}{1}+...+\binom{k}{k}=2^k\text{-}1 \text{ p's altogether}) .$$

Obviously,if the intersection spectrum is given,i.e. if a specific set of the p's is given,the non-zero value $txp\left[\binom{n}{r} + \sum_{s=1}^{k}(-1)^s \sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k} \binom{p_{i_{\lambda_1} i_{\lambda_2} \cdots i_{\lambda_s}}}{r}\right]$

(a function of the p's)of the general term $N(A_{i_1 j_1} \cap A_{i_2 j_2} \cap \cdots \cap A_{i_k j_k})$ *will*

also be determined.So,the problem to determine the frequency of occurrence of this value (in the summation with regard to all the k-event tuples in eq.(1)) is transformed into the problem to determine how many k-event tuples are there whose intersection spectrum equals the intersection spectrum appearing in this value.[4]

To solve the transformed problem,consider the Venn diagram of the k sets $A_{i_1 j_1} \cdots A_{i_k j_k}$ *.In this diagram, the n-set V is divided into* 2^k *disjoint parts.Let's use the* 2^k *k-digit binary numbers to represent the* 2^k *disjoint parts,the* m^{th} *digit of any one of these numbers being 0 or 1 depending on whether the corresponding part is contained in* $A_{i_m j_m}$ *.The cardinality of the part represented by the binary number B will be denoted by* q_B*,and the* 2^k *q's will be called the Venn spectrum of the k sets.We recognize immediately that the Venn spectrum and the intersection spectrum of the k sets are in 1-1 correspondence,since,upon looking at the diagram,we easily see that it suffices to use only addition to get the intersection spectrum from the Venn spectrum; while one can exploit the inclusion/exclusion principle to do the reverse:*

$$q_{11\ldots1} = p_{i_1 i_2 \cdots i_k} ;$$

$$q_{01\ldots1} = p_{i_2 \cdots i_k} - p_{i_1 i_2 \cdots i_k}, \cdots, q_{1\ldots10} = p_{i_1 \cdots i_{k-1}} - p_{i_1 i_2 \cdots i_k} ;$$

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$$q_{(\text{the k-digit binary number whose } \nu_1^{th},\ \nu_2^{th} \ldots \nu_{k-f}^{th} \text{ digits are 1, the other f digits are 0})}$$

$$= p_{i_{\nu_1} i_{\nu_2} \cdots i_{\nu_{k-f}}} + \sum_{s=1}^{f} (-1)^s \sum_{\substack{1 \le \lambda_1 < \lambda_2 < \cdots < \lambda_s \le k \\ \lambda_m \ne \nu_q (\forall m=1\ldots s, \forall q=1\ldots k-f) \\ \text{there are } \binom{f}{s} \text{ terms in this summation}}} p_{i_{\nu_1} i_{\nu_2} \cdots i_{\nu_{k-f}} i_{\lambda_1} i_{\lambda_2} \cdots i_{\lambda_s}}$$

;

……

$$q_{00\ldots0} = n + \sum_{s=1}^{k} (-1)^s \sum_{1 \le \lambda_1 < \lambda_2 < \cdots < \lambda_s \le k} p_{i_{\lambda_1} i_{\lambda_2} \cdots i_{\lambda_s}} .$$

(*In fact,we have a stronger result:the 2^k disjoint parts and the k sets are in 1-1 correspondence.This result is also obvious.*)

Using this 1-1correspondence,we can further transform the original problem "what is the frequency of occurrence of the value

$$txp\left[\binom{n}{r}+\sum_{s=1}^{k}(-1)^s\sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k}\binom{p_{i_{\lambda_1}i_{\lambda_2}\cdots i_{\lambda_s}}}{r}\right]$$ *of the general term*

