NEW RESULTS ON THE NOISE FIGURE
OF HEMTs

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In this communication, an accurate mathematical model for the noise figure of a high electron mobility transistor is developed. This model represents a substantial improvement of the Fukui model. In fact, the Fukui approach can be considered as an approximation of our model under certain conditions.

The fundamental relationship that gives minimum noise figure is derived from a first-order linear partial differential equation, which is established by assuming reasonable statements based on experience.

1. INTRODUCTION

It is well-known that minimum noise figure of HEMTs depends strongly on frequency and the important parasitic parameters: gate and source resistances, and gate-source capacitance[1]. The values of these elements and the frequency can be considered as real variables so that noise figure can be expressed as a function depending on these variables; by virtue of Fukui approach this function depends linearly on the frequency for a given device[2,3].

In the following, we shall state a new formulation based on the properties of certain linear partial differential equations in order to generalize the mathematical formulation of HEMT's noise figure.

2. NOISE FIGURE MODELISATION

First, minimum noise figure can be expressed as follows[2,4]:

$$F_{min} = 1 + mf \tag{1}$$

where f is the frequency and m is proportional to $C_{gs}(R_s + R_g)^{1/2} \cdot g_m^{-1/2}$; here C_{gs} is the gate-source capacitance, and R_s and R_g are the source and gate resistances, respectively; g_m denotes transconductance.

It is possible to establish a partial differential equation whose complete solution coincides with the Eqn. (1). In order to obtain this equation, we note that the

relative variation of F_{min} is proportional to the relative variation of each variable in Eqn. (1); we define relative variation of a magnitude as the ratio: variation of this magnitude/value of it. From experimental data, we can claim that the proportionality "constant" involved in the above assumption is the same for the two variables. Therefore we have:

$$\frac{\Delta F_{min}}{F_{min}} = A \cdot \frac{\Delta m}{m} \tag{2}$$

$$\frac{\Delta F_{min}}{F_{min}} = A \cdot \frac{\Delta f}{f} \tag{3}$$

(with A depending on m and f) where Δ denotes variation (absolute variation), and A is the pseudoconstant mentioned above. Since $\lim_{\Delta m \to 0}(\Delta F_{min}/\Delta m) = \partial F_{min}/\partial m$ and $\lim_{\Delta f \to 0}(\Delta F_{min}/\Delta f) = \partial F_{min}/\partial f$. From (2) and (3) we obtain

$$m \frac{\partial F_{min}}{\partial m} = A(m, f) \tag{4}$$

and

$$f \frac{\partial F_{min}}{\partial f} = A(m, f) \tag{5}$$

and hence

$$m \frac{\partial F_{min}}{\partial m} = f \frac{\partial F_{min}}{\partial f} \tag{6}$$

Next, we will solve the first-order linear partial differential equation (6). To get this end, we formulate the differential system associated with Eq. (6) namely

$$\frac{dm}{m} = -\frac{df}{f} = \frac{dF_{min}}{0} \tag{7}$$

whose solution is

$$\left.\begin{matrix} mf = C_1 \\ F_{min} = C_2 \end{matrix}\right\} \tag{8}$$

where C_1 and C_2 are constants.
    Therefore, the general solution of Eq. (6) is given by

$$\Phi(C_1, C_2) = 0 \quad \text{or} \quad \Phi(mf, F_{min}) = 0 \tag{9}$$

On the other hand, a complete solution to Eq. (6) can be expressed as follows[5]:

$$v = k_1 u + k_2 \qquad (10)$$

where k_1 and k_2 are constants, and u and v depend on m, f, and F_{min} so that $u(m, f, F_{min}) = C_1$ and $v(m, f, F_{min}) = C_2$. Then, by considering (8), (9) and (10) we have

$$F_{min} = k_1 mf + k_2 \qquad (11)$$

Now, we compute k_2 by assuming that $F_{min} = 1$ when $mf = 0$; hence $k_2 = 1$. Moreover, when $mf = 1$ we have $F_{min} = 2$ and consequently $k_1 = 1$. Then, equation (11) reduces to $F_{min} = 1 + mf$, that is the relationship (1).

The crucial point of our research work is related to the fact that the complete integral of Eq. (6) is not unique. In fact, another complete solution is

$$F_{min} = e^{mf} \qquad (12)$$

In effect, it is easy to prove that Eq. (12) satisfies Eq. (6). The Mc. Laurin expansion of (12) is

$$F_{min} = \sum_{n=0}^{\infty} \frac{(mf)^n}{n!} \qquad (13)$$

When $mf \ll 1$, Eq. (13) can be reduced to

$$F_{min} \cong 1 + mf \qquad (14)$$

There exist infinite complete solutions to Eq. (6). These solutions obey Eq. (9), and Eq. (11) represents the standard complete integral (affine function in mf). Moreover, all the terms in the right-hand side of the expression (13) satisfy Eq. (6) and, since this equation is linear, the sum of these infinite terms converges to a function that satisfies Eq. (6); this function is given by Eq. (12). Therefore, it is clear that, in our context, the "best" solution is Eq. (12).

In addition, it is easy to see that the relationship (12) represents a minimum noise figure. By applying variational techniques to an adequate formula to describe noise, it is possible to get the previous result.

3. DISCUSSION

It is evident that the model developed here improves substantially the noise formulation of Fukui which is given usually by $F_{min}(dB) = 10 \log_{10} (1 + mf)$. When the inequality $mf \ll 1$ does not occur, the Fukui model is inadequate and Eq. (12) is really suitable for a typical GaAs/AlGaAs HEMT operating at frequencies greater than 5 GHz with $m \cong 0.02$ ns.

When we assume that A does not depend approximately on m and f, it is easy to see that $F_{min} \cong A\ln(mf) + C$ where C is a constant, that is, C depends on A, and does not depend on m and f. The above expression is incorrect because it cannot be proven experimentally. Then, we conclude that the dependence on m and f is considerable. Eq. (12) is in good agreement with experimental work[6] and, therefore, it represents a full solution to the noise problem with respect to high electron mobility transistors.

On the other hand, the following fact is remarkable: in order to determine the constant k_1 in Eq. (11), $F_{min} = 2$ so that $F_{min} = 2$ (= 3 dB) corresponds to f = 1/m (cutoff frequency with respect to noise figure).

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