

Group Analysis of Born-Infeld Equation

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Abstract

Lie symmetry group method is applied to study the Born-Infeld equation. The symmetry group and its optimal system are given, and group invariant solutions associated to the symmetries are obtained. Finally the structure of the Lie algebra symmetries is determined.

Key words: Born-Infeld theory; Lie symmetry; Partial differential equation.

1 Introduction

The method of point transformations are a powerful tool in order to find exact solutions for nonlinear partial differential equations. It happens that many PDE's of physical importance are nonlinear and Lie classical symmetries admitted by nonlinear PDE's are useful for finding invariant solutions.

In physics, the Born-Infeld theory is a nonlinear generalization of electromagnetism [6,7]. The model is named after physicists Max Born (1882-1970) and Leopold Infeld (1898-1968) who first proposed it. In physics, it is a particular example of what is usually known as a nonlinear electrodynamics. It was historically introduced in the 30's to remove the divergence of the electron's self-energy in classical electrodynamics by introducing an upper bound of the electric field at the origin. The Born-Infeld electrodynamics possesses a whole series of physically interesting properties: First of all the total energy of the electromagnetic field is finite and the electric field is regular everywhere. Second it displays good physical properties concerning wave propagation, such as the absence of shock waves and birefringence. A field theory showing this property is usually called completely exceptional and Born-Infeld theory is the only completely exceptional regular nonlinear electrodynamics. Finally (and more technically) Born-Infeld theory can be seen as a covariant generalization of Mie's theory, and very close to Einstein's idea of introducing a nonsymmetric metric tensor with the symmetric part corresponding to the usual metric tensor and the antisymmetric to the electromagnetic field tensor. During the 1990s there was a revival of interest on Born-Infeld theory and its nonabelian extensions as they were found in some limits of string theory.

2 Lie Symmetries of the Equation

A PDE with p -independent and q -dependent variables has a Lie point transformations

$$\tilde{x}_i = x_i + \varepsilon \xi_i(x, u) + \mathcal{O}(\varepsilon^2), \quad \tilde{u}_\alpha = u_\alpha + \varepsilon \varphi_\alpha(x, u) + \mathcal{O}(\varepsilon^2)$$

where $\xi_i = \left. \frac{\partial \tilde{x}_i}{\partial \varepsilon} \right|_{\varepsilon=0}$ for $i = 1, \dots, p$ and $\varphi_\alpha = \left. \frac{\partial \tilde{u}_\alpha}{\partial \varepsilon} \right|_{\varepsilon=0}$ for $\alpha = 1, \dots, q$. The action of the Lie group can be considered by its associated infinitesimal generator

$$\mathbf{v} = \sum_{i=1}^p \xi_i(x, u) \frac{\partial}{\partial x_i} + \sum_{\alpha=1}^q \varphi_\alpha(x, u) \frac{\partial}{\partial u_\alpha} \quad (1)$$

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on the total space of PDE (the space containing independent and dependent variables). Furthermore, the characteristic of the vector field (1) is given by

$$Q^\alpha(x, u^{(1)}) = \varphi_\alpha(x, u) - \sum_{i=1}^p \xi_i(x, u) \frac{\partial u^\alpha}{\partial x_i},$$

and its n -th prolongation is determined by

$$\mathbf{v}^{(n)} = \sum_{i=1}^p \xi_i(x, u) \frac{\partial}{\partial x_i} + \sum_{\alpha=1}^q \sum_{\#J=j=0}^n \varphi_\alpha^J(x, u^{(j)}) \frac{\partial}{\partial u_J^\alpha},$$

where $\varphi_\alpha^J = D_J Q^\alpha + \sum_{i=1}^p \xi_i u_{J,i}^\alpha$. (D_J is the total derivative operator describes in (3)).

The aim is to analysis the Lie point symmetry structure of the Born-Infeld equation, which is

$$(1 - u_t^2)u_{xx} + 2u_x u_t u_{xt} - (1 + u_x^2)u_{tt} = 0, \quad (2)$$

where u is a smooth function of (x, t) . Let us consider a one-parameter Lie group of infinitesimal transformations (x, t, u) given by

$$\tilde{x} = x + \varepsilon \xi_1(x, t, u) + \mathcal{O}(\varepsilon^2), \quad \tilde{t} = t + \varepsilon \xi_2(x, t, u) + \mathcal{O}(\varepsilon^2), \quad \tilde{u} = u + \varepsilon \eta(x, t, u) + \mathcal{O}(\varepsilon^2),$$

where ε is the group parameter. Then one requires that this transformations leaves invariant the set of solutions of the Eq. (2). This yields to the linear system of equations for the infinitesimals $\xi_1(x, t, u)$, $\xi_2(x, t, u)$ and $\eta(x, t, u)$. The Lie algebra of infinitesimal symmetries is the set of vector fields in the form of

$$\mathbf{v} = \xi_1(x, t, u) \partial_x + \xi_2(x, t, u) \partial_t + \eta(x, t, u) \partial_u.$$

