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Cascades with Adjoint Matter: Adjoint Transitions

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Abstract: A large class of duality cascades based on quivers arising from non-isolated singularities enjoy adjoint transitions - a phenomenon which occurs when the gauge coupling of a node possessing adjoint matter is driven to strong coupling in a manner resulting in a reduction of rank in the non-Abelian part of the gauge group and a subsequent flow to weaker coupling. We describe adjoint transitions in a simple family of cascades based on a $\mathbb{Z}_2$-orbifold of the conifold using field theory. We show that they are dual to Higgsing and produce varying numbers of $U(1)$ factors, moduli, and monopoles in a manner which we calculate. This realizes a large family of cascades which proceed through Seiberg duality and Higgsing. We briefly describe the supergravity limit of our analysis, as well as a prescription for treating more general theories. A special role is played by $\mathcal{N} = 2$ SQCD. Our results suggest that additional light fields are typically generated when UV completing certain constructions of spontaneous supersymmetry breaking into cascades - potentially leading to instabilities.

Contents

1. Introduction

Quiver gauge theories possessing $\mathcal{N} = 1$ supersymmetry arising from fractional brane configurations at non-isolated singularities (QNISs) provide a potentially large class of examples realizing spontaneous supersymmetry breaking in string theory [1–5]. It is commonly believed that these examples may be embedded into duality cascades, which by the AdS/CFT correspondence would be dual to warped throats with spontaneous supersymmetry breaking deep in the interior. This is desirable as supersymmetry breaking throats are ubiquitous in a variety of string phenomenological applications to cosmology and particle physics [6–11].

To realize such a construction, an improved understanding of the duality cascades arising from $\mathcal{N} = 1$ QNISs is needed. A challenge is posed by the presence of adjoint matter. QNISs typically contain one or more nodes whose field content is formally that of an $\mathcal{N} = 2$ theory, and the associated adjoints have $\mathcal{N} = 2$ couplings to the rest of the theory. Thus, while some of the steps in the cascade can be understood in terms of Seiberg duality, the steps involving strongly coupled adjoint nodes will involve some other dynamics, referred to as an 'adjoint transition'. Our purpose here is to understand the adjoint transitions and plot out the zoo of possible cascade-like flows using the simplest case of a certain $\mathbb{Z}_2$ orbifold of the conifold as a prototype. We expect that this will help clarify questions related to existence of meta-stable supersymmetry breaking configurations at the bottom of such cascades.

Cascades enjoying adjoint transitions have been studied in detail for $\mathcal{N} = 2$ QNISs both in supergravity and in field theory [12–17]. In these cases the adjoint transitions proceed through Higgsing. There is a large moduli space of possible transitions, resulting in reductions in the rank of the non-Abelian part of the gauge group in varying degrees. It is quite plausible that similar dynamics persists in $\mathcal{N} = 1$ QNISs. This has been noted in [16]. Here we explore cascades with adjoint transitions in $\mathcal{N} = 1$ theories using the techniques of supersymmetric field theory, in the simplest case of cascades based on a $\mathbb{Z}_2$-orbifolded conifold.

To begin, we find a family of cascade-like flows which holomorphically interpolate between regimes where field theory techniques are expected to be useful and where supergravity is expected to be useful. We construct these flows as relevant deformations of a family of conformal field theories. The different flows are holomorphically connected by moving along the ultraviolet fixed-manifold. By the standard lore that superymmetric field theories undergo no phase transitions, we may thus learn about either regime by studying the field theory regime [18, 19]. In particular this should be sufficient to uncover the field theory mechanism behind the adjoint transition in either regime, and give information on the variety of possible infrared effective field theories. The definition of this family of flows is the subject of section 2.

In section 3 we describe a discrete family of fixed points and establish a few results which are needed in section 4.

In section 4 we study flows in the field theory regime. The flows in the field theory regime spend most of their time hovering near fixed points, with shorter flows in between which transition between the fixed points. The transitions gradually reduce the number of degrees of freedom available to the non-Abelian sector of the theory. This is similar in spirit to the original analysis due to Strassler of the conifold model [20]. There the transitions between fixed points could be described using Seiberg duality. In the flows we study only half of the transitions can be understood in this manner. The other half occur on segments of the flow where there is a seemingly indefinite growth of a coupling associated to an adjoint node. These correspond to adjoint transitions. We find that the growth of the coupling is regulated, as it is in $\mathcal{N}=2$ SQCD on its Coulomb branch, by the appearance of instantons which spontaneously break the gauge group down to an infrared free or conformal subgroup [21]. The adjoint transitions correspond to choosing a vacuum on a Coulomb branch in a manner which preserves some amount of non-Abelian gauge symmetry. In fact, the properties of such vacua are directly controlled by the physics of $\mathcal{N}=2$ SQCD in a manner which we desrcibe. After the adjoint transition, the theory is pushed back to a known interacting fixed point, however with a non-trivial number of $U(1)$ factors, moduli, and in certain cases monopoles present, forming a decoupled sector of the theory (of course, the monopoles are only massless for specially tuned values of the moduli). Thus we find a zoo of possible cascading renormalization group flows which proceed through a combination of Seiberg duality and Higgsing, producing a number of deconfined degrees of freedom along the way.

In section 5 we discuss the supergravity limit of our analysis. Using what is known of the gauge-gravity dictionary we construct a qualitative picture of the resulting supergravity background. We find a warped throat containing a sequence of singular points at which explicit fractional brane sources are present, as in [22]. These are required to realize the U(1) factors and moduli predicted from field theory at each Higgsing event. The resulting picture agrees well with previous work [22].

In section 6 we discuss the generalization of our results to other quivers. Just as understanding the mechanism behind cascades at the simplest example of an isolated singularity, the conifold [23], appears to be sufficient in understanding the mechanism behind cascades arising at a variety of isolated singularities [24], we expect that the understanding we have obtained in the specific example of the orbifolded conifold is sufficient for understanding the mechanism behind cascades arising at a large class of non-isolated singularities which we describe. In this class, we suggest a prescription for tracking the field theory in the supergravity regime analogous to the prescription applied to cascades proceeding through Seiberg duality [24].

We present some conclusions in section 7.

Table of symbols:

$\mathcal{F}_N$	*five complex dimensional family of CFTs described in §2.*
$\mathcal{S}_N$	*three complex dimensional family of CFTs described in §2.*
S_{N,P_2,P_4}	*a member of the discrete family of CFTs described in §3.*
$F_{l,s}$	l $\mathcal{N}=2$ *vectors interacting with* s *charged hypermultiplets.*
τ_i	*holomorphic gauge coupling of node* i.
λ_i	*'t Hooft coupling of node* i.
Λ_i	*strong coupling scale of node* i.

