

SOME REMARKS ON QUASI-SOLITONS IN OPTICAL FIBERS

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In this paper a unique analysis of solitary waves corresponding to the solutions of the non-linear cubic-quintic Schrödinger equation for optical fibers is performed. These results imply that for an intensity-squared dependent refractive index, the behavior of the solitary waves is at most quasi-soliton. In our context there is not soliton behavior so that the existence of Raman solitons is not possible. However, under certain conditions it is reasonable to expect the existence of Raman quasi-solitons in the anomalous-dispersion regime.

The mathematical aspects involved are emphasized because the theoretical research is very important in the context of non-linear optics to clarify various, often confusing ideas derived from certain experimental works.

1. INTRODUCTION

It is well-known that non-linear partial differential equations of the Schrödinger type are relevant in non-linear optics. A cubic-quintic Schrödinger equation appears when the refractive index of the non-linear medium is an even fourth-order polynomial in terms of the electric field modulus. This polynomial derives from the representation of the refractive index function as a Taylor series. On the other hand, we will assume an optical fiber with an isotropic core.

Pushkarov et al. [1] obtained solitary-wave solutions of the cubic-quintic Schrödinger equation. When the higher order term of this equation is equal to zero, the equation reduces to the well-known non-linear cubic Schrödinger equation, which represents the Kerr effect. Theoretical-experimental background corresponding to this equation has been exposed in various well-known books by using standard methodology. In contrast, in the following we will use an elegant procedure that gives precise results.

2. DERIVATION OF THE LOSSLESS CUBIC-QUINTIC SCHRÖDINGER EQUATION

A lossless non-linear cubic-quintic Schrödinger equation can be deduced from [1], [2], and by considering the following statements. At first, for an isotropic and non-linear optical medium, the refractive index is given by a Taylor's expansion in

function of the electric field namely:

$$n(E) \cong \sum_{i=0}^N \frac{1}{i!} \left[\frac{d^i n}{dE^i} \right]_{E=0} E^i \quad (1)$$

where n is the refractive index and E is the modulus of the electric field. Under the fourth-order approximation and by assuming the cancellation of first and third derivatives, when the electric field vanishes we have:

$$n \cong n_0 + n_2 E^2 + n_4 E^4 \quad (2)$$

by taking $N = 4$.

In (2) we identify:

$$n_0 \cong n(0), \quad n_2 \cong \frac{1}{2!} \left[\frac{d^2 n}{dE^2} \right]_{E=0},$$

$$n_4 \cong \frac{1}{4!} \left[\frac{d^4 n}{dE^4} \right]_{E=0}$$

The second fact to consider consists of the assumption: $\vec{E} = (E_1, E_2, E_3)$ where, $E_1 = E_2 = 0$,

$$E_3 = \text{Re}\{\xi(x, t) \exp[j(kx - \omega t)]\}(\sqrt{-1} = j) \quad (3)$$

where $\xi(x, t)$ is the amplitude of the electric field, k the wave number, and ω the radian frequency. Then the differential equation in question is:

$$\frac{1}{2} \frac{\partial^2 \psi}{\partial \tau^2} + j \frac{\partial \psi}{\partial y} \pm 2|\psi|^2 \psi + \alpha |\psi|^4 \psi = 0 \quad (4)$$

neglecting damping and higher order dispersion, and assuming [3]:

$$y \cong 10^{-9} x / \lambda, \quad \tau \cong \frac{10^{-9/2}}{\sqrt{\lambda |k''|}} \left(t - \frac{x}{v_g} \right)$$

$$\psi \cong 10^{9/2} (\pi n_2)^{1/2} \xi, \quad \alpha \cong 10^{-9} (2n_4 / \pi n_2^2)$$

where λ is the wavelength, v_g is the group velocity namely $v_g^{-1} = (\partial k / \partial \omega) \cong k'$ and k'' is the group dispersion; here $k'' < 0$.

On the other hand, we recall that the geometric frame of reference associated with the above exposition moves with the group velocity.

3. SOLUTIONS OF THE EQUATION

The following considerations refer to the calculation of the solitary-wave solutions of eq. (4). These solutions can be obtained by considering:

$$\psi(y, \tau) = e^{j(ay + b\tau)} \varphi(\tau - by) \quad (5)$$

where a and b are real constants and φ is a real function. Replacing (5) into (4), we obtain:

$$\varphi'' - (2a + b^2)\varphi \pm 4\varphi^3 + 2\alpha\varphi^5 = 0 \quad (6)$$

assuming $\varphi = \varphi(u)$ with $u = \tau - by$

Before solving (6) we will examine the behavior of the function φ . Some authors claim that $\varphi' \rightarrow 0$ when $u \rightarrow \infty$ and that $\varphi \rightarrow 0$ when $u \rightarrow \infty$. The second condition is necessary in the context in question but the first is redundant. It is possible to deduce this condition from the limit assumptions on the function and its second derivative (φ is a C^2 -function). Taking $u \rightarrow \infty$ in (6) we have $\varphi'' \rightarrow 0$ since $\varphi \rightarrow 0$ when $u \rightarrow \infty$.

