Qualitative Analysis in Locally Coupled Neural Oscillator Network

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Abstract. The paper investigates a locally coupled neural oscillator autonomous system qualitatively. To obtain analytical results, we choose an approximation method and obtain the set of parameter values for which an asymptotically stable limit cycle exists, and then give sufficient conditions on the coupling parameters which can guarantee asymptotically global synchronization of oscillators given the same external input. The above results are potentially useful to analytical and numerical work on the binding problem in perceptual grouping and pattern segmentation.

Keywords: neural network, limit cycle, synchronization, dynamic system

1 Introduction

A fundamental aspect of perception is to bind spatially separate sensory features to form coherent objects. There is also wide experiment evidence that perception of a single object (especially in the visual scene) involves distributed in a highly fragmented over a large spatial region. The problem thus arise of how the constituent features are correctly integrated together to represent a single object.

Some authors [1, 3, 5] assume that these features of an object are grouped based on the temporal correlation of neural activities. Thus neurons that fire in synchronization would signal features of the same object, and groups desynchronized from each other represent different objects. Experimental observations of the visual cortex of animals show that synchronization indeed exists in spatially remote columns and phase-locking can also occur between the striate cortex and extratriate cortex, between the two striate cortices of the two brain hemisphere, and across the sensorimotor cortex. These findings have concentrated the attention of many researchers on the use of neural oscillators such as Wilson-Cowan oscillators and so on. In this scheme, neural oscillators that are in phase would represent a single object (binding), while neural groups with no phase lock would represent different objects. Though there are some analysis results [4, 5, 7, 8, 10] on Wilson-Cowan neural network, the results on autonomous Wilson-Cowan network system are still less. To make use of oscillation in phase, it is necessary to study the autonomous Wilson-Cowan network system. In this paper, we use a neural network based on locally coupled Wilson-Cowan oscillators to analyse the binding problem. To solve the binding problem, it is necessary to determine the conditions under which neural oscillators would exhibit periodical behavior and synchronize asymptotically..

The paper is organized as follows. In Section 2, the mathematical model is described. Our main theoretical results are given in Section 3. Conclusions are given in Section 4.

2 Mathematical model

In the model each oscillator is described by means of simplified Wilson-Cowan equations. Such a model consists of two nonlinear ordinary differential equations representing the interactions between two populations of neurons that are distinguished by the fact that their synapses are either excitatory or inhibitory. Thus, each oscillator consists of a feedback loop between an excitatory unit x_i and inhibitory unit y_i that obey the equations:

$$\frac{dx_i}{dt} = -r_1 x_i + r_1 H(ax_i - cy_i + I_i - \phi_x) + \alpha[\gamma_{i-1,i}(x_{i-1} - x_i) + \gamma_{i+1,i}(x_{i+1} - x_i)]$$

$$\frac{dy_i}{dt} = -r_2 y_i + r_2 H(bx_i - dy_i - \phi_y) + \beta[\gamma_{i-1,i}(y_{i-1} - y_i) + \gamma_{i+1,i}(y_{i+1} - y_i)]$$
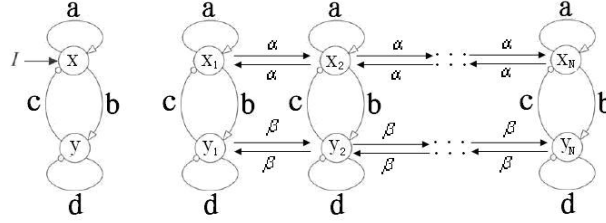
(1)

Fig.1. A single oscillator and an open chain of coupled oscillators

Both x_i and y_i variables are interpreted as the proportion of active excitatory and inhibitory neurons respectively, which are supposed to be continuous variables and their values may code the information processed by these populations. Especially, the state $x_i = 0$ and $y_i = 0$ represents a background activity. The parameters have the following meanings: a is the strength of the self-excitatory connection, d is the strength of the self-inhibitory connection, b is the strength of the coupling from x to y, c is the strength of the coupling from y to x. Both ϕ_x and ϕ_y are thresholds, r_1 and r_2 modify the rate of change of the x and y unit respectively. Fig.1 shows the connections for single oscillator and the structure of an open chain of coupled oscillators. All these parameters have nonnegative values, I_i is external input to the oscillator in position i which corresponds to a pixel in object. $H(\cdot)$ is a sigmoid activation function defined as: $H(z) = 1/(1 + e^{-z/T})$, T is a parameter that sets the central slope of the sigmoid relationship, α and β represent the strength of the connection between neurons.