2. A family of conformal field theories

The quiver gauge theories that we consider arise by placing N D3 branes at the origin of a certain $\mathbb{Z}_2$-orbifold of the conifold, which we may write as a hypersurface in $\mathbb{C}^4$:

$$\mathcal{Z} := \{(x_1, x_2, x_3, x_4) \in \mathbb{C}^4 \mid x_1 x_2 - (x_3 x_4)^2 = 0\}. \tag{2.1}$$

This gives rise to a five-complex dimensional family of conformal field theories $\mathcal{F}_N$ with gauge group $\mathcal{G}$ and matter content [26]:[1]

$\mathcal{G}$	$U(N)_1$	$U(N)_2$	$U(N)_3$	$U(N)_4$
Φ_{22}	1	adj	1	1
Φ_{44}	1	1	1	adj
X_{12}, X_{21}	$\square, \overline{\square}$	$\overline{\square}, \square$	1	1
X_{23}, X_{32}	1	$\square, \overline{\square}$	$\overline{\square}, \square$	1
X_{34}, X_{43}	1	1	$\square, \overline{\square}$	$\overline{\square}, \square$
X_{41}, X_{14}	$\overline{\square}, \square$	1	1	$\square, \overline{\square}$

(2.2)

The exactly marginal couplings of the conformal field theory map to supersymmetric deformations of the IIB string background.

Throughout this work, we consider only a three-complex dimensional subspace of $\mathcal{F}_N$, which we denote by $\mathcal{S}_N$. This will suffice for studying a variety of cascades enjoying adjoint transitions, as well as interpolating between supergravity and field theory regimes.

A three-complex dimensional family of fixed points

Consider a family of effective field theories which are nearly free at some ultra-violet scale μ with gauge group and matter content as in (2.2) and superpotential:

$$\begin{aligned} \mathcal{W} = \; & h_1 \left(\Phi_{22} X_{21} X_{12} - \Phi_{22} X_{23} X_{32}\right) + h_2 \left(\Phi_{44} X_{41} X_{14} - \Phi_{44} X_{43} X_{34}\right) \\ & + (\eta/\mu) \left(X_{23} X_{34} X_{43} X_{32} - X_{12} X_{21} X_{14} X_{41}\right). \end{aligned} \tag{2.3}$$

The parameters h_1, h_2, η as well as the holomorphic gauge couplings, evaluated at μ, are tunable parameters.

For appropriate values of $\{\tau_1, \tau_3\}$, this theory is invariant under a transformation $\mathcal{J}_-$ which acts as a product of a discrete R-transformation and an outer automorphism of the gauge group, where the R-transformation acts multiplicatively with a factor of

[1] In order to see the adjoints one must be in the appropriate duality frame.

i on the gauginos and chiral superspace coordinates while keeping fixed the bottom component of all chiral matter superfields, and the outer automorphism exchanges the first and third gauge groups in (2.2).

Classically the holomorphic gauge couplings τ_1, τ_3 are simply exchanged, and thus the classical condition for $\mathcal{J}_-$ invariance is simply $\tau_1 = \tau_3$. However, due to an anomaly in the discrete R-symmetry, we have at the quantum level:

$$\mathcal{J}_- : (\Lambda_1^N, \Lambda_3^N) \to (-)^N (\Lambda_3^N, \Lambda_1^N). \tag{2.4}$$

Thus the action of $\mathcal{J}_-$ is only trivial if $\Lambda_1^N = (-)^N \Lambda_3^N$. We henceforth restrict to this subspace.

In the infrared, we assume the theory flows to a non-trivial fixed point. Imposing the vanishing of the beta-functions results in non-trivial constraints on the anomalous dimensions, and hence the ultra-violet parameters. We have:

$$\begin{aligned} \gamma_{\Phi_{44}} + 2\gamma_{X_{14}} &= 0 \\ \gamma_{\Phi_{22}} + 2\gamma_{X_{12}} &= 0 \\ 1 + 2\gamma_{X_{12}} + 2\gamma_{X_{14}} &= 0 \end{aligned} \tag{2.5}$$

where γ_ψ denotes the anomalous dimensions of a given field, ψ. The anomalous dimensions of the other fields are related to those appearing in (2.5) by symmetries:

$$\gamma_{X_{23}} = \gamma_{X_{12}}, \quad \gamma_{X_{34}} = \gamma_{X_{14}}, \quad \gamma_{X_{ij}} = \gamma_{X_{ji}}. \tag{2.6}$$

The conditions for a fixed point thus comprise three real constraints. Up to phase redefinitions of the fields, the family of effective field theories we consider has nine real parameters. Thus, we expect to obtain a complex three-dimensional family of conformal field theories, which we define to be $\mathcal{S}_N$.

The elements of $\mathcal{S}_N$ are acted on trivially by $\mathcal{J}_-$.

2.1 A holomorphic family of flows

Using $\mathcal{S}_N$ we define a family of flows. This is accomplished by deforming it's elements by relevant perturbations.

We may think of $\mathcal{S}_N$ as being parameterized by three of the original six holomorphic couplings, for example, $\{\tau_2, \tau_4, \eta\}$. We define two different regimes in this space of couplings:

$$\begin{aligned} &\textbf{Supergravity regime}: \quad N >> \operatorname{Im}\tau_2,\ \operatorname{Im}\tau_4 >> 1, \quad \eta \sim 1 \\ &\textbf{Field theory regime}: \quad \infty\ >\ \operatorname{Im}\tau_2,\ \operatorname{Im}\tau_4 >> N, \quad \eta << 1. \end{aligned} \tag{2.7}$$

Since the family of conformal field theories denoted by $\mathcal{S}_N$ holomorphically interpolate between these two regimes, we sometimes refer to its elements as 'interpolating' conformal field theories.

$\mathcal{S}_N$ gives rise to a family of renormalization group flows by turning on vacuum expectation values for the adjoint fields. Each element in $\mathcal{S}_N$ is arranged to result in a rank $(N, N-P_2, N, N-P_4)$ theory at some common scale ν, for some $P_2, P_4 > 0$. Furthermore we take $P_2, P_4 << N$, as we are interested in discussing cascades.

A flow is said to be in the field theory regime, supergravity regime or neither according to the element in $\mathcal{S}_N$ from which it arises.

We note that these flows are also acted on trivially by $\mathcal{J}_-$. This will play an important role in the discussion of non-perturbative corrections.

3. A discrete family of fixed points

Here we describe a discrete family of fixed points which play an important role in the analysis of §4.

