Noncoherent Interference Alignment: Trade Signal Power for Diversity Towards Multiplexing

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Abstract—This paper proposes the first known universal interference alignment scheme for general $(1 \times 1)^K$ interference networks, either Gaussian or deterministic, with only 2 symbol extension. While interference alignment is theoretically powerful to increase the total network throughput tremendously, no existing scheme can achieve the degree of freedom upper bound exactly with finite complexity. This paper starts with detailed analysis of the diagonality problem of naive symbol extension in small $(1 \times 1)^3$ networks, a technique widely regarded as necessary to achieve interference alignment with insufficient diversity. Then, a joint bandpass noncoherent demodulation and interference alignment scheme is proposed to solve the diagonality problem by trading signal power for increased system diversity, which is further traded for multiplexing improvement. Finally, the proposed noncoherent interference alignment scheme is extended to general $(1 \times 1)^K$ cases and is proven to achieve the degree of freedom upper bound exactly. Simulation results verify the correctness and powerfulness of the proposed scheme and show significant degree of freedom improvement compared to the conventional orthogonal transmission scheme.

Index Terms—Interference alignment, noncoherent transmission, bandpass modulation, degree of freedom.

I. INTRODUCTION

THE exact capacity region of the general interference network has been an open problem to information theorists for decades. Even for the two-user case, capacity region is only known for special cases such as those with strong and very strong interference [1], [2]. The best known result for the general two-user Gaussian interference network can determine the capacity region within 0.5 bit for real cases or 1 bit for complex cases [3], [4] by using a modified version of the Han-Kobayashi scheme [5].

For the general K-user interference network, where $K > 2$, most research focused on the degree of freedom (DoF) region [6], [7], which characterizes the capacity scaling behavior with respect to the signal-to-noise ratio (SNR). In [7], Cadambe and Jafar showed that the sum capacity of the general $(1 \times 1)^K$ interference network can be approximated as

$$C(\text{SNR}) = \frac{K}{2} \log(\text{SNR}) + o(\log(\text{SNR})), \qquad (1)$$

where $f(x) = o(g(x))$ denotes $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$. The DoF characterization $\frac{K}{2}$, which is also known as the multiplexing gain, becomes increasingly accurate as $o(\log(\text{SNR}))$ tends to be negligible compared to $\frac{K}{2} \log(\text{SNR})$ in the high SNR regime.

The scheme used in [7] to achieve the DoF upper bound is interference alignment, which controls the interference contamination such that all interference signals are aligned into a certain signal subspace and leaves the remaining signal subspace interference-free for desired signal. Equation (1) implies that on average, each user can almost achieve half the rate as if there were no interference at all, no matter how many of them share the resource. Thus, in the high SNR regime, the sum capacity scales linearly with the number of users.

Prosperous research works follow to construct interference alignment solutions using various techniques [8]–[15] and to apply similar ideas to several different applications [16], [17]. However, although being theoretical powerful, interference alignment may not be feasible for certain network configurations. In [18], the feasibility conditions for interference alignment were analyzed. The interference alignment problem was viewed as a multivariate polynomial system, and a $(M \times N)^K$ interference network[1] is feasible to achieve interference alignment without symbol extension only if

$$M + N \geqslant (K + 1)d, \qquad (2)$$

because only under this condition, the number of variables exceeds the number of equations so that a solution may exist.

For single-antenna interference networks, no practical scheme exists that can achieve the DoF upper bound exactly with finite complexity. Symbol extension is widely regarded as necessary to asymptotically approach the DoF upper bound [7]. However, this is only true for time varying or frequency selective fading channels. For deterministic networks with constant channel coefficients, simple symbol extension will generate a scaled identity matrix, which can not be used to separate the desired and interference signals into different signal subspaces.

Although interference alignment mainly focused on improving the multiplexing gain, the other important asset a system possess is the diversity gain. In conventional point-to-point multiple-input multiple-output (MIMO) channels, it has been proved that there is a fundamental tradeoff between the achievable multiplexing and diversity gains of a communication system [19]. Similarly, in network level transmission strategy designs, one can also purposely tradeoff one asset for the other in order to maximize the desired network performance [20].

From (2), it is easy to see that the reason for the infeasible interference alignment system is because there is not enough

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[1] $(M \times N)^K$ is used to denote a K-user interference network, where each transmitter has M antennas, each receiver has N antennas, and each user wants to achieve d DoF per channel use.

Fig. 1. An indirect approach to achieve interference alignment via varying signal power to provide more diversity.

diversity, i.e., M and N are too small[2]. Similar problem exists for the interference alignment schemes with naive symbol extension [7]. In those schemes, simple symbol extension generates sparse channel matrix with only diagonal or block diagonal elements. As a result, the scheme in [7] is only able to asymptotically achieve the DoF upper bound with infinitely large symbol extension, i.e., when diversity is high enough. The diversity insufficiency problem becomes even more catastrophic for deterministic interference networks with constant channel coefficients. In such deterministic cases, symbol extension does not even asymptotically achieve the DoF upper bound because the extension itself does not increase diversity.

Inspired by the research about diversity and multiplexing tradeoff (DMT) [19], [20], we adopt an indirect approach to obtain the DoF benefit offered by interference alignment as shown in Fig. 1. Firstly, we trade signal power for diversity improvement by using noncoherent transmissions with random phase offsets at both transmitters and receivers, which is done by distinctly scaled signal between each transmitter-receiver pair. While diversity is not our ultimate goal, we then further trade the increased system diversity for multiplexing improvement in order to achieve the DoF upper bound promised by interference alignment.

A. System model

We focus on $(1 \times 1)^K$ interference networks with each transmitter or receiver having only 1 antenna. $\mathbf{H}^{[ji]}$ is used to denote the channel matrix between the i-th transmitter S_i and the j-th receiver D_j after symbol extension, for $1 \leqslant i, j \leqslant K$. The diagonal elements in $\mathbf{H}^{[ji]}$ are independent real Gaussian distributed scalars for Gaussian interference networks or real constant scalars for deterministic interference networks. In this paper, we mainly consider real Gaussian channel coefficients if not particularly specified, while the real deterministic cases will be discussed separately.

