

Modeling Gravitational Recoil Using Numerical Relativity

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Abstract. We review the developments in modeling gravitational recoil from merging black-hole binaries and introduce a new set of 20 simulations to test our previously proposed empirical formula for the recoil. The configurations are chosen to represent generic binaries with unequal masses and precessing spins. Results of these simulations indicate that the recoil formula is accurate to within a few km/s in the similar mass-ratio regime for the out-of-plane recoil.

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1. Introduction

The field of Numerical Relativity (NR) has progressed at a remarkable pace since the breakthroughs of 2005 [1, 2, 3] with the first successful fully non-linear dynamical numerical simulation of the inspiral, merger, and ringdown of an orbiting black-hole binary (BHB) system. In particular, the ‘moving-punctures’ approach, developed independently by the NR groups at NASA/GSFC and at RIT, has now become the most widely used method in the field and was successfully applied to evolve generic BHBs. This approach regularizes a singular term in space-time metric and allows the black holes (BHs) to move across the computational domain. Previous methods used special coordinate conditions that kept the BHs fixed in space, which introduced severe coordinate distortions that caused orbiting-BHB simulations to crash. Recently, the generalized harmonic approach method, first developed by Pretorius [1], has also been successfully applied to accurately evolve generic BHBs for tens of orbits with the use of pseudospectral codes [4, 5].

Since then, BHB physics has rapidly matured into a critical tool for gravitational wave (GW) data analysis and astrophysics. Recent developments include: studies of the orbital dynamics of spinning BHBs [6, 7, 8, 9, 10, 11, 12], calculations of recoil velocities from the merger of unequal mass BHBs [13, 14, 15], and the surprising discovery that very large recoils can be acquired by the remnant of the merger of two spinning BHs [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 9, 29, 30, 31], empirical models relating the final mass and spin of the remnant with the spins of the individual BHs [32, 33, 34, 35, 36, 37, 38, 39], comparisons of waveforms and orbital dynamics of BHB inspirals with post-Newtonian (PN) predictions [40, 41, 42, 43, 44, 45, 46, 47], and simulations reaching mass ratios [48] $q = 1/100$.

1.1. Kicks and Super-kicks: A brief history

The first in-depth modeling of the recoil from the merger of non-spinning asymmetric BHBs was done in Ref. [15], where it was shown that the maximum recoil is limited to $\approx 175 \text{ km s}^{-1}$. Soon after, other groups showed that the maximum recoil for spinning binaries is much larger. In Ref. [16] and [30] (which were released within a few days of each other), it was shown that the maximum recoil for a spinning binary with one BH spin aligned with the orbital angular momentum and other anti-aligned is 475 km s^{-1} . Within a day of the initial release of the preprint for Ref. [16], our group released a preprint [17] for more generic spin and mass configurations, showing that the recoil is considerably larger if the spins are anti-aligned and pointing in the orbital plane. In [17] we measured recoil velocities beyond the maximum predicted for the configurations in [16, 30].

An initial analysis of the results in [17] indicated that the maximum velocity exceeded 1300 km s^{-1} for spins lying in the orbital plane. This initial estimate was based on two ‘generic’ configurations. A subsequent analysis that incorporated the angular dependence of the projection of the spin on the orbital plane, showed that

the maximum recoils is $\sim 4000 \text{ km s}^{-1}$ [49]. Based on our conclusions in the preprint of [17], the group at Jena performed a set of two simulations in this ‘maximum-kick’ configuration and measured recoils of around 2600 km s^{-1} . Our modeling showed that this recoil velocity actually depends sinusoidally on the angle that the spins make with the infall direction. By evolving a set of configurations with spins at different initial angles in the orbital plane, we found that the recoil reaches a maximum of $\sim 4000 \text{ km s}^{-1}$ [18, 63]. In Ref. [49], as part of our analysis, we proposed an empirical formula for the recoil based on post-Newtonian expressions for the radiated linear momentum. The formula correctly predicts the sinusoidal dependence of the ‘maximum-kick’ recoil.

In Ref. [26] the recoil for unequal-mass, spinning binaries, with total spin equal to zero and individual spins lying in the initial orbital plane, was measured. These $S = 0$ configurations are preserved under numerical evolution and lead to minimal precession of the orbital pane. Interestingly, they found that the recoil scales with the cube of the mass ratio q , rather than the expected q^2 seen in post-Newtonian expressions for the recoil. In a subsequent study [19], our group found that the recoil scales as q^2 for the more astrophysically important precessing case.