$N(A_{i_1j_1}\cap A_{i_2j_2}\cap\cdots\cap A_{i_kj_k})$ *?"into a new one "How many k-event tuples are there whose Venn spectrum equals the Venn spectrum corresponding to the intersection spectrum appearing in this value? "The solution of the latter problem seems to be given by(using the stronger result mentioned above)the number of ways of partition of V into 2^k disjoint parts,whose cardinalities are specified by the Venn spectrum corresponding to the intersection spectrum appearing in the value.i.e.The solution is*

$$n!/\prod_{\text{all } 2^k \text{ k-digit binary number B}} q_B=\frac{n!}{p_{i_1i_2\cdots i_k}!}\times$$

$$\frac{1}{\prod_{f=1}^{k-1}\prod_{1\le\nu_1<\nu_2<\cdots<\nu_{k-f}\le k}[p_{i_{\nu_1}i_{\nu_2}\cdots i_{\nu_{k-f}}}+\sum_{s=1}^{f}(-1)^s\sum_{\substack{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k\\ \lambda_m\ne\nu_q(\forall m=1\ldots s,\forall q=1\ldots k-f)\\ \text{there are }\binom{f}{s}\text{ terms in this summation}}}p_{i_{\nu_1}i_{\nu_2}\cdots i_{\nu_{k-f}}i_{\lambda_1}i_{\lambda_2}\cdots i_{\lambda_s}}]!}$$

$$\times\frac{1}{(n+\sum_{s=1}^{k}(-1)^s\sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k}p_{i_{\lambda_1}i_{\lambda_2}\cdots i_{\lambda_s}})!}\qquad(4)$$

However,the above solution is not correct.The number of ways of partition of V given by (4) is in general ***larger*** *than the number of unordered k-event tuples corresponding to a particular intersection spectrum due to the occurrence of permutation symmetries【or,more plainly,eq.(4) counts the number of* ***ordered*** *2^k -part tuples.In usual case, this is correct:ordered 2^k -part tuples and unordered k-set tuples are in 1-1 correspondence(see the stronger result at the top of this page).However,when the cardinalities of some of the 2^k parts (these parts have the* *same number of 1's in their binary representation,so that they are of the same type)are equal(i.e.when some numbers in the Venn spectrum of the k sets are equal),this 1-1correspondence may break down:under some appropriate cyclic permutation of these parts,one can get the same unordered k-set tuple.】 The appearance of permutation symmetries are caused by the "coincidence" that some of the k sets are "on an equal footing".The sets* (i_μ, j_μ) *and* (i_ν, j_ν) *are said to be on an equal footing iff they intersect with the other sets in the same fashion.More accurately,let's define the "symmetry discriminant" as follows:*

$$\delta(i_\mu, i_\nu) = \delta_{i_\mu i_\nu} \prod_{\substack{\lambda=1 \\ \lambda \neq \mu,\ \nu}}^{k} \delta_{p_{i_\mu i_\lambda},\ p_{i_\nu i_\lambda}} \prod_{\substack{1 \le \lambda < \xi \le k \\ \lambda,\ \xi \neq \mu,\ \nu}} \delta_{p_{i_\mu i_\lambda i_\xi},\ p_{i_\nu i_\lambda i_\xi}} \cdots$$

$$\cdots \prod_{\substack{1 \le \lambda_1 < \lambda_2 < \cdots < \lambda_{k-2} \le k \\ \lambda_\alpha \neq \mu,\ \nu\ (\alpha = 1 \cdots k-2)}} \delta_{p_{i_\mu i_{\lambda_1} \cdots i_{\lambda_{k-2}}},\ p_{i_\nu i_{\lambda_1} \cdots i_{\lambda_{k-2}}}} \qquad (5)$$

When $\delta(i_\mu, i_\nu) = 1$*,i.e.when the values of the p's satisfy the following conditions(*2^{k-2} *equations altogether)*

$$i_\mu = i_\nu (p_{i_\mu} = p_{i_\nu}), p_{i_\mu i_\lambda} = p_{i_\nu i_\lambda} (\lambda = 1 \ldots k; \lambda \neq \mu, \nu)$$

$$p_{i_\mu i_\lambda i_\xi} = p_{i_\nu i_\lambda i_\xi} (\lambda, \xi = 1 \ldots k; \lambda, \xi \neq \mu, \nu; \lambda < \xi) \cdots$$

$$p_{i_\mu i_{\lambda_1} \cdots i_{\lambda_{k-2}}} = p_{i_\nu i_{\lambda_1} \cdots i_{\lambda_{k-2}}} (1 \le \lambda_1 < \cdots < \lambda_{k-2} \le k; \lambda_\alpha \neq \mu, \nu \quad \alpha = 1 \cdots k-2)$$