$\mathbf{V}^{[i]}$ is used to denote the precoding matrix at S_i and $\mathbf{U}^{[j]}$ is used to denote the receiving matrix at D_j, for $1 \leqslant i, j \leqslant K$. $\mathbf{s}^{[i]}$ and $\hat{\mathbf{s}}^{[i]}$ are used to denote the baseband precoded symbols to be transmitted by S_i with short-term power constraint P and the symbols estimated by D_i, for $1 \leqslant i \leqslant K$. The additive noise at D_j is denoted as $\mathbf{n}^{[j]}$ and assumed to be Gaussian distributed with zero mean and covariance matrix $\sum_{\mathbf{n}^{[j]}} = \mathbf{I}_{N \times N}$, for $1 \leqslant j \leqslant K$. $\mathbf{x}^{[i]}$ and $\mathbf{y}^{[j]}$ are used to denote the bandpass transmitted signal at S_i and the bandpass received signal at D_j respectively, for $1 \leqslant i, j \leqslant K$.

We assume each transmitter or receiver uses advanced coding or decoding techniques in order to approach the Shannon

[2]The maximum single-user diversity gain increases as $M + N$ increases because $MN \leqslant \frac{(M+N)^2}{4}$.

limit. Moreover, pulse amplitude modulation (PAM) with sine wave being the carrier signal is adopted as the modulation scheme. For single-antenna interference networks, bandpass representation of the transmitted signal at S_i can be written as

$$
\begin{aligned}
x^{[i]} &= \mathrm{Re}\{s^{[i]} e^{j2\pi f_c t + \theta^{[i]}}\} \\
&= s^{[i]} \cos(2\pi f_c t + \theta^{[i]}),
\end{aligned} \tag{3}
$$

where f_c is the carrier frequency and $\theta^{[i]}$ is the random phase offset brought in by S_i. Similarly, the received signal (ignoring noise) from S_i to D_j can be written as

$$
y^{[ji]} = h^{[ji]} s^{[i]} \cos(2\pi f_c t + \theta^{[i]}). \tag{4}
$$

At D_j, a demodulator f with random phase offset $\varphi^{[j]}$ processes the received signal from S_i as

$$
\begin{aligned}
&f_\varphi^{[j]}(y^{[ji]}) \\
&= 2 \int_0^T y^{[ji]} \cos(2\pi f_c t + \varphi^{[j]}) dt \\
&= 2 \int_0^T h^{[ji]} s^{[i]} \cos(2\pi f_c t + \theta^{[i]}) \cos(2\pi f_c t + \varphi^{[j]}) dt \\
&= \cos(\varphi^{[j]} - \theta^{[i]}) h^{[ji]} s^{[i]},
\end{aligned} \tag{5}
$$

where T is a whole symbol interval.

Conventionally, a coherent demodulator should track the phase change and reproduce the carrier signal such that $\theta^{[i]}$ and $\varphi^{[j]}$ are as close as possible. We will later show that it is these intentionally chosen noncoherent random phase offsets that provides us more diversity. Roughly speaking, if the transmitter and receiver phase offsets between two antennas happen to be close to each other, the channel gain between them is large; On the other hand, if they happen to be far away, the channel gain between them is small.

II. THE DIVERSITY INSUFFICIENCY PROBLEM

From (2), we know that interference alignment is not feasible without symbol extension for some network configurations because the diversity is not large enough to be traded for multiplexing improvement to achieve the DoF upper bound. Thus, it is widely conjectured that symbol extension, either in time or frequency domain, is needed to bridge the information theoretically powerful interference alignment to practical applications. In particular, symbol extension is mostly desired in the following two common scenarios:

1) In cases that the feasibility condition (2) for a MIMO interference network is not satisfied, it is natural to consider symbol extension (either in time or frequency domain) to increase M and/or N.
2) For single-antenna interference networks, one may want to use symbol extension to increase M and N (and thus

the resultant equivalent channel matrices are MIMO) so that MIMO interference alignment schemes can be applied to achieve the DoF upper bound exactly.

A. Problems of naive symbol extension

Although conceptually simple, there are certain limitations so that naive symbol extension themselves are not able to resolve all the problems:

1) Naive symbol extension increase $M + N$ and d proportionally, and thus an original infeasible system remains infeasible.
2) More importantly, the equivalent MIMO channel matrices after naive symbol extension possess a diagonal or block diagonal structure such that conventional MIMO interference alignment schemes are not feasible to produce proper interference alignment precoding and receiving matrices.

The first problem is straightforward from (2). To better illustrate the second problem, let us consider a $(1 \times 1)^3$ interference network. With 2 symbol extension in the time domain, the equivalent channel matrix between S_i and D_j becomes

$$\mathbf{H}^{[ji]} = \begin{bmatrix} h_1^{[ji]} & 0 \\ 0 & h_2^{[ji]} \end{bmatrix}, \text{ for } 1 \leqslant i, j \leqslant 3, \quad (6)$$

where $h_\tau^{[ji]}$ is used to denote the channel coefficient value between S_i and D_j at time instant τ, for $1 \leqslant \tau \leqslant 2$. Since the received signal at each receiver is a 2×1 vector, the interference alignment conditions can be written as

$$\mathbf{H}^{[12]}\mathbf{V}^{[2]} = \alpha\mathbf{H}^{[13]}\mathbf{V}^{[3]}, \quad (7)$$

$$\mathbf{H}^{[21]}\mathbf{V}^{[1]} = \beta\mathbf{H}^{[23]}\mathbf{V}^{[3]}, \quad (8)$$

$$\mathbf{H}^{[31]}\mathbf{V}^{[1]} = \gamma\mathbf{H}^{[32]}\mathbf{V}^{[2]}, \quad (9)$$

where α, β and γ are three scalers. From (7)-(9), it is easy to see:

$$\mathbf{V}^{[2]} = (\mathbf{H}^{[12]})^{-1}\alpha\mathbf{H}^{[13]}\mathbf{V}^{[3]}, \quad (10)$$

$$\mathbf{V}^{[3]} = (\beta\mathbf{H}^{[23]})^{-1}\mathbf{H}^{[21]}\mathbf{V}^{[1]}, \quad (11)$$

$$\mathbf{V}^{[1]} = (\mathbf{H}^{[31]})^{-1}\gamma\mathbf{H}^{[32]}\mathbf{V}^{[2]}. \quad (12)$$

