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SOME COMMENTS ON LONDON THEORY FOR SUPERCONDUCTORS

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The basic formulae of London theory for superconductors are reviewed. Moreover, an expression for the spatial charge density in a type-II superconductor is obtained; this equation is associated with sinusoidal oscillations. Considerations on both penetration depth and coherence length are exposed.

1. INTRODUCTION

In recent years, superconductor alloys have been investigated extensively. However, theoretical work on this subject is scant and basic concepts are exposed inadequately in a considerable part of literature. In reality, efforts in experimental research have not given rise to an increasing knowledge on superconductivity mechanisms; many of them remain unclear. In order to clarify superconductor behavior, in the following an analytical study will be developed from a fundamental equation due to F. and H. London[1,2]. Phenomenological London theory predicts values of penetration depth smaller than the values observed in real superconductors[1]. In contrast, Pippard nonlocal theory predicts more realistic values and also introduces coherence length. However, London theory is applicable to certain cases as, for example, magnetic force microscopy[3,4,5]. In particular, ref. [5] deals with magnetic force microscopy of Abrikosov vortices in type-II superconductors.

2. THEORY

First let us consider the well known equation of F. and H. London namely[1,2]:

$$\vec{J} = -\frac{n_0 e^2}{m^* c}\vec{A} \tag{1}$$

where $\vec{J}$ is the current density vector, n_0 the initial electron density, e stands for electron charge, m^* is effective mass, c is speed of light in a vacuum, and $\vec{A}$ means magnetic vector potential. London theory is consistent with Maxwell equations;

we shall use these equations in gaussian units. Since $\vec{H} = \text{curl}\,\vec{A}$, eq. (1) becomes:

$$\vec{H} = -\frac{m^*c}{n_0e^2}\,\text{curl}\,\vec{J} \tag{2}$$

$\vec{H}$ being magnetic field intensity.

By using $\text{curl}\,\vec{H} = (4\pi/c)\,\vec{J}$, from eq. (2) it is deduced:

$$\vec{H} = -\frac{m^*c^2}{4\pi n_0e^2}\,(\text{grad}(\text{div}\,\vec{H}) - \nabla^2\vec{H})$$

but $\text{div}\,\vec{H} = 0$ and we get:

$$\nabla^2\vec{H} = \frac{4\pi n_0e^2}{m^*c^2}\,\vec{H} \tag{3}$$

In a one-dimensional case, eq. (3) becomes $\nabla^2H_x = 4\pi n_0e^2(m^*c^2)^{-1}H_x$ and if H_x depends on x(depth) and t(time) only, we have:

$$\frac{\partial^2H_x}{\partial x^2} = \frac{4\pi n_0e^2}{m^*c^2}\,H_x \tag{4}$$

At any rate, we can assume a magnetostatic situation (see ref. [1]) so that $\partial^2H_x/\partial x^2$ can be replaced by d^2H_x/dx^2. Solving eq. (4), a physically admissible solution is:

$$H_x(x) = H_0\exp(-x/\lambda_L) \tag{5}$$

where H_0 is the surface field and λ_L the London penetration depth namely:

$$\lambda_L = \frac{c}{2e}\left(\frac{m^*}{\pi n_0}\right)^{1/2} \tag{6}$$

We can claim that $\lambda_L \sim 10^{-6}$ cm; λ_L is smaller than the penetration depth in real superconductors. In contrast, Pippard theory[1] gives values of penetration depth more realistic than those of London theory. In fact, Pippard penetration depth is given by[1]:

$$\lambda = \left(\frac{\xi_0}{\xi}\right)^{1/2}\cdot\lambda_L \tag{7}$$

for $\xi << \lambda$ and

$$\lambda = \left(\frac{\sqrt{3}}{2\pi}\,\xi_0\lambda_L^2\right)^{1/3} \tag{8}$$

for $\xi >> \lambda$.

In eqs. (7)–(8), ξ stands for coherence length. ξ depends on the electron mean free path that we denote by s; we have $\xi_0 = \lim_{s\to\infty}\xi(s)$[1]. The coherence length constitutes a concept associated with the fact that the Pippard theory is a non-local theory. On the other hand, eq. (8) indicates that $\lambda > \lambda_L$; this equation is satisfied by purest superconductors.

Next we shall derive an important equation involving $\vec{J}$. To this end, we take the partial derivative with respect to time at both sides in eq. (2) as follows:

$$\frac{\partial \vec{H}}{\partial t} = -\frac{m^* c}{n_0 e^2} \operatorname{curl}\left(\frac{\partial \vec{J}}{\partial t}\right) \tag{9}$$

Moreover, we can write:

$$\operatorname{curl} \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \tag{10}$$

where $\vec{E}$ stands for electric field strength.

By combining eqs. (9) and (10), we get:

$$\frac{\partial \vec{J}}{\partial t} = \frac{n_0 e^2}{m^*} \vec{E} \tag{11}$$

Finally, we shall obtain an interesting expression for the spatial charge density in the London theory. To this end, we shall use the continuity equation namely:

$$\operatorname{div} \vec{J} = -\frac{\partial \rho}{\partial t} \tag{12}$$

with $\rho(x, y, z, t)$ being spatial charge density.

Furthermore, we shall consider:

$$\operatorname{div} \vec{E} = 4\pi\rho \tag{13}$$

Since $\rho(x, y, z, t) = en(x, y, z, t)$, by combining eqs. (11), (12) and (13) we obtain:

$$\frac{\partial^2 n}{\partial t^2} + \omega^2 n = 0 \tag{14}$$

or

$$\frac{\partial^2 \rho}{\partial t^2} + \omega^2 \rho = 0 \tag{15}$$

with $\omega^2 = 4\pi n_0 e^2/m^*$.

Solving eqs. (14) and (15) we get:

$$n(x, y, z, t) = n_1(x, y, z) \sin[\omega t + \theta(x, y, z)] \tag{16}$$

$$\rho(x, y, z, t) = \rho_1(x, y, z) \sin[\omega t + \theta(x, y, z)] \tag{17}$$

with $\rho_1 = en_1$, $n_0 \equiv n(x, y, z, 0)$.

Then by (16) we have: $\theta = \sin^{-1}(n_0/n_1)$. On the other hand, $\omega = 2e(\pi n_0/m^*)^{1/2}$ represents the plasma angular frequency and it is related to λ_L (see eq. (6)) by the following simple formula: $\omega\lambda_L = c$.

3. CONCLUDING REMARKS

Previously, we have developed the fundamental concepts in the context of London theory. These concepts have been examined in a mathematical form by using Maxwell equations. In this way, we have found the important result expressed in eq. (11). In addition, eqs. (14), (15), (16) and (17) constitute interesting results from the point of view of plasma oscillations. Finally, we recall the usefulness of the theory of London in order to analyse some recent problems related to magnetic force microscopy.

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