Masking a singularity with $k-$essence fields in an emergent gravity metric

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It is known that dynamical solutions of the k-essence equation of motion change the metric for the perturbations around these solutions and the perturbations propagate in an emergent spacetime with metric $\tilde{G}^{\mu\nu}$ different from the gravitational metric $g^{\mu\nu}$. We show that for observers travelling with the perturbations, there exist field configurations for the lagrangian $L = [\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi]^{\frac{1}{2}}$ for which a singularity in the gravitational metric $g^{\mu\nu}$ can be masked or hidden for such observers. This is shown for the Schwarzschild and the Reissner-Nordstrom metrics.

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1.Introduction

Present day observations have established that the universe consists of roughly 25 percent dark matter, 70 percent dark energy , about 4 percent free hydrogen and helium with the remaining one percent consisting of stars, dust, neutrinos and heavy elements. Actions with non-canonical kinetic terms have been shown to be strong candidates for dark matter and dark energy. A theory with a non-canonical kinetic term was first proposed by Born and Infeld in order to get rid of the infinite self-energy of the electron [1]. Similar theories were also studied in [2, 3]. Cosmology witnessed these models first in the context of scalar fields having non-canonical kinetic terms which drive inflation. Subsequently $k-$essence models of dark matter and dark energy were also constructed [4–11]. Effective field theories arising from string theories also have non-canonical kinetic terms [12–15].

An approach to understand the origins of dark matter and dark energy involve setting up lagrangians for what are known as $k-$essence fields in a Friedman-Robertson-Walker metric with zero curvature constant. In one approach [16] it is possible to unify the dark matter and dark energy components into a single scalar field model with the scalar field ϕ having a non-canonical kinetic term. These scalar fields are the $k-$essence fields mentioned above. The general form of the lagrangian for these $k-$essence models is assumed to be a function $F(X)$ with $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, and do not depend explicitly on ϕ to start with. In [16], X was shown to satisfy a general scaling relation,*viz.* $X(\frac{dF}{dX})^2 = Ca(t)^{-6}$ with C a constant (similar expression was also derived in [17]).

Recently a lagrangian for the $k-$essence field has been set up [18] in a homogeneous and isotropic universe where there are two generalised coordinates $q(t) = ln\ a(t)$ ($a(t)$ is the scale factor) and a scalar field $\phi(t)$ with a complicated polynomial interaction between them. In the lagrangian, q has a standard kinetic term while ϕ does not have a kinetic part and occurs purely through the interaction term. [18] incorporates the scaling relation of [16] and shows that the the lagrangian has a $\sqrt{X}$ dependence. Classical solutions of this lagrangian gave very good results for relevant cosmological parameters [18]. So addressing questions relating to (quantum) fluctuations became meaningful. In [19] questions regarding the amplitude of a scale factor at some epoch evolving to a different value at a later epoch was addressed for the above lagrangian at times close to the big bang (very small scale factor). As the scale factor is inversely proportional to the temperature at a particular epoch, these amplitudes provided an estimate of quantum fluctuations of the temperature.

Relativistic field theories with canonical kinetic terms differ from lagrangian theories of k-essence in that non-trivial dynamical solutions of the k-essence equation of motion not only spontaneously break Lorentz invariance but also change the metric for the perturbations around these solutions [20]. The perturbations propagate [20, 21] in an emergent spacetime with metric $\tilde{G}^{\mu\nu}$ different from (and also not conformally equivalent to) the gravitational metric $g^{\mu\nu}$.

Now $g^{\mu\nu}$ can contain physical singularities. The motivation of this work is to investigate whether scenarios can be constructed where the singularity in $g^{\mu\nu}$ can be "masked" to observers travelling piggy-back on the perturbations of the k-essence scalar fields. In the context of cosmological perturbations ,it has been shown in [20] that for purely kinetic k-essence theories there exist lagrangians which are proportional to $\sqrt{X}$. We show here that a simple model lagrangian *viz.* $L = \sqrt{X}$ has k-essence field configurations (for both the Schwarzschild and the Reissner-Nordstrom metrics) for which the singularities can be masked for observers sitting on the scalar field perturbations. The plan of the paper is as follows. In Section 2 a brief summary is given of emergent gravity concepts as developed in [20]. In Section 3 the Schwarzschild metric is considered while Section 4 deals with the Reissner-Nordstrom case. Section 5 is the conclusion.