Thus, in order to align all interference at each receiver into the same signal subspace, the precoding matrices need to be designed as

$$\mathbf{V}^{[1]} = \mathbf{E}\mathbf{V}^{[1]}, \quad (13)$$

$$\mathbf{V}^{[2]} = \mathbf{F}\mathbf{V}^{[1]}, \quad (14)$$

$$\mathbf{V}^{[3]} = \mathbf{G}\mathbf{V}^{[1]}, \quad (15)$$

where

$$\mathbf{E} = (\mathbf{H}^{[31]})^{-1}\gamma\mathbf{H}^{[32]}(\mathbf{H}^{[12]})^{-1}\alpha\mathbf{H}^{[13]}(\beta\mathbf{H}^{[23]})^{-1}\mathbf{H}^{[21]}, \quad (16)$$

$$\mathbf{F} = (\gamma\mathbf{H}^{[32]})^{-1}\mathbf{H}^{[31]}, \quad (17)$$

$$\mathbf{G} = (\beta\mathbf{H}^{[23]})^{-1}\mathbf{H}^{[21]}. \quad (18)$$

Therefore, $\mathbf{V}^{[1]}$ must be a linearly scaled version of an eigenvector of $\mathbf{E}$. Since $\mathbf{E}$ is a diagonal matrix, its eigenvectors

are $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Whichever eigenvector $\mathbf{V}^{[1]}$ is related to, one of its entries is 0, i.e., S_1 is silent in one of the two time instants. So do S_2 and S_3 because of the diagonal structure of $\mathbf{F}$ and $\mathbf{G}$.

Thus, with diagonal equivalent channel matrices through naive 2 symbol extension, every transmitter transmits in one time instant and keeps silent in the other time instant. Consequently, every receiver receives superpositioned desired and interference signal in one time instant and receive nothing but noise in the other time instant. Although all interference is aligned into the same signal subspace, the desired signal is in the same signal subspace and inseparable from the interference.

B. The insufficiency of coherent demodulation

From the last section, we see that the sparse diagonal channel matrix is infeasible to achieve interference alignment because of the lack of diversity. One technique to resolve the diagonality problem is to add some scaled versions of the received signal across several symbol extension together, in order to artificially generate the non-diagonal terms for the equivalent MIMO channel matrices. However, such operations will scale the desired and all interference signal equally so that although individual channel matrix is not diagonal, $\mathbf{E}$, $\mathbf{F}$ and $\mathbf{G}$ in (16)-(18) are still diagonal.

Let us consider again the $(1 \times 1)^3$ interference network with 2 symbol extension in the time domain. As a first step to resolve the diagonality problem, at each receiver, we add a scaled version of the received signal in the first time instant to the received signal in the second time instant. Thus, the equivalent channel matrix between S_i and D_j becomes

$$\mathbf{H}^{[ji]} = \begin{bmatrix} h_1^{[ji]} & 0 \\ \lambda_j^i h_1^{[ji]} & h_2^{[ji]} \end{bmatrix}, \text{ for } 1 \leqslant i, j \leqslant 3, \quad (19)$$

where λ_j^i is the scaling factor at D_j for received signal from S_i. Because the desired and interference signal are superpositioned to each other and inseparable at this stage, we must have $\lambda_j^1 = \lambda_j^2 = \lambda_j^3$.

At D_j, we have

$$(\mathbf{H}^{[jp]})^{-1}\mathbf{H}^{[jq]}$$

$$= \begin{bmatrix} h_1^{[jp]} & 0 \\ \lambda_j^p h_1^{[jp]} & h_2^{[jp]} \end{bmatrix}^{-1} \begin{bmatrix} h_1^{[jq]} & 0 \\ \lambda_j^q h_1^{[jq]} & h_2^{[jq]} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{h_1^{[jp]}} & 0 \\ -\frac{\lambda_j^p}{h_2^{[jp]}} & \frac{1}{h_2^{[jp]}} \end{bmatrix} \begin{bmatrix} h_1^{[jq]} & 0 \\ \lambda_j^q h_1^{[jq]} & h_2^{[jq]} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{h_1^{[jq]}}{h_1^{[jp]}} & 0 \\ -\frac{\lambda_j^p h_1^{[jq]}}{h_2^{[jp]}} + \frac{\lambda_j^q h_1^{[jq]}}{h_2^{[jp]}} & \frac{h_2^{[jq]}}{h_2^{[jp]}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{h_1^{[jq]}}{h_1^{[jp]}} & 0 \\ 0 & \frac{h_2^{[jq]}}{h_2^{[jp]}} \end{bmatrix}, \text{ for } 1 \leqslant j, p, q \leqslant 3, j \neq p, q. \quad (20)$$

Thus, from (16) and (20), it is easy to see $\mathbf{E}$ still possesses a diagonal structure and $\mathbf{V}^{[1]}$ must have a zero entry. Similarly

to the argument in the last section, since $\mathbf{F}$ and $\mathbf{G}$ are both diagonal matrices as $\mathbf{E}$, $\mathbf{V}^{[2]}$ and $\mathbf{V}^{[3]}$ both have zero entries in the same position as in $\mathbf{V}^{[1]}$. Therefore, although 2 symbol extension is used, only 1 time instant is used to transmit information by each transmitter and the received desired and interference signal at each receiver is still inseparable.

From (20), we know the reason that simple artificial superposition technique does not work properly is $-\frac{\lambda_j^p h_1^{[jq]}}{h_2^{[jp]}}$ and $\frac{\lambda_j^q h_1^{[jq]}}{h_2^{[jp]}}$ cancels each other because $\lambda_j^p = \lambda_j^q$. In other words, the main problem is such operations scale all interference components equally.

Thus, it is obvious that finding a function which can provide unequal scaling to different components in superpositioned signal is very important. If we restrict ourselves to the baseband representation of the received signal (ignoring noise), at time instant τ, we have

$$y_\tau^{[j]} = y_\tau^{[j1]} + y_\tau^{[j2]} + y_\tau^{[j3]}, \tag{21}$$

where $y_\tau^{[j]}$ is the overall received signal at D_j and $y_\tau^{[ji]}$ is the individual received signal at D_j from S_i, for $1 \leqslant i, j \leqslant 3$. What we need is to find a function f such that

$$f(y_\tau^{[j]}) = \lambda_j^1 y_\tau^{[j1]} + \lambda_j^2 y_\tau^{[j2]} + \lambda_j^3 y_\tau^{[j3]} \tag{22}$$

and

$$\lambda_j^1 \neq \lambda_j^2 \neq \lambda_j^3, \text{ for } 1 \leqslant j \leqslant 3. \tag{23}$$

(22) implies that such a function f should only produce linear combinations of different signal components, with unequal combining coefficients. Unfortunately, to our best knowledge, such a function f does not exist under baseband signal representation after coherent demodulation, where the phase offsets at all transmitters and receivers are equal, i.e, $\theta^{[i]} = \varphi^{[j]}$, $1 \leqslant i, j \leqslant 3$.