Consider the theory with matter content and superpotential:

$\mathcal{G}$	$U(N)_1$	$U(N+M_2)_2$	$U(N)_3$	$U(N+M_4)_4$
Φ_{22}	1	adj	1	1
Φ_{44}	1	1	1	adj
X_{12}, X_{21}	$\square, \overline{\square}$	$\overline{\square}, \square$	1	1
X_{23}, X_{32}	1	$\square, \overline{\square}$	$\overline{\square}, \square$	1
X_{34}, X_{43}	1	1	$\square, \overline{\square}$	$\overline{\square}, \square$
X_{41}, X_{14}	$\overline{\square}, \square$	1	1	$\square, \overline{\square}$

(3.1)

$$\mathcal{W} = \ h_1(\Phi_{22}X_{21}X_{12} - \Phi_{22}X_{23}X_{32}) + h_2(\Phi_{44}X_{41}X_{14} - \Phi_{44}X_{43}X_{34}), \qquad (3.2)$$

in the limit $\mathrm{Im}\,\tau_3 = \mathrm{Im}\,\tau_3 = \infty$.

This system allows for a fixed point for a wide range of $\{N, M_2, M_4\}$. We refer to such fixed points as S_{N,M_2,M_4} conformal field theories. Their union comprises the discrete family which play an important role in §4.

The S_{N,M_2,M_4} fixed points consist of three decoupled sectors. Two of them correspond to a copy of free $\mathcal{N}=1$ SYM. Since these just go on for the ride and so we ignore them in the following discussion. The third sector consists of an interacting fixed point involving nodes 1 and 3 of the quiver.

The fixed point is such that there is a symmetry that exchanges nodes 1 and 3, involving also a flip in sign of the superpotential - $\mathcal{J}_-$ invariance. This symmetry guarantees that:

$$\gamma_{12} = \gamma_{23}, \quad \gamma_{14} = \gamma_{34}. \tag{3.3}$$

Note however that it is not necessarily the case that $\gamma_{12} = \gamma_{14}$. The symmetry that could guarantee this is broken for $M_2 \neq M_4$. We will consider the case $M_2 = M_4$ separately. For now we assume $M_2 \neq M_4$. After imposing the symmetry constraints, the non-trivial conditions for a fixed point are:

$$\begin{aligned} \gamma_{22} + 2\gamma_{12} &= 0 \\ \gamma_{44} + 2\gamma_{14} &= 0 \\ N(1 + 2\gamma_{12} + 2\gamma_{14}) - M_2(1 - 2\gamma_{12}) - M_4(1 - 2\gamma_{14}) &= 0 \end{aligned} \tag{3.4}$$

Thus we have three constraints on four unknowns. This is not enough to determine the anomalous dimensions. We can solve this problem using the technique of a-maximization [28]. The analysis is simplified a great deal in the regime $M_i/N << 1$ which is the regime of interest to us.

Let us label the R-charge of X_{12} by t. The R-charges of the other fields are determined in terms of t via the symmetries and constraints. The value of t corresponding to the superconformal $U(1)_R$ maximizes $a(R)$,

$$a(R) := 3\mathrm{Tr}R^3 - \mathrm{Tr}R \tag{3.5}$$

where R is the generator of the candidate superconformal $U(1)_R$, and the trace is over fermion species. The result of this maximization procedure is:

$$t = 1/2 + \frac{5M_2 + 4M_4}{18N} - \frac{(11M_2^2 + 16M_2M_4 + 9M_4^2)}{72N^2} + \mathcal{O}(M_i^3/N^3), \tag{3.6}$$

where we have included the $\mathcal{O}(M_i^2/N^2)$ term although we do not make use of it in our analysis in §4. It will be needed in the next subsection. In the case of $M_2 = M_4 := M$ we may impose a symmetry that guarantees:

$$\gamma_{12} = \gamma_{23} = \gamma_{14} = \gamma_{34} =: \gamma_X \tag{3.7}$$

In this case the anomalous dimensions are uniquely determined to be:

$$\gamma_{X_{ij}} = 1/2 - \frac{3N}{2(2N + 2M)}, \quad \gamma_{\Phi_{ii}} = 1/2 + \frac{3P}{2(N + M)}. \tag{3.8}$$

Of course this formula only holds in the region $3N/2 < 2N + 2M < 3N$ where it satisfies unitarity bounds [27].

3.1 Duality

In §4 it will seem important to our derivations that we can Seiberg dualize nodes 1 and 3 independently at a $S_{N,-P_2,-P_4}$ fixed point, even though they are strongly mixed by the superpotential interactions. We can justify this in the following way. Consider the theory with superpotential:

$$\mathcal{W} = \; h_1 \Phi_{22} X_{21} X_{12} + h_2 \Phi_{44} X_{41} X_{14} \tag{3.9}$$

and vanishing $\lambda_{2,4}$. In this case nodes 1 and 3 are completely decoupled. Node 3 is ordinary $\mathcal{N} = 1$ SQCD. We take it to sit at it's Seiberg fixed point [27]. It enjoys the usual Seiberg duality. On the other hand, node 1 is similar to magnetic SQCD but with the off-diagonal mesons "Φ_{24}, Φ_{42}" deleted. We refer to it as the magnetic SQCD-like theory. It is quite plausible that this theory harbors a fixed point. The conditions for this are:

$$\begin{aligned} \gamma_{22} + 2\gamma_{12} &= 0 \\ \gamma_{44} + 2\gamma_{14} &= 0 \\ N(1 + 2\gamma_{12} + 2\gamma_{14}) + P_2(1 - 2\gamma_{12}) + P_4(1 - 2\gamma_{14}) &= 0 \end{aligned} \tag{3.10}$$

Note that because of the unequal ranks of nodes 2 and 4 there is no reason for γ_{12} and γ_{14} to be equal. As in the previous section we have three constraints on four unknowns and we must use a-maximization to pin down the anomalous dimensions. If we label the R-charge of X_{12} by t we have:

$$t = 1/2 - \frac{11P_2 + 7P_4}{36N} - \frac{17P_2^2 + 14P_2P_4 + 5P_4^2}{72N^2} + \mathcal{O}(P_i^3/N^3). \tag{3.11}$$

The R-charges of the other fields at node 1 are determined in terms of t via (3.10). For $P_2 = P_4 =: P$ the anomalous dimensions have a compact expression:

$$\gamma_{X_{ij}} = 1/2 - \frac{3N}{2(2N - 2P)}, \quad \gamma_{\Phi_{ii}} = 1/2 + \frac{3P}{2(N - P)}. \tag{3.12}$$

where we require $3N/2 < 2N - 2P < 3N$ in order to be consistent with unitarity bounds on dimensions of gauge-invariant operators.