3 Model analysis

3.1 Oscillating conditions for single oscillator

Consider the following system of a single oscillator:

$$\tau_1 \frac{dx_i}{dt} = -x_i + H(ax_i - cy_i + I_i - \phi_x)$$

$$\tau_2 \frac{dy_i}{dt} = -y_i + H(bx_i - dy_i - \phi_y)$$

(2)

Where $\tau_1 = 1/r_1, \tau_2 = 1/r_2$. In order to study the equation of the model, we choose the following piece-wise linear function to approximate the sigmoid function in (2):

$$G(z) = \begin{cases} 0, & z < -2T \\ \dfrac{z}{4T} + \dfrac{1}{2}, & -2T \le z \le 2T \\ 1, & z > 2T \end{cases} \qquad (3)$$

Thus, the system (2) may be described as:

$$\begin{aligned} \tau_1 \dot{x}_i &= -x_i + \eta \\ \tau_2 \dot{y}_i &= -y_i + \gamma \end{aligned} \qquad (4)$$

where $\eta, \gamma \in \left\{ 0, 1, \dfrac{ax_i - cy_i + I_i + 2T - \phi_x}{4T}, \dfrac{bx_i - dy_i + 2T - \phi_y}{4T} \right\}$.

In order to solve the binding problem, we aim to find the conditions under which the oscillators keep silent when I_i equal to zero and the system will exist an asymptotic stable limit cycle when I_i adopts proper value. Through analysis, it is easy to find the necessary conditions for the above result.

$(1)\phi_y + 2T < b; (2)I_i - \phi_x - 2T > 0; (3)a - \phi_x + 2T + I_i < c; (4)d + \phi_y - 2T > 0;$

$(5)\phi_x - 2T > 0; (6)(a - 4T)r_1 > (d + 4T)r_2; (7)bc + (4T - a)(4T + d) > 0;$

$(8)\left| ax^* - cy^* + I_i - \phi_x \right| \le 2T; (9)\left| bx^* - dy^* - \phi_y \right| \le 2T$

where $x^* = \dfrac{d + 4T}{b} y^* + \dfrac{\phi_y - 2T}{b}; \quad y^* = \dfrac{b(\phi_x - I_i - 2T) - (a - 4T)(\phi_y - 2T)}{(a - 4T)(d + 4T) - bc}$.

On the base of the obtained conditions, it is easy to verify the fact that the above conditions satisfy Poincare-Bendixon Theorem. So an asymptotic stable limit cycle must exist. If you run the MatLab simulation, you will see that an asymptotic stable limit cycle does indeed exist and it is plotted in Fig. 2.
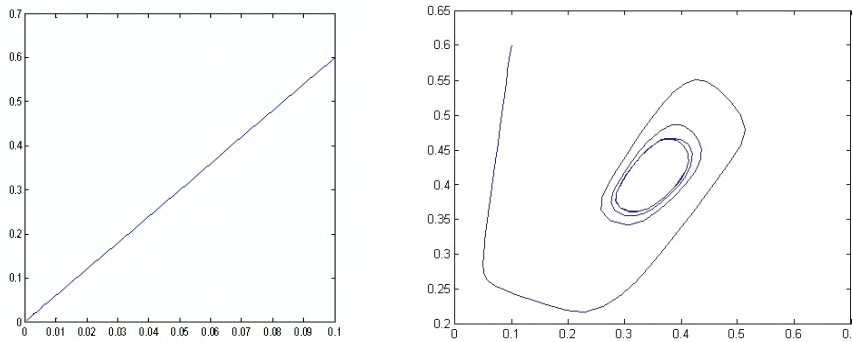

Fig.2. Two phase diagrams (an asymptotically stable zero solution and an asymptotically stable limit cycle). The parameters are: $a = 1, b = 1, c = 2, d = 0.5, I_i = 0, I_i = 0.65$ and $T = 0.025, r_1 = r_2 = 1, \phi_x = 0.2, \phi_y = 0.15$.

3.2 Synchronization of locally coupled Wilson—Cowan oscillators:

The boundary conditions of the coupled system (1) are as follows:

$$x_0 = x_1, \quad x_{N+1} = x_N; \quad y_0 = y_1, \quad y_{N+1} = y_N$$

$$r_{ij} = \begin{cases} 1, & if \left| I_i - I_j \right| < \phi \\ 0, & otherwise \end{cases}$$

where ϕ is a threshold. Based on the above restrictions, we will give the synchronization conditions of the coupled system in (1).

Theorem 1: Consider an open chain coupled oscillators receiving the same input $I_i = I$, $i = 1, 2, \cdots N$ in (1), the synchronization state is asymptotically stable if the following conditions hold: $0 < r_2 \le r_1, \beta \le \alpha$,

$$\alpha > \frac{(2a+c)r_1 + (b-8T)r_2}{16T(1-\cos\frac{\pi}{N})}, \quad \beta > \frac{(2a+c)r_1 + (b-8T)r_2}{16T} + \alpha\cos\frac{\pi}{N}$$

4 Conclusions

The paper presented a qualitative analysis on locally coupled autonomous Wilson-Cowan neural network, and gave the conditions under which the oscillator can oscillate and synchronize asymptotically. Note they are only sufficient conditions to achieve oscillation and synchronization. Some authors [5, 7, 8] give some analysis results on chaotic synchronization in Wilson-Cowan neural network and put these into image segmentation. And some results [10] lack of the generality and effectiveness in application more or less. In contrast to them, the obtained results in the paper are more convenient to solve the binding problem. In the future research, the authors will apply the proposed model to image segmentation and test its performance.

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