III. JOINT NONCOHERENT DEMODULATION AND INTERFERENCE ALIGNMENT

We have seen that baseband signal representation after coherent demodulation does not have enough freedom to be manipulated to meet our needs. However, noncoherent demodulation with random phase offset at each transmitter or receiver provides us extra diversity (opportunity of unequal scaling) between each transmitter-receiver pair.

Coming back to the previous $(1 \times 1)^3$ interference network with 2 symbol extension in the time domain. If $s_1^{[i]}$ and $s_2^{[i]}$ are the baseband precoded signal to be transmitted by S_i across two time instants, the following transmitting strategy is used:

1) In the first time instant, S_i modulates $s_1^{[i]}$ with random phase offset $\theta_1^{[i]}$.
2) In the second time instant, S_i modulates $s_2^{[i]}$ with random phase offset $\theta_2^{[i]}$.

Correspondingly, if $y_1^{[j]}$ and $y_2^{[j]}$ are the bandpass received signal across two time instants, D_j creates 2 artificial signalling branches and the following receiving strategy is used:

1) In the first signalling branch, D_j firstly demodulates $y_1^{[j]}$ with random phase offset $\varphi_{1,1}^{[j]}$ and $y_2^{[j]}$ with random phase offset $\varphi_{1,2}^{[j]}$. Then, it adds the two signal together to generate baseband received signal $\tilde{y}_1^{[j]}$.
2) In the second signalling branch, D_j firstly demodulates $y_1^{[j]}$ with random phase offset $\varphi_{2,1}^{[j]}$ and $y_2^{[j]}$ with random phase offset $\varphi_{2,2}^{[j]}$. Then, it adds the two signal together to generate $\tilde{y}_2^{[j]}$.

Thus, at D_j and the first time instant, if we apply a function $f_{\varphi_{1,1}^{[j]}}$ and $f_{\varphi_{1,2}^{[j]}}$ to the overall bandpass received signal $y_1^{[j]}$ and $y_2^{[j]}$ respectively, we have

$$
\begin{aligned}
&\tilde{y}_1^{[j]} \\
&= f_{\varphi_{1,1}^{[j]}}(y_1^{[j]}) + f_{\varphi_{1,2}^{[j]}}(y_2^{[j]}) \\
&= f_{\varphi_{1,1}^{[j]}}(y_1^{[j1]} + y_1^{[j2]} + y_1^{[j3]}) + f_{\varphi_{1,2}^{[j]}}(y_2^{[j1]} + y_2^{[j2]} + y_2^{[j3]}) \\
&= \sum_{i=1}^{3} [\cos(\varphi_{1,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]}s_1^{[i]} + \cos(\varphi_{1,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]}s_2^{[i]}]
\end{aligned} \tag{24}
$$

Similarly, at D_j and the second time instant, if we apply functions $f_{\varphi_{2,1}^{[j]}}$ and $f_{\varphi_{2,2}^{[j]}}$ to the overall received signal $y_1^{[j]}$ and $y_2^{[j]}$, we have

$$\tilde{y}_2^{[j]} = \sum_{i=1}^{3} [\cos(\varphi_{2,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]}s_1^{[i]} + \cos(\varphi_{2,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]}s_2^{[i]}]. \tag{25}$$

Therefore, the equivalent channel matrix between S_i and D_j across these 2 symbol extension should be

$$\mathbf{H}^{[ji]} = \begin{bmatrix} \cos(\varphi_{1,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]} & \cos(\varphi_{1,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]} \\ \cos(\varphi_{2,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]} & \cos(\varphi_{2,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]} \end{bmatrix}, \tag{26}$$

for $1 \leqslant i, j \leqslant 3$.

It is easy to see that with joint noncoherent demodulation and interference alignment, the scaling factor at D_j for the received signal from S_i becomes

$$\lambda_j^i = \frac{\cos(\varphi_{2,1}^{[j]} - \theta_1^{[i]})}{\cos(\varphi_{1,1}^{[j]} - \theta_1^{[i]})}. \tag{27}$$

Because each transmitter or receiver has a unique random phase offset, (27) implies that with joint noncoherent demodulation and interference alignment, $\lambda_j^1 \neq \lambda_j^2 \neq \lambda_j^3$ and the diagonality problem is now fully resolved.

Besides the conventional channel diversity, the unequal scaling of different signal components in superpositioned signal here uses the extra phase diversity provided by each distinct transmitter-receiver pair. It is also worth mentioning that the proposed joint noncoherent demodulation and interference alignment scheme also works for deterministic interference networks, where symbol extension themselves do not provide extra diversity. This is because with random phase offsets for each symbol extension at all transmitters and receivers, we literately improved the system diversity by the distinct phase difference of each transmitter-receiver pair at each symbol extension use.

IV. GENERALIZED NONCOHERENT INTERFERENCE ALIGNMENT FOR $(1 \times 1)^K$

Up to this point, we have used the $(1 \times 1)^3$ interference network to illustrate why to use symbol extension, the diagonality problem of naive symbol extension because of diversity insufficiency and how our proposed joint noncoherent demodulation and interference alignment scheme resolves the problem by jointly considering bandpass modulation/decomulation and interference alignment. This section generalizes the scheme to general $(1 \times 1)^K$ interference networks.

A. How many artificial signalling branches are needed

Let us restrict ourselves to use 2 symbol extension only (in time or frequency domain). Since each user has only 1 transmit or receive antenna, total achievable DoF upper bound is $\frac{K}{2}$, i.e., on average, each user wants to achieve $\frac{1}{2}$ DoF in one channel use or 1 DoF across 2 symbol extension, i.e., $d = 1$.