1.2. Phenomenological modeling of the recoil

In [17] we introduced an empirical formula for the recoil, augmented in [39] which has the form.

$$\begin{aligned}
 \vec{V}_{\text{recoil}}(q, \vec{\alpha}) &= v_m \hat{e}_1 + v_{\perp} (\cos \xi \hat{e}_1 + \sin \xi \hat{e}_2) + v_{\parallel} \hat{n}_{\parallel}, \\
 v_m &= A \frac{\eta^2(1-q)}{(1+q)} [1 + B \eta], \\
 v_{\perp} &= H \frac{\eta^2}{(1+q)} \left[(1 + B_H \eta) (\alpha_2^{\parallel} - q\alpha_1^{\parallel}) + H_S \frac{(1-q)}{(1+q)^2} (\alpha_2^{\parallel} + q^2\alpha_1^{\parallel}) \right], \\
 v_{\parallel} &= K \frac{\eta^2}{(1+q)} \left[(1 + B_K \eta) |\alpha_2^{\perp} - q\alpha_1^{\perp}| \cos(\Theta_{\Delta} - \Theta_0) \right. \\
 &\quad \left. + K_S \frac{(1-q)}{(1+q)^2} |\alpha_2^{\perp} + q^2\alpha_1^{\perp}| \cos(\Theta_S - \Theta_1) \right], \tag{1}
 \end{aligned}$$

where $\eta = q/(1+q)^2$, the index \perp and \parallel refer to perpendicular and parallel to the orbital angular momentum respectively, \hat{e}_1, \hat{e}_2 are orthogonal unit vectors in the orbital plane, and ξ measures the angle between the unequal mass and spin contribution to the recoil velocity in the orbital plane. The constants H_S and K_S can be determined from new generic BHB simulations as the data become available. The angles, Θ_{Δ} and Θ_S , are the angles between the in-plane component of $\vec{\Delta} = M(\vec{S}_2/m_2 - \vec{S}_1/m_1)$ or $\vec{S} = \vec{S}_1 + \vec{S}_2$ and the infall direction at merger. Phases Θ_0 and Θ_1 depend on the initial separation of the holes for quasicircular orbits. A crucial observation is that the dominant contribution to the recoil is generated near the time of formation of the common horizon of the merging BHs (See, for instance Fig. 6 in [50]). The formula (1) above describing the recoil applies at this moment (or averaged coefficients around this maximum generation of recoil), and has proven to represent the distribution of

velocities with sufficient accuracy for astrophysical applications. Recently, in Ref. [51] a very similar formula was proposed.

The most recent published estimates for the above parameters can be found in [19] and references therein. The current best estimates are: $A = 1.2 \times 10^4 \text{ km s}^{-1}$, $B = -0.93$, $H = (6.9 \pm 0.5) \times 10^3 \text{ km s}^{-1}$, $K = (6.0 \pm 0.1) \times 10^4 \text{ km s}^{-1}$, and $\xi \sim 145^\circ$.

Note that v_{\parallel} is maximized when $q = 1$ (i.e. equal masses), the in-plane spins are maximal ($\alpha_1 = \alpha_2 = 1$), and the two spins are anti-aligned.

In this paper, we use a new set of simulations of precessing binaries to test the recoil formula and obtain a new estimate for K_S .

2. Numerical Techniques

To compute the numerical initial data, we use the puncture approach [52] along with the `TWOPUNCTURES` [53] thorn. In this approach the 3-metric on the initial slice has the form $\gamma_{ab} = (\psi_{BL} + u)^4 \delta_{ab}$, where ψ_{BL} is the Brill-Lindquist conformal factor, δ_{ab} is the Euclidean metric, and u is (at least) C^2 on the punctures. The Brill-Lindquist conformal factor is given by $\psi_{BL} = 1 + \sum_{i=1}^n m_i^p / (2|\vec{r} - \vec{r}_i|)$, where n is the total number of ‘punctures’, m_i^p is the mass parameter of puncture i (m_i^p is *not* the horizon mass associated with puncture i), and \vec{r}_i is the coordinate location of puncture i . We evolve these BHB data-sets using the `LAZE` [54] implementation of the moving puncture approach [2, 3] with the conformal factor $W = \sqrt{\chi} = \exp(-2\phi)$ suggested by [11]. For the runs presented here we use centered, eighth-order finite differencing in space [55] and an RK4 time integrator (note that we do not upwind the advection terms).