For $P_2 \neq P_4$ can obtain the $S_{N,-P_2,-P_4}$ fixed points of the previous section via RG flow from the fixed points considered here. This is achieved by perturbing the theory by the superpotential interactions:

$$\delta W = -h_1' \Phi_{22} X_{23} X_{32} - h_2' \Phi_{44} X_{43} X_{34} \tag{3.13}$$

For $P_2 \neq P_4$ one of these two couplings is always relevant and thus induces a non-trivial RG flow. Because no accidental $U(1)$s are generated along the hypothetical flow between this fixed point and the $S_{N,-P_2,-P_4}$ fixed point, it must be the case that the value of the anomaly coefficient a decreases along the flow - yielding a consistency check on the existence of such a flow [28].[2] We can check that this is indeed the case. A straightforward computation yields:

$$\begin{aligned} a_{\rm IR} &= N^2 \left(1 - \frac{4}{N}(P_2+P_4) - \frac{63(P_2+P_4)^2 - (P_2-P_4)^2}{36N^2} + \mathcal{O}(P_i^3/N^3) \right) \\ a_{\rm UV} &= N^2 \left(1 - \frac{4}{N}(P_2+P_4) - \frac{63(P_2+P_4)^2 - 2(P_2-P_4)^2}{36N^2} + \mathcal{O}(P_i^3/N^3) \right) \end{aligned} \tag{3.14}$$

Thus,

$$a_{\rm IR} = a_{\rm UV} - N^2 \left(\frac{1}{36N^2}(P_2-P_4)^2 + \mathcal{O}(P_i^3/N^3) \right) \tag{3.15}$$

and so indeed $a_{\rm IR} < a_{\rm UV}$.

Now we come to the main point. The UV fixed point enjoys Seiberg duality acting on node 3. It is natural to conjecture that Seiberg duality holds on node 1 as well. Since this duality is exact in the UV it is exact everywhere along the subsequent flow, including at the bottom.[3] Thus, assuming that Seiberg duality holds in the UV at node 1, then Seiberg duality holds independently on the two nodes of the $S_{N,-P_2,-P_4}$ fixed point.

Similar reasoning holds in the case $P_2 = P_4$. In this case, using the techniques of Leigh and Strassler we see that this fixed point is part of a larger fixed line [30]. This fixed line includes the $S_{N,-P_2,-P_4}$ theory. If Seiberg duality applies node-wise at one point on this fixed line it will hold node-wise on any other point on the fixed line, in particular at the point corresponding to $S_{N,-P_2,-P_4}$.

Thus all we are left to prove is that Seiberg duality holds for the magnetic SQCD-like theory at node 1. To do so consider magnetic SQCD at it's fixed point. It has superpotential:

$$\mathcal{W} = \Phi_{22}X_{21}X_{12} + \Phi_{24}X_{41}X_{12} + \Phi_{42}X_{21}X_{14} + \Phi_{44}X_{41}X_{14}. \tag{3.16}$$

Consider adding a free sector consisting of singlets M_{24} and M_{42}. The theory consisting of both sectors trivially enjoys Seiberg duality with the duality acting trivially on

[2]When accidental symmetries are generated along the flow it is possible to construct a counter-example to the hypothesis that $a_{\rm IR} < a_{\rm UV}$ always [29].

[3]Exact Seiberg duality along RG flows appears, for example, in the Klebanov-Strassler model [20].

the free sector. Now consider perturbing the fixed point by the following relevant superpotential interactions:

$$\delta W = M_{24}\Phi_{42} + M_{42}\Phi_{24}. \tag{3.17}$$

Upon integrating out the massive modes we are left exactly with the magnetic SQCD-like theory. On the other hand, the UV fixed point enjoys an exact Seiberg duality as it is ordinary magnetic SQCD coupled to a free sector. Because the duality is exact in the UV, it is exact everywhere along the flow and in particular at the bottom. Thus the magnetic SQCD-like theory enjoys Seiberg duality. This completes the proof that we are free to act with Seiberg duality independently on nodes 1 and 3 of the $S_{N,-P_2,-P_4}$ fixed point.

3.2 Flows

In this section we study perturbations of a $S_{N'P_4P_2}$ fixed point by λ_2, λ_4. As is seen in §4, this is the key to understanding the adjoint transitions. We are interested in the case $P_i > 0$ where λ_2, λ_4 are relevant.[4]

It is easily seen that the Coulomb branch of the $S_{N'P_4P_2}$ theory is not lifted in the presence of λ_2, λ_4 perturbations. Consider a vacuum of the form:

$$\Phi_{22} \;= \text{diag}\{0,\ldots,0,\phi_1,\ldots,\phi_{P_4+k_2}\}, \quad \Phi_{44} = \text{diag}\{0,\ldots,0,\phi'_1,\ldots,\phi'_{P_2+k_4}\} \tag{3.18}$$

for some $k_2, k_4 \geq 0$. An effective potential $\int d^2\theta\ \delta W_{\rm np}(\phi_i, \phi'_j)$ is odd under $\mathcal{J}_-$ and hence forbidden. Thus ϕ_i, ϕ'_j parameterize exactly flat directions. We note that for less symmetric flows we know of no principle which would prevent such corrections.[5]

Below the scale of spontaneous gauge symmetry breakdown λ_2 and λ_4 are infrared free or conformal. Subsequent flow results in a $S_{N',-k_2,-k_4} \times F_{k_2+P_4,0} \times F_{k_4+P_2,0}$ fixed point, where in general $F_{l,s}$ denotes the infrared limit of $\mathcal{N} = 2\ U(1)^l$ Abelian gauge

[4]In the vicinity of the fixed point, the beta functions are easily computed using (3.6):

$$\beta_{\lambda_{2,4}} = -\frac{\lambda_{2,4}^2 f_{2,4}}{8\pi^2}\left(\frac{3P_{4,2}}{N'} + \mathcal{O}(P_i^2/N'^2)\right),$$

where $f_{2,4}$ are positive but scheme dependent functions. Thus λ_2, λ_4 are relevant perturbations when $P_2, P_4 > 0$.

[5]In fact, the functional form appears in general undetermined by the selection rules and holomorphy. In such cases even inverse powers of the moduli appear to be allowed, providing the possibility for potentially dangerous instabilities.

theory interacting with s charged hypermultiplets. This is easily seen to be true for values of the moduli such that the Higgsing occurs at scales where λ_2, λ_4 are still weakly coupled. Holomorphy then implies that it remains true for generic values of the ϕ_i, ϕ'_j, including those for which λ_2, λ_4 deviate significantly from small values before the spontaneous breakdown occurs.