,the sets (i_μ, j_μ) *and* (i_ν, j_ν) *intersect with the other sets in the same fashion,so that they are on an equal footing.The* ***simultaneous*** *interchanges of the various corresponding disjoint parts of the two sets(the parts contained in their intersection being fixed) generate* ***2*** *ways of partition of V,which differ only in a permutation of the 2 sets* (i_μ, j_μ) *and* (i_ν, j_ν) *and hence correspond to the same unordered k-set tuple* $(A_{i_1 j_1}, A_{i_2 j_2}, \cdots, A_{i_k j_k})$*,indicating that (4) needs to be further*

divided by 2! to give the number of unordered k-event tuples corresponding to a particular intersection spectrum .In general,the case "y(y>2)sets are on an equal footing "can also occur.The condition for such a case to occur can be obtained by replacing the μ and ν in the above 2^{k-2} equations by μ_1, μ_2, ... μ_y,and demand that those 2^{k-2} equations hold for any pair (μ_i, μ_j) .When such a case occur,one can appropriately cyclic permuting the corresponding parts of the y sets simultaneously.If the net effect of this operation is just a permutation among the y sets,then the new after-operation partition of V still corresponds to the same unordered k-event tuple as the before-operation one.The number of such kind of permutation among the y sets is y!,so expression (4) needs to be further divided by y! to give the desired frequency of occurrence of a particular value.

I've talked a lot in the above,just want to point out that in some case eq.(4) needs to be further multiplied by a symmetry factor to give the desired frequency of occurrence of a particular value of the general term in the basic eq.(1).In general,the symmetry factor has a more complex form than that of 1/y!,and can be calculated according to the following algorithm:

(a) For a given unordered k-set tuple (a given intersection spectrum,i.e. a given set of values of the 2^k-1 p's),calculate for every two of the k sets the "symmetry discriminant"defined in eq.(5) and obtain altogether $\binom{k}{2}$ symmetry discriminants.[5]

(b) Two sets are "on an equal footing" iff their symmetry discriminant equals 1.Obviously the relation "being on an equal footing"is an equivalence relation that can be used to classify these k sets.Assume the k sets is classified into m classes,there being k_α sets in the α^{th} class. (α =1...m ; $\sum_{\alpha=1}^{m} k_\alpha = k$)

(c) The symmetry factor that need to be multiplied for these k sets (for this set of values of the 2^k-1 p's) is $\dfrac{1}{\prod_{\alpha=1}^{m} k_\alpha !}$ .For every set of values of the 2^k-1 p's that is allowed in the sum $\sum_{1\le i_1\le i_2\le\cdots\le i_k\le t}\sum_{\{p\}}$,the corresponding symmetry factor should be calculated according to steps (a) — (c) . (6)

5 Dummy varibles and the range of summation in the basic eq.

We have known the various values the general term $N(A_{i_1j_1}\cap A_{i_2j_2}\cap\cdots\cap A_{i_kj_k})$ in the basic eq.(1) can take and the frequency of *occurrence of these values. What we should do next is to sum them up, writing eq.(1) into the following form:*

$$\sum_{k=1}^{\binom{n}{p_1}+\binom{n}{p_2}+\cdots+\binom{n}{p_t}} (-1)^{k-1} \sum_{\text{value}} \text{value of } N(A_{i_1 j_1} \cap \cdots \cap A_{i_k j_k}) \times \text{frequency this value occurs}$$

$$= t^{\binom{n}{r}}$$

The various values are completely determined by the intersection spectrum of the k sets $A_{i_1 j_1} \cdots A_{i_k j_k}$ *(i.e. the value is a function of the* 2^k*-1 p's, see eq.(2)), hence the second summation in the above eq. with respect to (w.r.t) value can be written as a summation w.r.t intersection spectrum (i.e. a* 2^k*-1 fold summation with respect to all possible combinations of values of the* 2^k*-1 p's) In other words, we will write* $\sum_{\text{value}}$

as $\sum_{1\le i_1 \le i_2 \le \cdots \le i_k \le t} \sum_{\{p\}}$ *. Here, the first* $\sum_{1\le i_1 \le i_2 \le \cdots \le i_k \le t}$ *is just the* i_α *part of the*