We use the `Carp` [56] mesh refinement driver to provide a ‘moving boxes’ style mesh refinement. In this approach refined grids of fixed size are arranged about the coordinate centers of both holes. The `Carp` code then moves these fine grids about the computational domain by following the trajectories of the two BHs.

We use `AHFINDERDIRECT` [57] to locate apparent horizons. We measure the magnitude of the horizon spin using the Isolated Horizon algorithm detailed in [58]. This algorithm is based on finding an approximate rotational Killing vector (i.e. an approximate rotational symmetry) on the horizon φ^a . Given this approximate Killing vector φ^a , the spin magnitude is

$$S_{[\varphi]} = \frac{1}{8\pi} \int_{AH} (\varphi^a R^b K_{ab}) d^2V, \quad (2)$$

where K_{ab} is the extrinsic curvature of the 3D-slice, d^2V is the natural volume element intrinsic to the horizon, and R^a is the outward pointing unit vector normal to the horizon on the 3D-slice. We measure the direction of the spin by finding the coordinate line joining the poles of this Killing vector field using the technique introduced in [8]. Our algorithm for finding the poles of the Killing vector field has an accuracy of $\sim 2^\circ$ (see [8] for details). Note that once we have the horizon spin, we can calculate the horizon mass via the Christodoulou formula $m^H = \sqrt{m_{\text{irr}}^2 + S^2/(4m_{\text{irr}}^2)}$, where $m_{\text{irr}} = \sqrt{A/(16\pi)}$ and A is the surface area of the horizon. We measure radiated energy, linear momentum,

and angular momentum, in terms of ψ_4 , using the formulae provided in Refs. [59, 60]. However, rather than using the full ψ_4 , we decompose it into ℓ and m modes and solve for the radiated linear momentum, dropping terms with $\ell \geq 5$. The formulae in Refs. [59, 60] are valid at $r = \infty$. We obtain accurate values for these quantities by solving for them on spheres of finite radius (typically $r/M = 50, 60, \dots, 100$), fitting the results to a polynomial dependence in $l = 1/r$, and extrapolating to $l = 0$ [3, 61, 44, 62]. Each quantity Q has the radial dependence $Q = Q_0 + lQ_1 + \mathcal{O}(l^2)$, where Q_0 is the asymptotic value (the $\mathcal{O}(l)$ error arises from the $\mathcal{O}(l)$ error in $r\psi_4$). We perform both linear and quadratic fits of Q versus l , and take Q_0 from the quadratic fit as the final value with the differences between the linear and extrapolated Q_0 as a measure of the error in the extrapolations. We found that extrapolating the waveform itself to $r = \infty$ introduced phase errors due to uncertainties in the areal radius of the observers, as well as numerical noise. Thus when comparing Perturbative to numerical waveforms, we use the waveform extracted at $r = 100M$.

In order to model the recoil as a function of the orientation and magnitudes of the spins, we use the techniques introduced in [19] to locate the approximate orbital plane at merger and 3D rotation such that infall directions are the same for each simulation. Briefly, this technique uses three points on the trajectories, given by fiducial choices of the BH separations, to define the orbital plane and preferred orientation.

3. Simulations

Config	x_1/M	x_2/M	P/M	m_1^p	m_2^p	S_x/M^2	S_y/M^2
Q33TH000	4.882446	-1.607923	0.101163	0.171173	0.723529	0	0.045983
RQ33TH000	4.885558	-1.609045	0.101153	0.171170	0.723523	0	0.045982
Q50TH000	4.360493	-2.163334	0.119252	0.230648	0.618813	0	0.082098

Table 1. Initial data parameters for the quasi-circular configurations. The punctures are located at $\vec{r}_1 = (x_1, 0, 0)$ and $\vec{r}_2 = (x_2, 0, 0)$, with momenta $P = \pm(0, P, 0)$, spins $\vec{S}_1 = (S_x, S_y, 0)$, $\vec{S}_2 = -q\vec{S}_1$, mass parameters m^p . The configuration are denoted by QXXXTHYYY where XXX gives the mass ratio (0.33, 0.50) and YYY gives the angle in degrees between the initial spin direction and the y -axis. In all cases the initial orbital period is $M\omega = 0.05$ and the spin of the smaller BH is $\alpha = 0.72$. Initial data parameters for the Q50TH000 and Q33TH000 configurations are given. The remaining configurations are obtained by rotating the spin directions, keeping all other parameters the same. For the RQ33THxxx configurations, \vec{S}_2 is rotated by 90° with respect to the corresponding Q33THxxx configuration.