Also, with 2 symbol extension, the equivalent channel matrices are MIMO and in the form of $N \times 2$, where N is the total number of artificial signalling branches we need to create at each receiver. From (2), we know that interference alignment is feasible only if

$$N + 2 \geqslant (K+1)d$$
$$\Leftrightarrow \quad N \geqslant K - 1. \tag{28}$$

B. Generalized scheme description

- Before the start of transmission, S_i passes the original symbol through its unique interference alignment precoding matrix to generate the precoded signal to be transmitted across 2 symbol extension.
- In the first channel use, S_i modulates the first component of its precoded signal with random phase offset $\theta_1^{[i]}$.
- In the second channel use, S_i modulates the second component of its precoded signal with random phase offset $\theta_2^{[i]}$.
- D_j creates $K - 1$ artificial signalling branches. The output of the k-th branch, for $1 \leqslant k \leqslant K - 1$, is the addition of the demodulated signal of the received signal in first channel use with random phase offset $\varphi_{k,1}^{[j]}$ and the demodulated signal of the received signal in second channel use with random phase offset $\varphi_{k,2}^{[j]}$.
- After the demodulation process, D_j passes all the demodulated signal through its unique interference alignment receiving matrix to remove all interference from its undesired transmitters.

Correspondingly, the block diagram for the generalized scheme can be illustrated in Fig. 2 and Fig. 3.

C. DoF optimality

From the last subsection, it is easy see that the equivalent channel matrix between S_i and D_j of a $(1 \times 1)^K$ interference network using the noncoherent interference alignment scheme

becomes

$$\mathbf{H}^{[ji]} = \begin{bmatrix} \cos(\varphi_{1,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]} & \cos(\varphi_{1,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]} \\ \cos(\varphi_{2,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]} & \cos(\varphi_{2,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]} \\ \multicolumn{2}{c}{...} \\ \cos(\varphi_{K-1,1}^{[j]} - \theta_1^{[i]})h_1^{[ji]} & \cos(\varphi_{K-1,2}^{[j]} - \theta_2^{[i]})h_2^{[ji]} \end{bmatrix}, \tag{29}$$

for $1 \leqslant i, j \leqslant K$. Now, we need to show with such equivalent channel matrices, a total of K DoF can be achieved, such that on average, each user can achieve $\frac{1}{2}$ DoF per channel use. We assume the random phase offset at each transmitter or receiver is a rational multiple of π and the channel coefficients are Gaussian rational numbers. A phase offset of a rational multiple of π can be obtained by a finite precision sampling of the carrier wave. Similarly, a finite precision sampling of the received signal before the receiving matrix will result an equivalent quantized Gaussian rational distributed channel.

Lemma 1. $\cos(q\pi)$ *is a root of a monic polynomial with integer coefficients for any rational q.*

Proof: We only give a brief proof here for the self-containess of this paper, while interested readers can refer to [21] for details.

Define a linear fractional transformation and its similarity parameter as

$$f(x) = \frac{ax+b}{cx+d}, \text{ and } \sigma = \frac{(a+d)^2}{ad-bc}. \tag{30}$$

where a, b, c, d are complex constants. The iterations of two linear fractional transformations in the complex plane are geometrically similar if and only if they have the same similarity parameter. Now, consider the following function

$$g(x) = \frac{(1+m)x + (1-m)}{(1-m)x + (1+m)}, \tag{31}$$

where m is a complex constant. For an random initial value of x_0, the n-th iteration of $g(x)$ can be written as

$$g^n(x) = \frac{(1+m^n)x_0 + (1-m^n)}{(1-m^n)x_0 + (1+m^n)}. \tag{32}$$

If we require $g^n(x)$ to be cyclic with period p, then we must have $m^p = 1$ for an integer p. This means m must be of the form

$$m = e^{j2\pi\frac{q}{p}}, \text{ for } q = 1, 2, ..., p-1. \tag{33}$$

Thus, the similarity parameter of $g(x)$ is

$$\begin{aligned} \sigma_{g(x)} &= \frac{4(1+m)^2}{(1+m)^2 - (1-m)^2} \\ &= m + \frac{1}{m} + 2 \\ &= 2\cos(\frac{2q\pi}{p}) + 2 \\ &= 4\cos(\frac{q\pi}{p})^2, \text{ for } q = 1, 2, ..., p-1. \end{aligned} \tag{34}$$

Now, consider another linear fractional transformation with one of the similarity parameters in (34) as

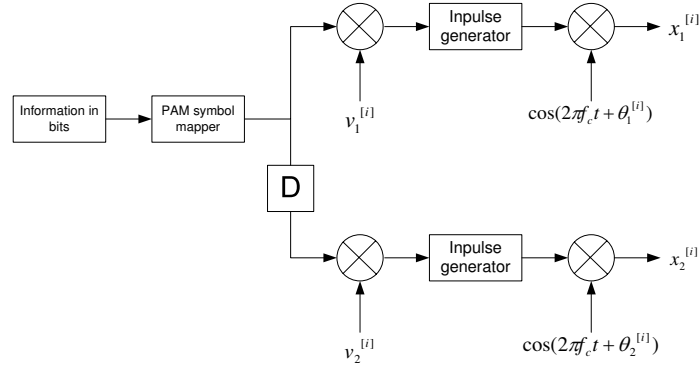$$h(x) = 1 - \frac{1}{\sigma_{g(x)}x}. \tag{35}$$