Thus for generic vacua of the form (3.18) the endpoint of the flow from the $S_{N'P_4P_2}$ fixed point perturbed by λ_2, λ_4 is a $S_{N',-k_2,-k_4} \times F_{k_2+P_4,0} \times F_{k_4+P_2,0}$ conformal field theory.

Non-generic points

For non-generic values of the ϕ_i, ϕ'_j the endpoint of the flow may differ from an $S_{N',-k_2,-k_4} \times F_{k_2+P_4,0} \times F_{k_4+P_2,0}$ conformal field theory. To understand the possibilities consider the theory with matter content as in (2.2) but with gauge group $\mathcal{G} = U(N')_1 \times U(N'+P_4)_2 \times U(N')_3 \times U(N'+P_2)_4$ and superpotential:

$$\mathcal{W} = \sqrt{2}\left(\Phi_{22}X_{21}X_{12} - \Phi_{22}X_{23}X_{32}\right) + \sqrt{2}\left(\Phi_{44}X_{41}X_{14} - \Phi_{44}X_{43}X_{34}\right). \quad (3.19)$$

We consider this theory to be near the free fixed point at some ultraviolet scale μ' which we imagine to be much larger than any other scale in the problem. We further impose $\mathcal{J}_-$ invariance, which implies $\Lambda_1^{N'} = (-)^{N'}\Lambda_3^{N'}$, as discussed in §2.

We define two regimes:

$$\begin{aligned} &\textbf{Regime A}: \ |\Lambda_{1,3}| >> |\Lambda_2|, |\Lambda_4| \neq 0, \\ &\textbf{Regime B}: \ |\Lambda_2|, |\Lambda_4| >> |\Lambda_{1,3}| \neq 0. \end{aligned} \quad (3.20)$$

In regime A the flow merges onto that of the $S_{N'P_4P_2}$ fixed point perturbed by λ_2, λ_4 non-vanishing. Thus the endpoints of the flow from the $S_{N'P_4P_2}$ fixed point perturbed by λ_2, λ_4 are contained in those of regime A.

Regime A is holomorphically connected to regime B and so the endpoints of flows from the $S_{N'P_4P_2}$ fixed point due to λ_2, λ_4 perturbations can be understood by studying flows in regime B. In fact, if the endpoints of flows in a given regime are isolated fixed points, then not only will the endpoints of flows of regime A and B be identical as phases, they will also be identical as conformal field theories.

The usefulness of these observations lies in the fact that regime B is described over a wide range of scales by two copies of $\mathcal{N} = 2$ SQCD, weakly perturbed by the non-zero gauge couplings at nodes 1 and 3. The breaking to $\mathcal{N} = 1$ can only be felt at much lower scales corresponding to Λ_1, Λ_3.

The infrared phases of asymptotically free $\mathcal{N}=2$ SQCD can be understood in terms of spontaneous breakdown of the gauge group [21]. The scale at which spontaneous breakdown occurs is bounded below by the strong coupling scale of the theory. In particular, an infrared free or conformal subgroup emerges at a scale of order the strong coupling scale. This emergence is an adjoint transition. The boundedness of scale of the adjoint transition implies a decoupling of the non-trivial $\mathcal{N}=2$ physics from the $\mathcal{N}=1$ physics represented by the non-vanishing of Λ_1, Λ_3. This in turn implies that the infrared phases of the flows of regime B are classified by a choice of vacuum in the theory consisting of two copies of $\mathcal{N}=2$ SQCD. This is a powerful observation because the vacuum structure of $\mathcal{N}=2$ SQCD is known exactly [21].[6]

We are interested in vacua consistent with cascades.[7] Thus, we restrict to phases arising from the Coulomb branch. Such vacua are classified by four non-negative integers $\{k_2, k_4; r, r'\}$ which specify the amount of dynamical gauge symmetry breaking and the number monopole hypermultiplets respectively [21].

Below the scale of the transition the theory consists of two irrelevantly coupled sectors. The first is a theory with identical form and superpotential as before the transition except with gauge group $\mathcal{G} = U(N')_1 \times U(N'-k_2)_2 \times U(N')_3 \times U(N'-k_4)_4$. The beta functions for the couplings at nodes 1 and 3 continue to be large and negative. Thus, the resulting flow drives this sector of the theory into an $S_{N',-k_2,-k_4}$ conformal field theory. The second sector is composed of $k_2 + P_4 + k_4 + P_2$ $\mathcal{N}=2$ Abelian vector multiplets and $r + r'$ monopole hypermultiplets.

Let us denote by $\{\psi_j\}_{j=1}^{k_2+P_4+k_4+P_2}$ and $\{e_l, \tilde{e}_l\}_{l=1}^{r+r'}$ the moduli and monopole fields respectively in the vicinity of a monopole point. Consider the extreme limit of regime B with $\Lambda_1, \Lambda_3 \equiv 0$. The moduli and monopoles have superpotential couplings:

$$W = \sqrt{2} \sum_{jl} Q_{jl} \psi_j e_l \tilde{e}_l, \tag{3.21}$$

where Q_{jl} is the charge matrix.[8] Because of $\mathcal{N}=2$ supersymmetry, (3.21) is exact. However, away from $\Lambda_1, \Lambda_3 \equiv 0$ this expression may receive non-perturbative corrections.

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[6] We note that not all vacua of the two copy $\mathcal{N}=2$ SQCD theory lead to stable vacua. For example, a generic point on the Coulomb branch gives masses to all of the quark fields. This results in gaugino condensation at nodes 1 and 3 which in turn generates a destabilizing potential for the moduli. This is possible because the value of the condensate is a function of the quark masses which in turn are a function of the moduli.

[7] By this we mean transitions which preserve the general form of the quiver and result in small rank reductions.

[8] These matrices are calculated in [21].

As discussed previously, away from $\Lambda_1, \Lambda_3 \equiv 0$ the moduli $\{\psi_j\}$ remain exactly massless and continue to parameterize flat directions. Thus, the only corrections which threaten to remove the massless monopole points as vacua in the theory are monopole billinears:

$$\delta W = \sum_i m_i e_i \tilde{e}_i, \tag{3.22}$$

where the m_i may in general be allowed to depend on the moduli.

However, in regime B such a mass term is necessarily small compared to the scale of the transition which is comparable to $\Lambda_{2,4}$ and thus may be self-consistently absorbed by a shift in the moduli if one so chooses. Thus the second sector flows into a $F_{k_2+P_4,r} \times F_{k_4+P_2,r'}$ factor. The possibilities for $\{r, r'\}$ as well as the monopole charge matrix are exactly determined by the two copies of $\mathcal{N} = 2$ SQCD.

Thus the result of the endpoints of flows in regime B are $S_{N',-k_2,-k_4} \times F_{k_2+P_4,r} \times F_{k_4+P_2,r'}$ conformal field theories. Analyticity implies that the same holds for flows in regime A.