This vector field has the second prolongation

$$\mathbf{v}^{(2)} = \mathbf{v} + \varphi^x \partial_{u_x} + \varphi^t \partial_{u_t} + \varphi^{xx} \partial_{u_{xx}} + \varphi^{xt} \partial_{u_{xt}} + \varphi^{tt} \partial_{u_{tt}}$$

with the coefficients

$$\begin{aligned} \varphi^x &= D_x(\varphi - \xi_1 u_x - \xi_2 u_t) + \xi_1 u_{xx} + \xi_2 u_{xt}, \\ \varphi^t &= D_t(\varphi - \xi_1 u_x - \xi_2 u_t) + \xi_1 u_{xt} + \xi_2 u_{tt}, \\ \varphi^{xx} &= D_x^2(\varphi - \xi_1 u_x - \xi_2 u_t) + \xi_1 u_{xxx} + \xi_2 u_{xxt}, \\ \varphi^{xt} &= D_x D_t(\varphi - \xi_1 u_x - \xi_2 u_t) + \xi_1 u_{xxt} + \xi_2 u_{xtt}, \\ \varphi^{tt} &= D_t^2(\varphi - \xi_1 u_x - \xi_2 u_t) + \xi_1 u_{xtt} + \xi_2 u_{ttt}, \end{aligned}$$

where the operators D_x and D_t denote the total derivative with respect to x and t :

$$\begin{aligned} D_x &= \partial_x + u_x \partial_u + u_{xx} \partial_{u_x} + u_{xt} \partial_{u_t} + \dots, \\ D_t &= \partial_t + u_t \partial_u + u_{tt} \partial_{u_t} + u_{xt} \partial_{u_x} + \dots. \end{aligned} \quad (3)$$

Using the invariance condition, i.e., applying the second prolongation $\mathbf{v}^{(2)}$ to Eq. (2), the following system of 10 determining equations yields:

$$\begin{aligned} \xi_{2xx} &= 0, \quad \xi_{2xu} = 0, \quad \xi_{2tt} = 0, \quad \xi_{2uu} = 0, \quad \xi_{1x} = \xi_{2t}, \\ \xi_{1t} &= \xi_{2x}, \quad \xi_{1u} = -\eta_x, \quad \xi_{2t} = \eta_u, \quad \xi_{2u} = \eta_t, \quad \xi_{2tu} = -\eta_{xx}. \end{aligned}$$

Table 1
Commutation relations of \mathfrak{g}

$[\cdot, \cdot]$	\mathbf{v}_1	\mathbf{v}_2	\mathbf{v}_3	\mathbf{v}_4	\mathbf{v}_5	\mathbf{v}_6	\mathbf{v}_7
\mathbf{v}_1	0	0	0	\mathbf{v}_3	\mathbf{v}_3	0	\mathbf{v}_1
\mathbf{v}_2	0	0	0	\mathbf{v}_1	0	\mathbf{v}_3	\mathbf{v}_2
\mathbf{v}_3	0	0	0	0	$-\mathbf{v}_1$	\mathbf{v}_2	\mathbf{v}_3
\mathbf{v}_4	$-\mathbf{v}_2$	$-\mathbf{v}_1$	0	0	\mathbf{v}_6	\mathbf{v}_5	0
\mathbf{v}_5	$-\mathbf{v}_3$	0	\mathbf{v}_1	$-\mathbf{v}_6$	0	\mathbf{v}_4	0
\mathbf{v}_6	0	$-\mathbf{v}_3$	$-\mathbf{v}_2$	$-\mathbf{v}_5$	$-\mathbf{v}_4$	0	0
\mathbf{v}_7	$-\mathbf{v}_1$	$-\mathbf{v}_2$	$-\mathbf{v}_3$	0	0	0	0