Fig. 2. Transmitter side model of the noncoherent interference alignment scheme, where "D" is the delay component, v are precoding coefficients and θ are transmitter random phase offsets.
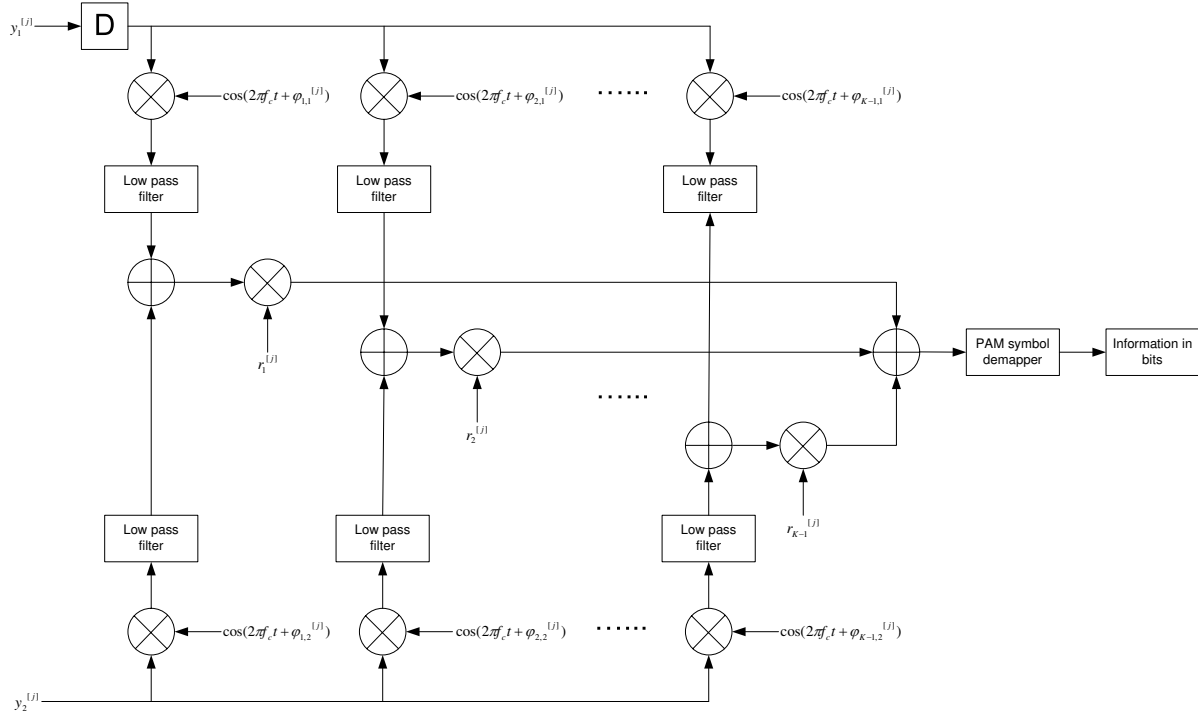

Fig. 3. Receiver side model of the noncoherent interference alignment scheme, where r are receiving coefficients and φ are receiver random phase offsets.

Therefore, we know $h^p(x) = x$ because of their geometric similarity. Let P_p denote a polynomial of $\sigma_{g(x)}$ with period p, then $h^p(x) = x$ implies (after some algebra)

$$\frac{-[\sigma_{g(x)}x^2 - \sigma_{g(x)}x + 1]P_p}{\sigma_{g(x)}[P_p x - P_{p-1}]} = 0, \text{ for odd } p \tag{36}$$

and

$$\frac{-[\sigma_{g(x)}x^2 - \sigma_{g(x)}x + 1]P_p}{[\sigma_{g(x)}P_p x - P_{p-1}]} = 0, \text{ for even } p. \tag{37}$$

Thus, the polynomials P_p satisfy

$$P_{p+1} = \begin{cases} P_p - P_{p-1}, & \text{for } p \text{ is odd}, \\ \sigma_{g(x)}P_p - P_{p-1}, & \text{for } p \text{ is even}. \end{cases} \tag{38}$$

It is easy to verify that $\sigma_{g(x)}$ with period p are roots to monic polynomials P_p with integer coefficients and this completes the proof. ■

Lemma 2. *Gauss's Lemma: Any root of a monic polynomial with integer coefficients must either be an integer or irrational number.*

Proof: Again, only brief proof is provided for the completeness of this paper. It is trivial to verify that polynomials of degree 1 with integer coefficients only have integer roots. Let us define the degree of a real number x as the degree of the minimal monic polynomial with integer coefficients having x as a root. For a degree k monic polynomial $f(x)$ with integer coefficients and constant C, define another polynomial $g(x)$ as

$$g(x) = \frac{f(x) - C}{x}. \tag{39}$$

It is easy to see $g(x)$ is a degree $k-1$ monic polynomial with integer coefficients.

Assume $f(x)$ has a rational non-integer root r. Our task now becomes to deriving a contradiction under this assumption. Let r be a non-integer root of $f(x)$ and it must be of degree k. Thus, we have $f(r) = 0$ and $g(r) = -\frac{C}{r}$, where $-\frac{C}{r}$ must be a non-integer.

Define N as the smallest integer such that $Ng(r)$ is an integer. Thus, the product of N and any polynomial in r of degree $k-1$ is an integer and we have $N' = N[g(r) - \lfloor g(r) \rfloor]]$ is an integer. Moreover, we have

$$
\begin{aligned}
N'g(r) &= N[g(r) - \lfloor g(r) \rfloor]]g(r) \\
&= Ng(r)^2 - \lfloor g(r) \rfloor Ng(r) \quad (40)
\end{aligned}
$$

also being an integer. This is because $g(r)^2$ can be expressed as a polynomial of degree $k - 1$ by reducing every higher power of r by substituting from the expression for r^k given by $f(r) = 0$. However, this contradicts the fact that N is the smallest integer such that $Ng(r)$ is an integer because $0 < [g(r) - \lfloor g(r) \rfloor]] < 1$ and $N' < N$. Thus, $f(x)$ does not have a non-integer rational root and this completes the proof. ■

Lemma 3. *The product of a rational number and an irrational number is irrational with probability* 1.

Lemma 4. *If all elements of fully connected channel matrices are irrational algebraic numbers, then the total achievable DoF is* $\frac{K}{2}$.

Proof: Please refer to Theorem 1 in [4]. ■

Theorem 1. *The noncoherent interference alignment scheme is DoF optimal such that each user can achieve $\frac{1}{2}$ DoF per channel use.*

Proof: The proof for the theorem is a combination of the previously introduced lemmas. Firstly, from Lemma 1 and Lemma 2, the terms $\cos(\varphi^{[j]} - \theta^{[i]})$ in (29) are irrational numbers. Then, from Lemma 3, we know the products $\cos(\varphi^{[j]} - \theta^{[i]})h^{[ji]}$ are also irrational with probability 1. Finally, from Lemma 4, the equivalent channel matrices offered by noncoherent interference alignment are able to achieve the DoF upper bound $\frac{K}{2}$. ■

D. Extensions to real deterministic interference networks

It is easy to see that the proposed noncoherent interference alignment scheme does not distinguish between Gaussian or deterministic interference networks. This is because either of them lacks of sufficient diversity to be traded for multiplexing improvement. The extra diversity we were manipulating comes from the distinct phase difference of each transmitter-receiver pair which does not depend on the underlying physical channels. Thus, noncoherent interference alignment works for Gaussian as well as deterministic interference networks.

