Two-variable Wiman-Valiron theory and PDEs Erratum

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Abstract: This note corrects Example 3.2 in a published paper with the same title.
Mathematics Subject Classification 32A15

Typographical errors and a careless comment on the Cauchy-Kovalevskaya theorem make Example 3.2 in our published paper [1] somewhat confusing. Below is a corrected version.

Example 3.2

Consider the n-th order linear PDE

$$\sum_{i+j\leq n} P_{i,j} f^{i,j} = 0, \tag{24}$$

where the $P_{i,j}$ are polynomials in two complex variables. We assume that f is an entire solution of (24). Clearly, as in Example 3.1, every such solution is transcendental.

To the best of our knowledge there have been no order estimates of entire solutions of (24). Our method can often obtain such results. To simplify matters, let us take the second order equation

$$f^{1,1} = Pf, \tag{25}$$

where P is a polynomial, and proceed as in Example 3.1. Using (15), (25) becomes

$$A\zeta_1\mathcal{F}^{1,0} + B\zeta_2\mathcal{F}^{0,1} + C\zeta_1^2\mathcal{F}^{2,0} + D\zeta_1\zeta_2\mathcal{F}^{1,1} + E\zeta_2^2\mathcal{F}^{0,2} = \zeta_1^4\zeta_2^2 P(\zeta_1^2\zeta_2, \zeta_1\zeta_2^2)\mathcal{F},$$

where A, B, C, D and E are constants. Using (11), we obtain

$$A\mathcal{N}_1 + B\mathcal{N}_2 + C\mathcal{N}_1^2 + D\mathcal{N}_1\mathcal{N}_2 + E\mathcal{N}_2^2 = (1+o(1))\zeta_1^4\zeta_2^2 P(\zeta_1^2\zeta_2, \zeta_1\zeta_2^2), \tag{26}$$

and therefore

$$\mathcal{N}^{*2} \geq (c+o(1))|\zeta_1|^4|\zeta_2|^2|P(\zeta_1^2\zeta_2, \zeta_1\zeta_2^2)|, \tag{27}$$

where c is a positive constant. As in Example 3.1, this implies that $\rho(\mathcal{F}) \geq 3 + 3d/2$, where d is the degree of P, and thus $\rho(f) \geq 1 + d/2$.

We remark also that the reference after equation (7) should read [2, p. 228].

References

1. Fenton, P.C. and Rossi, John, Two Variable Wiman-Valiron Theory and PDEs, Ann. Acad. Sci. Fenn. Math 35 (2010), 571-580.

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