$\sum_{(1,1)\le(i_1,j_1) < (i_2,j_2) < \cdots < (i_k,j_k) \le (t,\binom{n}{p_t})}$ *(which sums all* (i_α, j_α) *) in eq.(1). Alternatively speaking, it represents the sum w.r.t all possible combinations of values of the* $\binom{k}{1}$ p_{i_μ} *'s of the k sets.* $1 \le \mu \le k$ *[6]; the second* $\sum_{\{p\}}$ *represents the sum w.r.t the* $\binom{k}{2}$ $p_{i_\mu i_\nu}$ *'s;* $\binom{k}{3}$ $p_{i_\mu i_\nu i_\lambda}$ *'s;* $\binom{k}{4}$ $p_{i_\mu i_\nu i_\lambda i_\xi}$ *'s...* $\binom{k}{k}$ $p_{i_1 i_2 \cdots i_k}$ *, in which* $p_{i_\mu i_\nu}$ *is summed from 0 to* $r-1+\delta_{i_\mu i_\nu}(p_{i_\mu}-r+1)$ *;* $p_{i_\mu i_\nu i_\lambda}$ *from 0 to* $\min(p_{i_\mu i_\nu}, p_{i_\nu i_\lambda}, p_{i_\lambda i_\mu})$ *;*

$p_{i_\mu i_\nu i_\lambda i_\xi}$ *from 0 to* $\min(p_{i_\mu i_\nu i_\lambda}, p_{i_\nu i_\lambda i_\xi}, p_{i_\lambda i_\xi i_\mu}, p_{i_\xi i_\mu i_\nu})$ ... $p_{i_1 i_2 \cdots i_k}$ *from 0 to* $\min_f \{p_{i_1 \cdots i_{f-1} i_{f+1} \cdots i_k}\}_{f=1}^{k}$ *. The other ranges of summation are easy to* *understand,but why do we sum* $p_{i_\mu i_\nu}$ *from 0 to* $r-1+\delta_{i_\mu i_\nu}(p_{i_\mu}-r+1)$ *?The compatibility condition we derived in p5 explains this.The validity of eq.(2) (the value)entails that any 2 of the k sets,say* $A_{i_s j_s}$ *and* $A_{i_q j_q}$ *, must not have common r-subsets when* $i_\mu \neq i_\nu$ *,i.e. the cardinality of their intersection* $p_{i_\mu i_\nu}$ *must not exceed r-1.If* $p_{i_\mu i_\nu}$ *exceed r-1,eq.(2) is no longer valid and must not be added into the sum,at that time what should be added into the sum is actually a zero.(and hence don't need to be added at all)Hence,when* $i_s \neq i_q$ *,the range of summation w.r.t* $p_{i_\mu i_\nu}$ *is from 0 to r-1.When* $i_\mu = i_\nu$ *,however,* $A_{i_\mu j_\mu}$ *and* $A_{i_\nu j_\nu}$ *can have common r-subsets,so we sum* $p_{i_\mu i_\nu}$ *from 0 to its greatest possible value,* p_{i_μ} *.*