V. SIMULATION RESULTS

In this section, we investigate the performance of our proposed noncoherent interference alignment scheme. The modulation scheme we employ is PAM, interference alignment
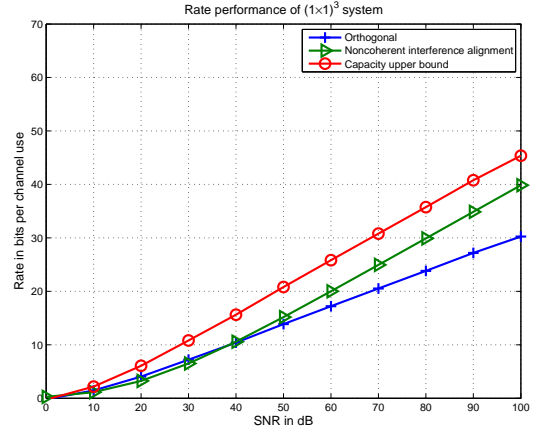

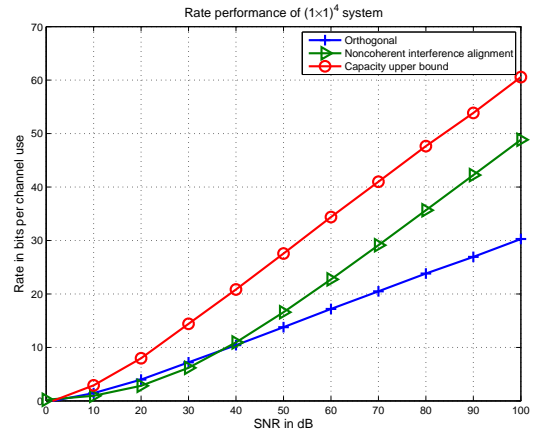Fig. 4. Rate performance of noncoherent interference alignment for $(1 \times 1)^3$.

Fig. 5. Rate performance of noncoherent interference alignment for $(1 \times 1)^4$.

precoding and receiving matrices are derived from closed-form solution for three-user MIMO interference channel in [7] and iterative zero-forcing and max-SINR solutions in [8]. The capacity upper bound we use is from (1) with the trivial $o(\log(\text{SNR}))$ term being ignored. Such approximation becomes increasingly accurate in the medium-to-high SNR regime.

From the simulation results in Fig. 4 and Fig. 5, we can see our proposed noncoherent interference alignment scheme is able to achieve the DoF upper bound, i.e., its achievable throughput increases as the SNR with slope $\frac{K}{2}$. However, we obverse that the achievable throughput is better than that of the orthogonal transmission scheme only in the high SNR regime. This is because the noncoherent interference alignment scheme (actually almost every other interference alignment scheme) is only DoF optimal but not capacity optimal. Thus, there is a constant gap (in the medium-to-high SNR regime where interference rather than noise is the dominating factor that affects the throughput) between the achievable throughput and the capacity upper bound. The reasons for the gap is explained in detail as follows.

Firstly, for any interference alignment scheme, one has to sacrifice some signal subspaces in order to align and remove all interference signals from undesired transmitters.

This, however, will at the same time remove the desired signal in those signal subspaces and the overall signal energy is almost definitely reduced. Fortunately, this only results in a fixed SNR offset and does not affect the achievable DoF. Actually, one important contribution of our work is to propose the first known scheme to tradeoff fixed power (or SNR offset) for DoF improvement.

Secondly, the use of random phase offsets in noncoherent interference alignment results in energy loss of desired signal in the demodulation process. From (5), we know desired signal energy is always reduced to some extent in order to create the "unequal scaling". In the simulation results presented above, the phase offset between each transmitter-receiver pair is drawn from a continuous uniform distribution set $[0^o, 360^o)$. It is easy to verify that such operations halve the average received signal power at each receiver compared to the case when coherent detection is used.

Finally, in the process of creating artificial signalling branches in order to meet the equivalent MIMO channel feasibility condition, we raised the noise level. From the noncoherent interference alignment scheme description, it is easy to see that the output of the k-th, for $1 \leqslant k \leqslant K - 1$, signalling branch is the addition of the demodulated signal of the received signal across two channel uses. The addition operation includes the addition of desired and interference signals and also the addition of noise signals across two channel uses. While all interference signals can be removed, nothing can be done about the increased random noise.

Next, we consider the bit error rate (BER) performance of our proposed noncoherent interference alignment scheme. As shown in Fig. 6, due to the reasons explained in the last several paragraphs, and the fact that our proposed noncoherent interference alignment scheme trades the diversity gain for the multiplexing improvement, the BER performance is not good in its original form. In order to recover the diversity benefit in the finite rate case, we employ the Fischer-Huber (FH) loading algorithm [22]. In particular, we formulate an optimization problem to maximize the minimum distance of different PAM symbols so that the BER is minimized, subject to the total rate constraint. The rate and power allocation is applied to the equivalent parallel channels after and before and interference alignment precoding and receiving matrices respectively. For our real (PAM) case, the rate and power for each channel under the FH algorithm is allocated as

$$R_i = \frac{R_T}{d_i} + \frac{1}{2d_i} \log\left(\frac{|h_i|^{2d_i}}{\prod_{l \in I} |h_l|^2}\right), \qquad (41)$$

and

$$P_i = \frac{\frac{P_T 2^{2^R Q_i}}{|h_i|^2}}{\sum_{l \in I} \frac{2^{2^R Q_l}}{|h_l|^2}}, \qquad (42)$$

where d_i, R_{Q_i} and P_i are the DoF, rate and power respectively from S_i to D_i and h_i and Q_i are the channel gain and the precoding matrix of the i-th parallel channel, for $1 \leqslant i \leqslant K$. All the above variables are considered only in I which corresponds to the set of channels actually in use after the loading algorithms. Simulation results in Fig. 6 and
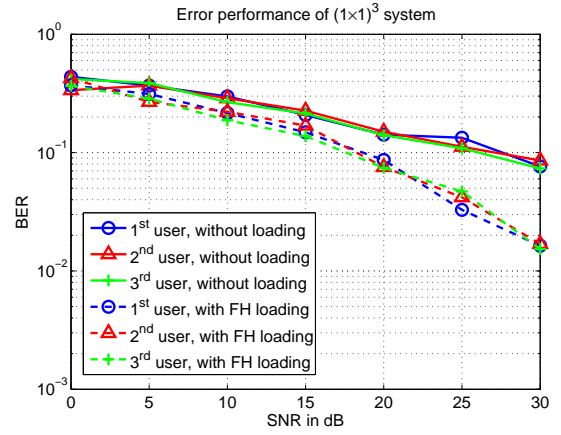