The solution of the above system gives the following coefficients of the vector field \mathbf{v} :

$$\xi_1 = c_1 + c_4 t - c_5 u + c_7 x, \quad \xi_2 = c_2 + c_4 x + c_6 u + c_7 t, \quad \eta = c_3 + c_5 x + c_6 t + c_7 t,$$

where c_1, \dots, c_7 are arbitrary constants, thus the Lie algebra \mathfrak{g} of the Born-Infeld equation is spanned by the seven vector fields

$$\begin{aligned} \mathbf{v}_1 &= \partial_x, & \mathbf{v}_2 &= \partial_t, & \mathbf{v}_3 &= \partial_u, & \mathbf{v}_4 &= t\partial_x + x\partial_t, \\ \mathbf{v}_5 &= -u\partial_x + x\partial_u, & \mathbf{v}_6 &= u\partial_t + t\partial_u, & \mathbf{v}_7 &= x\partial_x + t\partial_t + u\partial_u, \end{aligned}$$

which $\mathbf{v}_1, \mathbf{v}_2$ and \mathbf{v}_3 are translation on x, t and u , \mathbf{v}_5 is rotation on u and x and \mathbf{v}_7 is scaling on x, t and u . The commutation relations between these vector fields is given by the table 1, where entry in row i and column j representing $[\mathbf{v}_i, \mathbf{v}_j]$.

The one-parameter groups G_i generated by the base of \mathfrak{g} are given in the following table.

$$\begin{aligned} G_1 &: (x + \varepsilon, t, u), & G_2 &: (x, t + \varepsilon, u), \\ G_3 &: (x, t, u + \varepsilon), & G_4 &: \left(x \cosh \varepsilon + t \sinh \varepsilon, x \sinh \varepsilon + t \cosh \varepsilon, u \right) \\ G_5 &: (-u \sin \varepsilon + x \cos \varepsilon, t, x \sin \varepsilon + u \cos \varepsilon), & G_6 &: \left(x, t \cosh \varepsilon + u \sinh \varepsilon, t \sinh \varepsilon + u \cosh \varepsilon \right), \\ G_7 &: (xe^\varepsilon, te^\varepsilon, ue^\varepsilon). \end{aligned}$$

Since each group G_i is a symmetry group and if $u = f(x, t)$ is a solution of the Born-Infeld equation, so are the functions

$$\begin{aligned} u_1 &= f(x + \varepsilon, t), & u_2 &= f(x, t + \varepsilon), & u_3 &= f(x, t) - \varepsilon, \\ u_4 &= f\left(x \cosh \varepsilon - t \sinh \varepsilon, -x \sinh \varepsilon + t \cosh \varepsilon\right), & u_5 &= \sec \varepsilon f(u \sin \varepsilon + x \cos \varepsilon, t) + x \sin \varepsilon, \\ u_6 &= \operatorname{sech} \varepsilon f(x, t \cosh \varepsilon - u \sinh \varepsilon) + t \sinh \varepsilon, & u_7 &= e^{-\varepsilon} f(e^{-\varepsilon} x, e^{-\varepsilon} t). \end{aligned}$$

where ε is a real number. Here we can find the general group of the symmetries by considering a general linear combination $c_1 \mathbf{v}_1 + \dots + c_1 \mathbf{v}_6$ of the given vector fields. In particular if g is the action of the symmetry group near the identity, it can be represented in the form $g = \exp(\varepsilon_7 \mathbf{v}_7) \dots \exp(\varepsilon_1 \mathbf{v}_1)$.

3 Symmetry reduction for Born-Infeld equation

The first advantage of symmetry group method is to construct new solutions from known solutions. Neither the first advantage nor the second will be investigated here, but symmetry group method will be applied to the Eq. (2) to be connected directly to some order differential equations. To do this, a particular linear combinations of

infinitesimals are considered and their corresponding invariants are determined. The Born-Infeld equation expressed in the coordinates (x, t) , so to reduce this equation is to search for its form in specific coordinates. Those coordinates will be constructed by searching for independent invariants (y, v) corresponding to an infinitesimal generator. so using the chain rule, the expression of the equation in the new coordinate allows us to the reduced equation. Here we will obtain some invariant solutions with respect to symmetries. First we obtain the similarity variables for each term of the Lie algebra \mathfrak{g} , then we use this method to reduced the PDE and find the invariant solutions. All results are coming in the following table.