We evolve a set of configuration that initially have $\Delta = 0$, as well as a set of configuration with one spin rotated by 90° for mass ratios $q = 1/3$ and $q = 1/2$. The initial data parameters are summarized in Table 3. Note that the $\Delta = 0$ configurations are unstable in the sense that the system quickly evolved towards a nontrivial Δ . In particular, at merger, where most of the recoil asymmetry is generated,

the binary reaches a generic configuration regarding the spin orientations given the strong differential precession of each BH spins.

4. Results

In Table 2 we summarize the results of the simulations. The table shows the radiated energy and recoil (prior to any rotation). Note that these results can be used in additional fits of the final remnant BH formulae for the mass, spin, and recoil velocity of the remnant, as was done in Ref. [39] using the then currently available results in the literature.

Config	$100(\delta E/M)$	V_x	V_y	V_z
Q50TH000	2.858 ± 0.018	76.89 ± 2.47	234.40 ± 3.12	167.95 ± 3.29
Q50TH045	2.817 ± 0.017	-40.47 ± 131.03	10.46 ± 99.56	-228.49 ± 5.76
Q50TH090	2.778 ± 0.016	189.23 ± 0.77	97.60 ± 0.34	-543.78 ± 2.29
Q50TH130	2.795 ± 0.017	306.54 ± 1.23	319.10 ± 0.02	-762.32 ± 0.29
Q50TH180	2.858 ± 0.018	78.03 ± 2.66	235.61 ± 3.24	-167.47 ± 3.37
Q50TH210	2.831 ± 0.017	43.42 ± 1.32	108.60 ± 2.94	131.29 ± 3.42
Q50TH315	2.831 ± 0.017	295.55 ± 1.26	341.01 ± 0.78	741.05 ± 0.66
Q33TH000	1.90243 ± 0.012	119.18 ± 0.36	212.92 ± 2.78	$144.191.08$
Q33TH045	1.884 ± 0.012	67.29 ± 0.59	132.08 ± 2.51	-134.74 ± 0.94
Q33TH090	1.878 ± 0.012	02.92 ± 1.87	132.08 ± 1.51	-337.04 ± 0.88
Q33TH130	1.897 ± 0.012	210.61 ± 1.34	235.72 ± 0.71	-497.12 ± 1.34
Q33TH180	1.902 ± 0.012	119.21 ± 0.36	212.83 ± 2.78	-144.04 ± 1.08
Q33TH210	1.889 ± 0.012	73.21 ± 0.11	150.46 ± 2.73	59.42 ± 1.02
Q33TH260	1.878 ± 0.012	73.45 ± 1.79	127.45 ± 1.58	272.87 ± 1.14
Q33TH315	1.901 ± 0.013	210.14 ± 1.45	249.05 ± 0.87	489.34 ± 0.76
RQ33TH000	1.885 ± 0.013	229.17 ± 1.07	132.08 ± 1.81	539.21 ± 1.66
RQ33TH090	1.865 ± 0.012	45.62 ± 0.11	162.27 ± 1.00	-74.423 ± 0.08
RQ33TH130	1.860 ± 0.012	106.45 ± 1.16	82.27 ± 0.45	-427.92 ± 1.43
RQ33TH210	1.886 ± 0.012	182.61 ± 0.67	188.75 ± 2.00	-361.19 ± 1.04
RQ33TH315	1.861 ± 0.012	121.99 ± 1.42	79.03 ± 0.45	462.96 ± 1.68

Table 2. The radiated energy and recoil velocities for each configuration. Note that some of the error estimates, which are based on the differences between a linear and quadratic extrapolation in $l = 1/r$ of the observer location, are very small. This indicates that the differences between the extrapolation can underestimate the true error. All quantities are given in the coordinate system used by the code (i.e. the untransformed system).

Table 3 gives the components of the radiated angular momentum in the original x, y, z frame (that of the initial data) using the Cartesian decomposition as in Ref. [60].