On the other hand, we will contribute an original result with respect to the behavior of φ' . By virtue of Taylor's expansion theorem we can write:

$$\varphi'(u) = \frac{\varphi(u + 2h) - \varphi(u)}{2h} - h\varphi''(u + 2h\theta) \quad (7)$$

with $0 < \theta < 1$ and $h > 0$.

Now, when we take $u \rightarrow \infty$ in (7) the result is $\varphi' \rightarrow 0$. Next we will solve (6); multiplying (6) by $d\varphi$ and integrating we find:

$$\varphi(\tau - by) = C_1\{C_2 \cosh[2C_1(\tau - by)] \pm 1\}^{-1/2} \quad (8)$$

According to (5) and (8) the solutions of (4) are obtained. Moreover, the constants C_1 and C_2 are given by:

$$\begin{aligned} C_1 &= (2a + b^2)^{1/2} \\ C_2 &= [1 + \frac{2}{3}\alpha(2a + b^2)]^{1/2} \end{aligned} \quad (9)$$

4. RESULTS AND DISCUSSION

The solutions (5) (with (8)) are solitary waves propagating with a velocity less than v_g for $b > 0$; when $b < 0$ the waves-velocity is larger than v_g .

Next, we will define the so-called ratio of the fifth-to-third-order contributions, namely:

$$\rho = \frac{|\alpha||\psi|^4\psi}{2|\psi|^2\psi} = \frac{1}{2}|\alpha||\psi|^2 \quad (10)$$

This ratio is an important tool in order to analyze the stability of the solutions of (4). In our case (optical fibers) we have $n_2 > 0$ and $n_4 > 0$, and therefore $\alpha > 0$. For example, in silica fibers we have $n_2 \cong 2.3 \times 10^{-22} \text{ m}^2/\text{v}^2$ for $\lambda \cong 1.5 \mu\text{m}$. If $\rho \ll 1$ it is obvious that the behavior of the solitary-waves corresponding to (4) is

nearly the behavior of the solitary-wave solutions to (4) with $\alpha = 0$. It is well-known [2] that the behavior of these solutions is soliton when the cubic term of our equation is positive and this behavior is radiative in the negative case. Thus, we claim that when $\rho \ll 1$, the behavior of the solitary waves is quasi-soliton in the positive case. This result is confirmed when we perform a numerical simulation of (4). This numerical simulation is based on an explicit scheme with central-difference-approximation.

On the other hand, when the condition $\rho \ll 1$ does not occur, no quasi-soliton behavior is expected. In fact, by means of the simulation mentioned above, we have obtained an explosive behavior for $\rho \geq 2$, that is, $\alpha|\psi|^2 = \alpha\varphi^2 \geq 4$. Moreover, a very weak stability is expected for $\rho \cong 1$, i.e., $\alpha\varphi^2 \cong 2$. This discussion is confirmed, also, by calculations we have performed related to Bäcklund transformations, topological index and the following important invariants [4]:

$$I_1 = \int_{-\infty}^{+\infty} \varphi^2 d\tau$$

$$I_2 = \int_{-\infty}^{+\infty} \left(\left| \frac{\partial \psi}{\partial \tau} \right|^2 - 2|\psi|^4 - \frac{2}{3} \alpha |\psi|^6 \right) d\tau \quad (11)$$

With respect to the soliton effects in Stimulated Raman Scattering (SRS), we claim that when the wavelength of a pump pulse falls in the anomalous-dispersion regime of the optical fiber ($k'' < 0$), both the pump pulse and the SRS-generated Raman pulse can behave, under suitable conditions, so that almost all of the pump pulse energy is delivered to the Raman pulse; this pulse propagates undistorted as a fundamental soliton [5]. In our context (strongly non-linear monomode optical fibers), when $\rho \ll 1$, the existence of Raman quasi-solitons is expected for input peak powers in the range 50–900 W; for $k'' < 0$, Raman quasi-solitons of approximately 100 fs-width are, in principle, expected under certain conditions (see references [6], [7]).

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