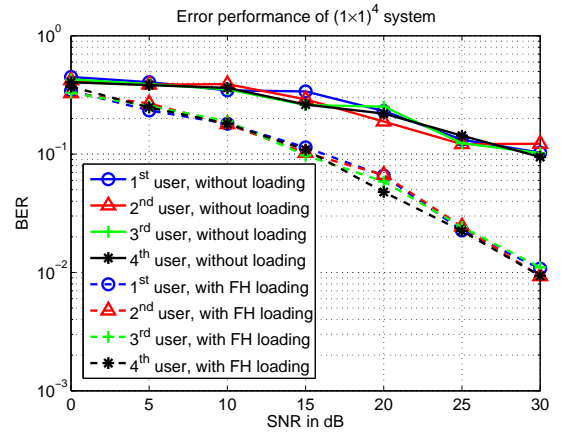Fig. 6. Error performance of noncoherent interference alignment for $(1 \times 1)^3$.

Fig. 7. Error performance of noncoherent interference alignment for $(1 \times 1)^4$.

Fig. 7 show that with rate and power loading algorithms, the proposed interference alignment scheme can achieve the DoF upper bound in the high SNR regime, while at the same time maintain acceptable BER performance in the low SNR regime.

VI. CONCLUSION

This paper proposed a practical and universal interference alignment scheme for general $(1 \times 1)^K$ interference networks, which trades signal power for intermediate diversity improvement towards the ultimate multiplexing requirement. The problems of naive symbol extension, which is a conventional diversity increasing technique to do interference alignment, was analyzed in details. It was identified that lack of diversity is the main problem such that there is not enough freedom to be manipulated to meet desired alignment conditions for some network configurations. An noncoherent interference alignment scheme with joint noncoherent band-pass modulaiton/demodulation and interference alignment was then proposed to resolve the diagonality problem of naive symbol extension and the simple superposition technique. This scheme was then generalized to $(1 \times 1)^K$ interference networks, either Gaussian or deterministic, and was proven to be DoF optimal. Simulation results verified its correctness and showed significant DoF improvement in the high SNR regime.

As a conclusion to this paper, we want to emphasize that although noncoherent energy loss of desired signal may cost us SNR offset, energy increase of interference signal will cause damaging error floor and decrease the slope of the achievable rate curve. Therefore, noncoherent interference alignment is preferable in the wide sense. An interesting future work would be analysis of the optimal region and distribution of the random phase offsets such that the total SNR loss is minimized. The challenge is to derive the tradeoff between the noncoherent loss and signal subspace loss (less noncoherent loss will result more signal subspace loss due to the increased closeness of desired and interference signal vectors).

REFERENCES

[1] A. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. 21, no. 5, pp. 569–570, Sep. 1975.

[2] H. Sato, "The capacity of the gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.

[3] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[4] R. Etkin and E. Ordentlich, "On the degrees-of-freedom of the k-user gaussian interference channel," *CoRR*, vol. abs/0901.1695, 2009.

[5] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[6] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *Proceedings of IEEE International Symposium on Information Theory, 2005*, Sep. 2005, pp. 2065–2069.

[7] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the k-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[8] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *IEEE Global Telecommunications Conference, 2008*, 30 Nov.-4 Dec. 2008, pp. 1–6.

[9] S. W. Peters and R. W. Heath, "Interference alignment via alternating minimization," in *ICASSP '09: Proceedings of the 2009 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2009, pp. 2445–2448.

[10] I. Thukral and H. Bolcskei, "Interference alignment with limited feedback," in *IEEE International Symposium on Information Theory, 2009*, Jun. 28-Jul. 3 2009, pp. 1759–1763.

[11] H. Yu, J. Park, Y. Sung, and Y. Lee, "A least squares approach to joint beam design for interference alignment in multiuser interference channels," in *IEEE 10th Workshop on Signal Processing Advances in Wireless Communications, 2009*, Jun. 2009, pp. 593–597.

[12] A. Khandani, S. Motahari, and B. Nourani, "Relay-aided interference alignment for the quasi-static x channel," in *IEEE International Symposium on Information Theory, 2009*, Jun. 28-Jul. 3 2009, pp. 1764–1768.

[13] B. Nazer, S. Jafar, M. Gastpar, and S. Vishwanath, "Ergodic interference alignment," in *IEEE International Symposium on Information Theory, 2009*, Jun. 28-Jul. 3 2009, pp. 1769–1773.

[14] A. Ozgur and D. Tse, "Achieving linear scaling with interference alignment," in *IEEE International Symposium on Information Theory, 2009*, Jun. 28-Jul. 3 2009, pp. 1754–1758.

[15] H. Ning, C. Ling, and K. K. Leung, "Relay-aided interference alignment: Feasibility conditions and algorithm," in *IEEE International Symposium on Information Theory, 2010*, Jun. 13- Jun. 18 2009.

[16] R. Tresch and M. Guillaud, "Cellular interference alignment with imperfect channel knowledge," in *IEEE International Conference on Communications Workshops, 2009*, Jun. 2009, pp. 1–5.

[17] Y. Wu and A. Dimakis, "Reducing repair traffic for erasure coding-based storage via interference alignment," in *IEEE International Symposium on Information Theory, 2009*, Jun. 28-Jul. 3 2009, pp. 2276–2280.

[18] C. Yetis, T. Gou, S. Jafar, and A. Kayran, "Feasibility conditions for interference alignment," in *Global Telecommunications Conference, 2009*, Nov. 30-Dec. 4 2009, pp. 1–6.

[19] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[20] H. Ning, C. Ling, and K. K. Leung, "Generalized sequential slotted amplify and forward strategy in cooperative communications," *IEEE Trans. Info. Thoery*, to appear.

[21] S. Lang, *Algebraic number theory, Graduate Texts in Mathematics (2 ed.), New York: Springer-Verlag, 1994*.

[22] R. Fischer and J. Huber, "A new loading algorithm for discrete multitone transmission," in *Global Telecommunications Conference, 1996. GLOBECOM '96.*, vol. 1, Nov. 1996, pp. 724–728.