In Table 4 we give the recoil velocities in a frame rotated such that the orbital

Config	δJ_x	δJ_y	δJ_z
Q50TH000	0.02454 ± 0.00056	0.05954 ± 0.00012	0.18040 ± 0.00200
Q50TH045	-0.0277 ± 0.0041	0.0572 ± 0.0029	0.1808 ± 0.0017
Q50TH090	-0.05995 ± 0.00019	0.02354 ± 0.00079	0.1808 ± 0.0015
Q50TH130	-0.06155 ± 0.00029	-0.02016 ± 0.00047	0.1801 ± 0.00164
Q50TH180	-0.02454 ± 0.00056	-0.05954 ± 0.00012	0.1804 ± 0.0020
Q50TH210	0.00857 ± 0.00035	-0.06326 ± 0.00059	0.1805 ± 0.0018
Q50TH315	0.05981 ± 0.00026	0.02545 ± 0.00046	0.1802 ± 0.0017
Q33TH000	0.0180 ± 0.00011	0.03134 ± 0.00022	0.1309 ± 0.0012
Q33TH045	-0.00930 ± 0.00008	0.03504 ± 0.00010	0.1312 ± 0.0012
Q33TH090	-0.03151 ± 0.00002	0.01835 ± 0.00013	0.13262 ± 0.00081
Q33TH130	-0.03524 ± 0.00047	-0.00678 ± 0.00002	0.1325 ± 0.0010
Q33TH180	-0.01801 ± 0.00011	-0.03134 ± 0.00022	0.1309 ± 0.0012
Q33TH210	-0.00013 ± 0.00002	-0.03608 ± 0.00022	0.1309 ± 0.0012
Q33TH260	0.02797 ± 0.00003	-0.02366 ± 0.00014	0.13254 ± 0.00084
Q33TH315	0.03462 ± 0.00041	0.00994 ± 0.00002	0.1325 ± 0.0011
RQ33TH000	0.021251 ± 0.000001	-0.00413 ± 0.00003	0.1348 ± 0.0012
RQ33TH090	0.00462 ± 0.00017	0.02225 ± 0.00001	0.13479 ± 0.00091
RQ33TH130	-0.00981 ± 0.00001	0.01968 ± 0.00017	0.1346 ± 0.0010
RQ33TH210	-0.02076 ± 0.00014	-0.00734 ± 0.00008	0.1345 ± 0.0012
RQ33TH315	0.01143 ± 0.00001	-0.01857 ± 0.00022	0.1346 ± 0.0011

Table 3. The radiated angular momentum. Note that some of the error estimates, which are based on the differences between a linear and quadratic extrapolation in $l = 1/r$ of the observer location, are very small. This indicates that the differences between the extrapolation can underestimate the true error. All quantities are given in the coordinate system used by the code (i.e. the untransformed system).

plane coincides with the xy plane and the infall direction in this plane is fixed. In order to fit the data, we chose $K = 5.9 \times 10^4$ based on previous work [17, 19, 63] and then fit the Q50 simulations for K_S , Θ_0 , and Θ_1 . We found $K_s = -4.2 \pm 1.8$. While, K and K_S are fixed, Θ_0 and Θ_1 depend on the configuration. To obtain these angles we fit the the recoil formula for the Q33Txxx and RQ33Txxx configurations separately. We then compared the predicted recoil for each configuration with the measured recoil. The results are summarized in Table 4. Note that the errors are typically less than 6 km s^{-1} . The largest relative errors are $\sim 3.3\%$, with most errors lying between $\sim 1\%$ and $\sim 2\%$, rendering the empirical formula accurate for most astrophysical applications. We plot the measured out-of-plane recoil and prediction in Fig. 1. An attempt to fit the in-plane recoil produced errors of the order of 50 km s^{-1} . This can be traced to three main sources the errors. The uncertainty in how the the recoils produced by unequal masses and out-of plane spins contribute to the total in-plane recoil, the error in the measurement in the out-of-plane spine due to errors in measuring the orientation of the

orbital plane at merger, and, as pointed out in [19], effects of precession on the in-plane recoil. We note that the dominant out-of-plane recoil is well modeled.