vector field	invariant function	invariant transformations	invariant solution
\mathbf{v}_1	$\Phi(t, u)$	$v(y) = u(x, t), y = t$	$u = c_1 t + c_2$
\mathbf{v}_2	$\Phi(x, u)$	$v(y) = u(x, t), y = x$	$u = c_1 x + c_2$
\mathbf{v}_3	$\Phi(x, t)$	\dots	translation of all solution is the invariant solution
\mathbf{v}_4	$\Phi(-x^2 + t^2, u)$	$v(y) = u(x, t), y = -x^2 + t^2$	$u = \pm c_1 \arctan\left(\frac{x^2 - t^2 + 2c_1^2}{\sqrt{(x^2 - t^2)(-x^2 + t^2 - 4c_1^2)}}\right) + c_2,$
\mathbf{v}_5	$\Phi(t, x^2 + u^2)$	$v(y) = x^2 + u(x, t)^2, y = t$	$u = \pm \frac{1}{2} \sqrt{2c_1 e^{\frac{t+c_2}{c_1}} + 8c_1^3 e^{-\frac{t+c_2}{c_1}} - 4x^2 - 8c_1^2}$ $u = \pm \frac{1}{2} \sqrt{2c_1 e^{-\frac{t+c_2}{c_1}} + 8c_1^3 e^{\frac{t+c_2}{c_1}} - 4x^2 - 8c_1^2}$
\mathbf{v}_6	$\Phi(x, -t^2 + u^2)$	$v(y) = -t^2 + u(x, t)^2, y = x$	$u = \pm \frac{1}{2} \sqrt{2c_1 e^{\frac{x+c_2}{c_1}} + 8c_1^3 e^{-\frac{x+c_2}{c_1}} + 4x^2 + 8c_1^2}$ $u = \pm \frac{1}{2} \sqrt{2c_1 e^{-\frac{x+c_2}{c_1}} + 8c_1^3 e^{\frac{x+c_2}{c_1}} + 4x^2 + 8c_1^2}$
\mathbf{v}_7	$\Phi\left(\frac{t}{x}, \frac{u(x, t)}{x}\right)$	$v(y) = \frac{u(x, t)}{x}, y = \frac{t}{x}$	$u = c_1 x + c_2 t, u = \pm \sqrt{-x^2 + t^2}$

4 Optimal system of Born-Infeld equation

As is well known, the theoretical Lie group method plays an important role in finding exact solutions and performing symmetry reductions of differential equations. Since any linear combination of infinitesimal generators is also an infinitesimal generator, there are always infinitely many different symmetry subgroups for the differential equation. So, a mean of determining which subgroups would give essentially different types of solutions is necessary and significant for a complete understanding of the invariant solutions. As any transformation in the full symmetry group maps a solution to another solution, it is sufficient to find invariant solutions which are not related by transformations in the full symmetry group, this has led to the concept of an optimal system [3]. The problem of finding an optimal system of subgroups is equivalent to that of finding an optimal system of subalgebras. For one-dimensional subalgebras, this classification problem is essentially the same as the problem of classifying the orbits of the adjoint representation. This problem is attacked by the naive approach of taking a general element in the Lie algebra and subjecting it to various adjoint transformations so as to simplify it as much as possible. The idea of using the adjoint representation for classifying group-invariant solutions is due to [4] and [5].

The adjoint action is given by the Lie series

$$\text{Ad}(\exp(\varepsilon \mathbf{v}_i) \mathbf{v}_j) = \mathbf{v}_j - \varepsilon [\mathbf{v}_i, \mathbf{v}_j] + \frac{\varepsilon^2}{2} [\mathbf{v}_i, [\mathbf{v}_i, \mathbf{v}_j]] - \dots, \tag{4}$$

where $[\mathbf{v}_i, \mathbf{v}_j]$ is the commutator for the Lie algebra, t is a parameter, and $i, j = 1, \dots, 10$. Let $F_i^\varepsilon : \mathfrak{g} \rightarrow \mathfrak{g}$ defined by $\mathbf{v} \mapsto \text{Ad}(\exp(\varepsilon \mathbf{v}_i) \mathbf{v})$ is a linear map, for $i = 1, \dots, 7$. The matrices M_i^ε of F_i^ε , $i = 1, \dots, 7$, with respect to basis

$\{\mathbf{v}_1, \dots, \mathbf{v}_7\}$ are

$$\begin{aligned}
M_1^\varepsilon &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\varepsilon & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\varepsilon & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, & M_2^\varepsilon &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\varepsilon & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\varepsilon & 0 & 0 & 1 & 0 \\ 0 & -\varepsilon & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\
M_3^\varepsilon &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \varepsilon & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\varepsilon & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\varepsilon & 0 & 0 & 0 & 1 \end{pmatrix}, & M_4^\varepsilon &= \begin{pmatrix} \cosh \varepsilon & \sinh \varepsilon & 0 & 0 & 0 & 0 & 0 \\ \sinh \varepsilon & \cosh \varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cosh \varepsilon & -\sinh \varepsilon & 0 \\ 0 & 0 & 0 & 0 & -\sinh \varepsilon & \cosh \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\
M_5^\varepsilon &= \begin{pmatrix} \cos \varepsilon & 0 & \sin \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\sin \varepsilon & 0 & \cos \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \varepsilon & 0 & \sin \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\sin \varepsilon & 0 & \cos \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, & M_6^\varepsilon &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cosh \varepsilon & \sin \varepsilon & 0 & 0 & 0 & 0 \\ 0 & \sin \varepsilon & \cos \varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cosh \varepsilon & \sin \varepsilon & 0 & 0 \\ 0 & 0 & 0 & \sinh \varepsilon & \cosh \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\
M_7^\varepsilon &= \begin{pmatrix} e^\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^\varepsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},
\end{aligned}$$