However, this does not yet imply that all such phases are realized in the $S_{N'P_4P_2}$ theory perturbed by λ_2, λ_4. The reason is that upon continuation to regime A the shift in the moduli required to remove the monopole mass term may in principle run off to vacuum expectation values corresponding to scales larger than those where the flow in regime A begins to yield an accurate approximation of the $S_{N'P_4P_2}$ theory perturbed by λ_2, λ_4. It is easily seen that this does not occur:

First, assume that some of the m_i are non-vanishing. Consider the R-symmetry under which the microscopic variables Φ_{ii} and X_{ij} have charge $0, 1$ respectively. From (3.21) it is easily deduced that the monopole fields $e_i, \tilde{e}_i$ have charge $+1$ under this transformation. Thus the m_i must be neutral. However, under this R-symmetry the holomorphic strong coupling scales of nodes 2 and 4 are neutral, while those of 1 and 3 have positive definite charge. However, this is inconsistent with the requirement that the $m_i \to 0$ as $\Lambda_1, \Lambda_3 \to 0$ in various ways. Thus, the m_i must vanish identically.

Therefore the required shift in the moduli is trivial. Thus, all of the $S_{N',-k_2,-k_4} \times F_{k_2+P_4,r} \times F_{k_4+P_2,r'}$ phases are realized in the $S_{N'P_4P_2}$ theory perturbed by λ_2, λ_4.

We thus find that the adjoint transitions are correctly described by approximating each strongly coupled adjoint node by a copy of $\mathcal{N} = 2$ SQCD.

4. Flows in the field theory regime

We now turn to the problem of studying the flows in the field theory regime, defined in §2.

We consider first the simplest such flow corresponding to an element $s_0 \in \mathcal{S}_N$ with $\lambda_2 = \lambda_4 = \eta = 0$. The infrared limit of this theory is the $S_{N,-P_2,-P_4}$ fixed point defined in §3.

4.1 Seiberg transitions

Consider now flows originating not from s_0 but from nearby s_0 - from elements of $\mathcal{S}_N$ with small but non-zero $\{\lambda_2, \lambda_4, \eta\}$. Such flows can be arranged to pass arbitrarily closely to the $S_{N,-P_2,-P_4}$ fixed point. We denote the scale at which the theory is closest to this fixed point by μ_0.

We assume sufficiently small $\{\lambda_2, \lambda_4, \eta\}_{\mu_0}$ such that conformal perturbation theory is useful. The leading terms in the β-functions near the fixed point may be then computed using (3.9):[9]

$$\begin{aligned} \beta_{\lambda_{2,4}} &= \frac{\lambda_{2,4}^2 f_{2,4}}{8\pi^2}\left(\frac{3P_{2,4}}{N} + \mathcal{O}(P_i^2/N^2)\right) \\ \beta_\eta &= -\eta\left(\frac{3P_2+3P_4}{2N} + \mathcal{O}(P_i^2/N^2)\right). \end{aligned} \tag{4.1}$$

Thus, near the $S_{N,-P_2,-P_4}$ fixed point the 't Hooft couplings λ_2 and λ_4 are irrelevant while the quartic coupling η is relevant. The growth of η thus drives the flow away from the fixed point and conformal perturbation theory eventually ceases to be useful.

In order to understand how to follow the resulting flow let us consider the case of $\mathcal{N}=1$ SQCD with $3n_c/2 < n_f < 2n_c$ at it's interacting fixed point. We deform the fixed point by the quartic superpotential:

$$W = \eta\, Q\tilde{Q}Q\tilde{Q}. \tag{4.2}$$

Because $n_f < 2n_c$ the anomalous dimensions are such that this operator is relevant [20,27]. In order to understand the resulting flow we change duality frames to magnetic SQCD. The dual theory with the η-deformation has superpotential [20]:

$$W_{\text{dual}} = M\tilde{q}q + \eta M^2. \tag{4.3}$$

Thus η has been transformed into a mesonic mass term. When the physical value of η approximates unity we integrate out M, yielding:

$$W_{\text{dual}} = -\frac{1}{4\eta}\tilde{q}q\tilde{q}q. \tag{4.4}$$

[9]The f_i are scheme dependent functions of the couplings which are positive [20,27,31].

Because $n_f > 2(n_f - n_c)$ the anomalous dimensions are such that this quartic deformation is irrelevant and so flows to zero in the IR. Thus the theory flows to the interacting fixed point of $SU(n_f - n_c)$ SQCD. The singlets are no longer present.

Thus in order to follow the RG flow into and past the region where η becomes strong we switch duality frames, replacing nodes 1 and 3 by their Seiberg duals, and integrate out massive modes.[10] The result is:

$$\begin{aligned} W &= \tilde{\Phi}_{22}\tilde{X}_{21}\tilde{X}_{12} + \tilde{\Phi}_{44}\tilde{X}_{41}\tilde{X}_{14} - \tilde{\Phi}_{22}\tilde{X}_{23}\tilde{X}_{32} - \tilde{\Phi}_{44}\tilde{X}_{43}\tilde{X}_{34} \\ &+\tilde{\eta}\ \tilde{X}_{23}\tilde{X}_{34}\tilde{X}_{43}\tilde{X}_{32} - \tilde{\eta}\ \tilde{X}_{21}\tilde{X}_{14}\tilde{X}_{41}\tilde{X}_{12} \end{aligned} \tag{4.5}$$

with $\tilde{\Phi}_{22} = M_{22} = \tilde{M}_{22}$ and $\tilde{\eta} = -1/\eta$.

The theory is back to a similar form as before except now the flow is near an $S_{N'P_4P_2}$ fixed point, with $N' = N - P_2 - P_4$. As in our example with ordinary SQCD, the quartic couplings are now irrelevant and were it not for the non-zerodness of $\lambda_{2,4}$, the theory would flow to a fixed point. The beta functions are:

$$\begin{aligned} \beta_{\lambda_{2,4}} &= -\frac{\lambda_{2,4}^2 f_{2,4}}{8\pi^2}\left(\frac{3P_{4,2}}{N'} + \mathcal{O}(P_i^2/N'^2)\right) \\ \beta_{\tilde{\eta}} &= \tilde{\eta}\left(\frac{3P_2 + 3P_4}{2N'} + \mathcal{O}(P_i^2/N'^2)\right) \end{aligned} \tag{4.6}$$

Thus, while the quartic couplings are irrelevant, the adjoint node gauge couplings are relevant, and their growth appears to drive the theory away from a known fixed point. This is the onset of an adjoint transition.