Config	V_x	V_y	V_z	$V_z(\text{predict})$	error
Q50TH000	17.47	-164.42	-248.44	-245.851	2.591
Q50TH045	-71.85	-100.95	196.47	190.25	-6.222
Q50TH090	44.52	-180.82	553.49	551.759	-1.73
Q50TH130	31.47	-242.82	846.74	848.283	1.540
Q50TH180	18.34	-165.91	248.56	245.987	-2.575
Q50TH210	26.93	-153.45	-81.52	-84.3437	-2.823
Q50TH315	28.58	-242.02	-832.71	-834.713	-1.999
Q33TH000	117.61	-157.99	-203.81	-208.612	-3.311
Q33TH045	110.76	-132.11	102.02	105.811	1.937
Q33TH090	122.60	-120.81	334.67	337.689	-0.893
Q33TH130	144.38	-155.39	549.60	553.834	-2.540
Q33TH180	117.60	-157.99	203.63	208.286	3.140
Q33TH210	109.72	-138.43	-18.17	-18.7138	0.501
Q33TH260	118.62	-122.29	-258.97	-268.242	-6.040
Q33TH315	144.43	-160.93	-546.70	-551.418	2.112
RQ33TH000	168.76	-118.44	-564.09	-569.559	-5.469
RQ33TH090	116.46	-133.88	49.68	49.8794	0.203
RQ33TH130	121.09	-92.33	421.93	413.897	-8.033
RQ33TH210	155.20	-147.31	391.98	386.733	-5.246
RQ33TH315	125.67	-89.20	-460.12	-464.877	-4.757

Table 4. The recoil velocities after rotation and a comparison of the out-of-plane component of the recoil (in coordinates aligned with the orbital plane at merger) with the predictions of the empirical formula with coefficients $K = 5.9 \times 10^4$ and $K_S = -4.25401$.

5. Conclusion

We report here on a new set of generic simulations, with no symmetries and different choices of the binary’s mass ratio and spin direction and magnitudes. We compute the radiation emitted by this binaries, and in particular focus on the magnitude and direction of the final recoil of the merged BH. While this set of runs can be used for fitting to additional subleading terms in the empirical formulae for the remnant BH recoil, we choose to use them to test the previously fitted values, extended to include effects linear in the spin, as accurate determinations of these parameters will require many more simulations. In this paper we showed that the empirical recoil formula provides accurate predictions for the recoil velocity from BHB mergers for the new sets of configurations. These configurations have fewer symmetries than previous comparisons

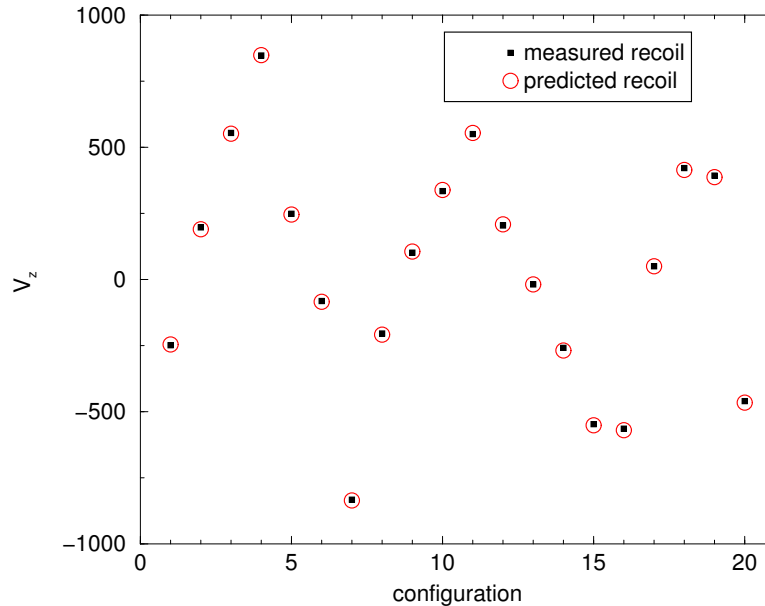


Figure 1. The predicted and measured out-of-plane recoil for each configuration in Table 4. The configurations are labeled 1-20 starting from Q50TH000 as ordered in Table 4.

and can be considered of generic nature regarding spin orientations and intermediate mass ratios and spin magnitudes. This shows that our empirical formula (1) can be used as a first approximation for astrophysical studies of statistical nature, as we did in Ref. [64] and also used for realistic recoil magnitudes and direction when modeling the observational effects of recoiling BHs in a gaseous environment such as accretion disks [65].

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