by acting these matrices on a vector field \mathbf{v} alternatively we can show that a one-dimensional optimal system of \mathfrak{g} is given by

$$\begin{aligned}
X_1 &= \mathbf{v}_1, & X_2 &= \mathbf{v}_3, \\
X_3 &= a(\mathbf{v}_1 - \mathbf{v}_7) + \mathbf{v}_2, & X_4 &= a\mathbf{v}_2 - b\mathbf{v}_3 + c\mathbf{v}_7, \\
X_5 &= \mathbf{v}_1 + a\mathbf{v}_2 + \mathbf{v}_3 - \mathbf{v}_6, & X_6 &= \mathbf{v}_1 + a\mathbf{v}_2 + b(\mathbf{v}_3 - \mathbf{v}_6), \\
X_7 &= a\mathbf{v}_1 + b\mathbf{v}_2 - c\mathbf{v}_3 - d\mathbf{v}_6, & X_8 &= a(\mathbf{v}_1 - \mathbf{v}_4) + b(\mathbf{v}_2 - \mathbf{v}_4) + \mathbf{v}_3 + c\mathbf{v}_4.
\end{aligned}$$

In the next section we will find the invariant solutions with respect to the symmetries and optimal system.

Table 2
Commutation relations of \mathfrak{g}

$[\cdot, \cdot]$	\mathbf{w}_1	\mathbf{w}_2	\mathbf{w}_3
\mathbf{w}_1	0	\mathbf{w}_3	\mathbf{w}_2
\mathbf{w}_2	$-\mathbf{w}_3$	0	\mathbf{w}_1
\mathbf{w}_3	$-\mathbf{w}_2$	$-\mathbf{w}_1$	0

5 Lie Algebra Structure

In this part, we determine the structure of symmetry Lie algebra of the Born-Infeld equation. The Lie algebra \mathfrak{g} is not solvable and semisimple, because if $\mathfrak{g}^{(1)} = \text{Span}_{\mathbb{R}}\{\mathbf{v}_i, [\mathbf{v}_i, \mathbf{v}_j]\}_{i,j}$ be the derived of \mathfrak{g} we have

$$\mathfrak{g}^{(1)} = \text{Span}_{\mathbb{R}}\{\mathbf{v}_1, \dots, \mathbf{v}_7\} = \mathfrak{g},$$

but it has a *Levi decomposition* in the form of

$$\mathfrak{g} = \mathfrak{r} \ltimes \mathfrak{g}_1, \quad (5)$$

where $\mathfrak{r} = \text{Span}_{\mathbb{R}}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_7\}$ is the radical (the largest solvable ideal) of \mathfrak{g} , and $\mathfrak{g}_1 = \text{Span}_{\mathbb{R}}\{\mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6\}$ is the semisimple and nonsolvable subalgebra of \mathfrak{g} . So the Levi decomposition of symmetry Lie algebra for Born-Infeld equation gives the quotient structure

$$\bar{\mathfrak{g}} = \mathfrak{g}/\mathfrak{r}. \quad (6)$$

If $\mathbf{w}_i = \mathbf{v}_i + \mathfrak{r}$ are the members of quotient algebra, the commutators table for $\bar{\mathfrak{g}}$ are given in table 2.

Finally, we have some analysis on the structure of (5) and (6) with some important objects in algebra. We know that the centralizer of a set of vectors \mathfrak{g} in a subalgebra \mathfrak{h} is the subalgebra of vectors in \mathfrak{h} which commute with all the vectors in \mathfrak{g} . With attentive to (5), \mathfrak{r} has no any nontrivial centralizer and it is the only minimal ideal containing itself. But this is not true for \mathfrak{g}_1 , because its centralizer has a member which is \mathbf{v}_7 , and the minimal ideal containing \mathfrak{g}_1 is spanned by $\{\mathbf{v}_1, \dots, \mathbf{v}_6\}$.

6 Conclusion

In this article group classification of Born-Infeld equation and the algebraic structure of the symmetry group is considered. Classification of one-dimensional subalgebra is determined by constructing one-dimensional optimal system. Some invariant solutions are found in the sequel and the Lie algebra structure of symmetries is found.

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