4.2 Adjoint transitions

We denote the scale at which the flow is closest to the $S_{N'P_4P_2}$ fixed point by μ_1. The theory at scales $\mu < \mu_1$ can be understood as a perturbation of this fixed point by $\{\lambda_2, \lambda_4, \tilde{\eta}\}_{\mu_1}$.

Since the quartic perturbation is initially small and irrelevant the problem of understanding the effects of the growth of λ_2, λ_4 reduces to to an analysis of $S_{N'P_4P_2}$ perturbed by $\{\lambda_2, \lambda_4, 0\}_{\mu_1}$. This problem is studied in §3 with the conclusion that as the couplings grow they induce spontaneous breakdown of the gauge symmetry. For example, if λ_2 is the first to reach strong coupling, then just below its strong coupling scale the theory has a description as a perturbation of $S_{N',-k_2,P_2} \times F_{P_4+k_2,r}$. Here $F_{l,s}$ denotes the infrared limit of $\mathcal{N} = 2$ $U(1)^l$ Abelian gauge theory interacting with s

[10] This is examined more carefully in §3.1.

charged hypermultiplets. The β-functions near the $S_{N',-k_2,P_2} \times F_{P_4+k_2,r}$ fixed point read:

$$\begin{aligned}
\beta_{\lambda_2} &= \frac{\lambda_2^2 f_2}{8\pi^2}\left(\frac{3k_2}{N'} + \mathcal{O}(P_2^2/N'^2, k_2^2/N'^2)\right) \\
\beta_{\lambda_4} &= -\frac{\lambda_4^2 f_4}{8\pi^2}\left(\frac{3P_2}{N'} + \mathcal{O}(P_2^2/N'^2, k_2^2/N'^2)\right) \\
\beta_{\tilde{\eta}} &= \tilde{\eta}\left(\frac{3P_2 - 3k_2}{2N'} + \mathcal{O}(P_2^2/N'^2, k_2^2/N'^2)\right).
\end{aligned} \tag{4.7}$$

Thus, after the transition λ_2 becomes an irrelevant perturbation, while λ_4 continues to be relevant.

If the quartic coupling remains non-relevant ($k_2 \leq P_2$) then we may continue to ignore it. The coupling λ_4 will grow until its strong coupling scale is reached and a second adjoint transition occurs. The resulting flow drives the theory to a perturbation of $S_{N',-k_2,-k_4} \times F_{P_4+k_2,r} \times F_{P_2+k_4,r'}$.

On the other hand, if the quartic coupling is relevant as a perturbation of $S_{N',-k_2,P_2} \times F_{P_4+k_2,r}$ ($k_2 > P_2$) and the strong coupling scale of λ_4 is sufficiently small, then it may grow to unity before the λ_4 transition occurs. In this case a Seiberg transition occurs as described in the previous section, taking the flow near a $S_{N'+P_2-k_2,-P_2,k_2} \times F_{P_4+k_2,r}$ fixed point. Around this fixed point λ_4 is the only remaining relevant coupling, and so the second adjoint transition occurs uninterrupted. The result is to take the theory near a $S_{N'+P_2-k_2,-P_2,-P_4'} \times F_{P_4+k_2,r} \times F_{k_2+P_4',r'}$ fixed point.

In either case the result of the flow from $S_{N'P_4P_2}$ perturbed by $\{\lambda_2, \lambda_4, \tilde{\eta}\}_{\mu_1}$ is to induce two adjoint transitions, one each on nodes 2 and 4, after which the flow is driven near a $S_{N'',-k_2,-k_4} \times F_{l,s} \times F_{l',s'}$ fixed point. The precise value of (N'', l, s, l', s') as a function of (N, P_2, P_4, k_2, k_4) is determined by which of the two cases described above is realized.

4.3 Subsequent cascade steps

As we have seen in the previous subsection, the result of the adjoint transitions is to take the flow back near a $S_{N'',-k_2,-k_4} \times F_{l,s} \times F_{l',s'}$ fixed point. We denote the scale at which the flow is closest to this fixed point by μ_2.

For $k_2 = k_4 = 0$, $\{\lambda_2, \lambda_4, \eta\}_{\mu_2}$ are exactly marginal perturbations and the flow terminates onto a conformal field theory. In fact, up to the $F \times F$ factor, the flow terminates onto an element of $S_{N'}$.

In the case of $k_2, k_4 > 0$, around μ_2 η is relevant while λ_2, λ_4 are irrelevant. The situation is identical to the situation around (4.1) except with the replacements $(N, P_2, P_4) \to (N'', k_2, k_4)$ and the additional $F \times F$ factor.

As in that situation, the resulting flow due to the growing quartics is best described by performing a Seiberg duality on nodes 1 and 3. Integrating out the massive modes takes the flow back to a $S_{N''k_4k_2} \times F \times F$ fixed point with $N''' = N'' - k_2 - k_4$. This is identical to the situation around (4.6) except with $(N', P_2, P_4) \to (N'', k_2, k_4)$ and the additional $F \times F$ factor. As in that situation $\lambda_{2,4}$ grow and additional adjoint transitions occur, and so on, resulting in a cascade.

Thus we find a family self-similar and cascade-like flows, which alternate between regions where the physics is close to an $S \times F \times F$ fixed point followed by regions where $\mathcal{N} = 2$ type dynamics is dominant and Higgsing occurs. Thus these cascades proceed through a combination of Seiberg duality and Higgsing in an alternating manner. When the ranks of the nodes are no longer large and nearly equal some of the approximations we have been making break down and a more refined analysis is needed.

Remarks

- In describing the RG flow, we have implicitly been assuming that a faithfull description is given by tracking a few superpotential couplings $\{\lambda_2, \lambda_4, \eta\}$. In reality, there are a large number of Kahler couplings generated along the RG flow which our analysis ignores. This is justified when such corrections are irrelevant. Thus, if $\delta\mathcal{K}$ is any non-redundant correction to the Kahler potential, then we require:

$$\Delta_{\delta\mathcal{K}} - 2 > 0. \tag{4.8}$$

This is easily satisfied if the theory is weakly interacting, and may be plausibly satisfied in the theories under consideration here.

We note that the irrelevance of such corrections in interacting theories is also implicit in Strassler's analysis of the conifold, which is similar in spirit to the analysis here [20]. A check on whether such corrections can be justifiably neglected is whether our conclusions are in qualitative agreement with supergravity. This is discussed in §5.

4.4 $\mathcal{J}_-$ non-invariant flows

In §2 we chose our flows to arise as relevant deformations of a special co-dimension 2 subspace $\mathcal{S}_N \subset \mathcal{F}_N$, which was defined by the property that the transformation $\mathcal{J}_-$, defined in §2, acted trivially on its elements. The more generic case corresponds to selecting an element from $\mathcal{F}_N \setminus \mathcal{S}_N$. The resulting flows are considerably more complicated as there are effectively five different couplings instead of three.