6 Conclusion

Now,we know the various values the general term in the basic eq.(1) can take,the frequencies of occurrence of these values and the dummy varibles and the range of summation in eq.(1).If we put all these ingredients into eq.(1),we get finally that the Ramsey number $R(p_1,p_2,\cdots p_t;r)$ *is the smallest positive integer* n *that satisfies the following equation:*

$$\sum_{k=1}^{\binom{n}{p_1}+\binom{n}{p_2}+\cdots+\binom{n}{p_t}} (-1)^{k-1} \sum_{1\le i_1\le i_2\le\cdots\le i_k\le t} \sum_{\{p\}} \mathrm{sym}(\{p\}) \times \frac{n!}{p_{i_1 i_2 \cdots i_k}!} \times$$

$$\frac{1}{\underbrace{\prod_{f=1}^{k-1} \prod_{1\le\nu_1<\nu_2<\cdots<\nu_{k-f}\le k}}_{\text{this is a } \binom{k}{f}\text{-factor product}} [p_{i_{\nu_1} i_{\nu_2}\cdots i_{\nu_{k-f}}} + \sum_{s=1}^{f}(-1)^s \underbrace{\sum_{\substack{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k \\ \lambda_m\neq\nu_q\ (\forall m=1\ldots s,\ \forall q=1\ldots k-f)}}}_{\text{there are } \binom{f}{s} \text{ terms in this summation}} p_{i_{\nu_1} i_{\nu_2}\cdots i_{\nu_{k-f}} i_{\lambda_1} i_{\lambda_2}\cdots i_{\lambda_s}}]!}$$

$$\times \frac{1}{(n+\sum_{s=1}^{k}(-1)^s \sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k} p_{i_{\lambda_1} i_{\lambda_2}\cdots i_{\lambda_s}})!}\cdot$$

$$\cdot txp\left[\binom{n}{r}+\sum_{s=1}^{k}(-1)^s \sum_{1\le\lambda_1<\lambda_2<\cdots<\lambda_s\le k}\binom{p_{i_{\lambda_1} i_{\lambda_2}\cdots i_{\lambda_s}}}{r}\right]$$

$$= t^{\binom{n}{r}} \qquad (7)$$

Here,the symmetry factor $\mathrm{sym}(\{p\})$ *corresponding to every intersection spectrum (i.e. every particular set of values of the* 2^k*-1 p's)in the summation should be calculated according to algorithm(6).*

7 Some remarks

Obviously eq.(15) cannot be an algebraic equation,since it is satisfied by any natural number n$\geqslant R(p_1,p_2,\cdots p_t;r)$ *.Maybe cancelling the huge common factor* $t^{\binom{n}{r}}$ *from both sides of eq.(15) can do a favor to solving it numerically;however it is not hard to imagine that even to verify whether a given integer is a solution is impossible to perform within polynomial time.Nevertheless,to give an optimal lower bound of* $R(p_1,p_2,\cdots p_t;r)$ *by using eq.(15) or other theoretical methods,and then to substitute every integer no less than this bound into eq.(15) seems the only method to find the value of* $R(p_1,p_2,\cdots p_t;r)$ *.*

8 Notes

[1] Here I assume some method has been devised to order the various p_i*-subsets(the various pairs* (i_α, j_α) *).For example,* $(i_1,j_1) < (i_2,j_2)$ *if* $i_1 < i_2$ *or* $i_1=i_2$ *and* $j_1 < j_2$ *.*

[2]A_{ij} *represents either set or event.This ambiguity will not bring confusion.On the contrary,it brings expository convenience.*

[3]Here, $p_{i_{\lambda_1} i_{\lambda_2} \cdots i_{\lambda_s}} \triangleq \left| \bigcap_{m=1}^{s} A_{i_{\lambda_m} j_{\lambda_m}} \right|$, *the more delicate notation*

$p_{i_{\lambda_1} j_{\lambda_1} i_{\lambda_2} j_{\lambda_2} \cdots i_{\lambda_s} j_{\lambda_s}}$ *is not used,because we will see later that these p's will eventually appear in a sum,becoming dummy varibles.It already suffices to use a row of i's as suffixes to distinguish different dummy varibles; a row of j 's do not appear because their appearance will fix the value of the dummy p's ,while,in a summation, they should not be fixed.*

[4]It is possible that two different intersection spectrum gives the same value,but this point is not important.

[5]I know according to (b),actually we don't need so much computation.

[6]Note that the value(eq.(2)) is dependent on these $\binom{k}{1}$ p_{i_μ} *'s.*

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