Furthermore, in such cases, we know of no principle which forbids corrections to the Coulomb branch of the kind ruled out in §3.2. The super-selection rules and dynamical considerations do not seem to sufficiently constrain the form of $\delta W_{\rm np}$. It would be interesting if such constraints could be found or the severity of the corrections better understood.

5. The supergravity regime

We discuss the supergravity limit of the above discussions. Explicit supergravity solutions for a class of flows similar to those considered here have been constructed in [22].

The cascades discussed above proceeded through a combination of Seiberg duality and the spontaneous breakdown of gauge symmetry. The spontaneous breakdown events were spread along the renormalization group scale in a hierarchical manner. This hierarchy is to a certain extent holomorphic, as the breakdown events are specified in terms of vacuum expectation values of holomorphic fields. Thus we expect this hierarchy to continue analytically to a hierarchy in the supergravity regime.

In the supergravity regime, the renormalization group scale is an emergent dimension along which physics is approximately local. The deconfined degrees of freedom produced at each Higgsing event (the monopoles, moduli, and U(1)s) must be localized along this dimension in a manner consistent with the stretch of RG time over which they deconfine and decouple - ie: "peel off" - from the original non-Abelian degrees of freedom which source the bulk geometry [32].

The bulk geometry in the region where the theory is cascading and approximately conformal has the metric [22]:

$$ds^2 = h(r)^{-1/2}dx^2 + h(r)^{1/2}(dr^2 + (dT_{1,1}/\mathbb{Z}_2)^2) \tag{5.1}$$

where $T_{11}/\mathbb{Z}_2$ denotes a $\mathbb{Z}_2$ orbifold of the space T_{11} as described in [22] and the coordinate r is related to the RG scale μ via $\mu = r/\alpha'$.

The orbifold produces a singularity stretching along r whose topology is locally $\mathbb{R} \times S^1 \cong \mathbb{C}$. Because adjoint fields are naturally identified with the motions of fractional branes along the non-isolated singularity [4, 16], it is natural to expect that the deconfined degrees of freedom are localized around specific radial locations along the singular locus of the bulk geometry. This is indeed seen in [22].

In [22] a common feature of all of the studied flows was that the adjoint transitions caused a reduction in rank of the gauge group in a manner matching the numerology of Seiberg duality. Such numerology is easily reproduced by the flows of the previous sections by choosing the Higgsing vacua appropriately.

Our field theory analysis, continued to the supergravity regime, adds to previous results in the supergravity regime in several ways. First, it establishes (at least for $\mathcal{J}_-$ invariant flows) the presence of exactly flat moduli and the existence of special points with massless monopoles in these string backgrounds. Second, our approach sheds light on the field theory mechanism behind the adjoint transitions, and allows the derivation of the precise low energy effective field theory for a large family of flows in a unified manner. This makes manifest the fact that the adjoint transitions need not follow the numerology of Seiberg duality; such flows are in fact more generic.

6. More general quivers and a prescription

We have seen that the adjoint transitions are well approximated by replacing each strongly coupled adjoint node by a copy $\mathcal{N} = 2$ SQCD. We expect this to hold quite generally for any $\mathcal{N} = 1$ QNIS.

When a cascade is in the supergravity regime, it is often still useful to have a field theory description for the renormalization group flow. For quivers based on isolated singularities, a prescription for keeping track of the field theory is to [24, 25]:

- Compute the beta functions.
- Run the inverse couplings until one approximately reaches zero.
- Apply a Seiberg duality on the node whose inverse coupling approximates zero.
- Integrate out massive mesons.
- Recompute the beta functions for updated matter content.
- Repeat.

When the node approaching strong coupling has adjoint matter, Seiberg duality cannot be applied. Instead our results point to the prescription:

- Approximate the strongly coupled node by a copy of $\mathcal{N} = 2$ QCD.
- Choose a point on the Coulomb branch.
- Integrate out massive modes, and update the Wilsonian effective action.
- Recompute the beta functions for updated matter content.
- Determine next node to hit strong coupling.

We have found strong evidence that this is the correct prescription for flows at the $\mathbb{Z}_2$ orbifolded conifold (at least for sufficiently symmetric flows), and it is natural to conjecture that it holds for any QNIS (at least for sufficiently symmetric flows).[11]

[11] We place the qualifier "for sufficiently symmetric flows" because we expect the necessity to impose a symmetry, in order to retain calculability, will continue in other quivers as well. For the flows

7. Conclusions

Our main interest in this paper was to understand the RG flows of $\mathcal{N}=1$ quiver gauge theories with adjoint matter, coupled in an $\mathcal{N}=2$ like manner to the rest of the theory. Such theories commonly arise from fractional brane configurations at non-isolated singularities and form an infinite class of theories.

The main hinderance in understanding these theories thus far has been the adjoint matter. The field theory dynamics in the regime where the gauge coupling associated to the adjoint becomes strong has not been well understood thus far. We argued for a solution to this problem in the concrete example of the $\mathbb{Z}_2$-orbifolded conifold, finding that the dynamics is correctly approximated by replacing the sector containing the adjoint by a copy of $\mathcal{N}=2$ SQCD. This is powerful because the IR structure of this theory is known exactly [21]. The resulting Higgsing produces various numbers of moduli, $U(1)$ factors, and monopoles which decouple from the remaining non-Abelian degrees of freedom in a calculable manner. Thus these cascades proceed through a combination of Seiberg duality and Higgsing.

By mapping our results into the supergravity regime, we argued that the geometry should contain a series of regions dispersed along the radial coordinate where these deconfined states are localized. We expect the $U(1)$ factors and moduli to arise from explicit factional brane sources in the geometry as in [16, 22], while the monopoles are expected to have a more 'non-perturbative' origin in terms of tensionless wrapped D3 branes.

Our results strongly suggest that the dynamics of the adjoint in more general quivers is also faithfully reproduced by $\mathcal{N}=2$ SQCD. This would open up the way for a detailed understanding of how to embed the metastable models of [1–4] into duality cascades, which was the primary motivation of this work. Via the gauge-gravity correspondence such an embedding would realize a meta-stable state inside of a warped throat geometry, and so could be of some interest. However, an interesting feature suggested by our analysis is that any to attempt to do so would yield additional light fields which survive in the IR, not present in the original models. This could affect the stabilization of these models as the scalars in these extra sectors may induce runaways. We plan to discuss this problem in forthcoming work [33].

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considered here this is briefly discussed in §4.4 and §3.2.

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