2. Emergent Gravity

Consider the $k-$essence scalar field ϕ minimallly coupled to the gravitational field $g_{\mu\nu}$. Then the $k-$essence action is

$$S_k[\phi, g_{\mu\nu}] = \int d^4x\sqrt{-g}L(X,\phi) \quad (1)$$

where $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ and ∇_μ means the covariant derivative asociated with the metric $g_{\mu\nu}$. The total action describing the dynamics of $k-$essence and gravity is

$$S[\phi, g_{\mu\nu}] = \int d^4x\sqrt{-g}[-\frac{1}{2}M_{\rm Pl}^2 R + L(X,\phi)] \quad (2)$$

where R is the Ricci scalar and $M_{\rm Pl}$ the reduced Planck mass. The energy momentum tensor for the $k-$essence field is (with $L_{\rm X} = \frac{dL}{dX}$, $L_{\rm XX} = \frac{d^2L}{dX^2}$)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_k}{\delta g^{\mu\nu}} = L_X\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}L \quad (3)$$

and the equation of motion for the $k-$essence field is

$$-\frac{1}{\sqrt{-g}}\frac{\delta S_k}{\delta\phi} = \tilde{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + 2XL_{X\phi} - L_\phi \quad (4)$$

where the effective metric $\tilde{G}^{\mu\nu}$ is

$$\tilde{G}^{\mu\nu} = L_X g^{\mu\nu} + L_{XX}\nabla^\mu\phi\nabla^\nu\phi \quad (5)$$

and is physically meaningful only when

$$\frac{1+2XL_{XX}}{L_X} > 0$$

When this condition holds everywhere the effective metric $\tilde{G}^{\mu\nu}$ determines the characteristics for $k-$essence [8, 22–24] .For the non-trivial configurations of the $k-$essence field $\partial_\mu\phi \neq 0$ and $\tilde{G}^{\mu\nu}$ is not conformally equivalent to $g^{\mu\nu}$. So the characteristics are different from canonical scalar fields whose lagrangians are linear in X. The characteristics determine the local causal structure of the spacetime at every point of the manifold. So the local causal structure for the $k-$essence field is different from those ones defined by $g_{\mu\nu}$.

It has been shown in [20] that a solution of the equation of motion (4) can be obtained as $L = f(\phi)\sqrt{X} - V(\phi)$. In this work we take $f(\phi)$ and $V(\phi)$ to be constants and so our lagrangian is effectively $L = \sqrt{X}$.

2. The Schwarzschild solution

The Schwarzschild metric is given by ($r_s = 2GM/c^2 \equiv 2GM$, taking $c = 1$)

$$ds^2 = (1-\frac{r_{\rm s}}{r})dt^2 - (1-\frac{r_{\rm s}}{r})^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\Phi^2) \quad (6)$$

and the emergent metric components $\tilde{G}^{\mu\nu}$ are related to the Schwarschild metric components $g^{\mu\nu}$ by (5) so that

$$\tilde{G}^{00} = \big(1-\frac{r_{\rm s}}{r}\big)L_{\rm X} + L_{XX}\big(\frac{\partial\phi}{dt}\big)^2 \quad (7)$$

$$\tilde{G}^{11} = \big(1-\frac{r_{\rm s}}{r}\big)^{-1}L_{\rm X} + L_{XX}\big(\frac{\partial\phi}{dr}\big)^2 \quad (8)$$

As we are concerned only with the singularity structure of the metrics we are not discussing the $\tilde{G}^{22}$ and $\tilde{G}^{33}$ components as g^{22} and g^{33} are well behaved for $r \to 0$.

Note that at $r = 0$ (7) has a singularity in the term $\big(1-\frac{r_{\rm s}}{r}\big)L_{\rm X}$ while the first term on the right hand side of (8) merely goes to zero. So to avoid the physical singularity we must have

$$L_{\rm X}(1-\frac{r_{\rm s}}{r}) = h_1(r) \quad (9)$$

Let us assume the function h_1 to be a constant. Choose this constant to be $c_{\rm s}$ and the k-essence scalar field to be spherically symmetric. With the lagrangian $L = \sqrt{X}$, the equation to be solved is then $L_{\rm X}(1-\frac{r_{\rm s}}{r}) = c_{\rm s}$ i.e. $X = g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi = \frac{1}{2c_{\rm s}^2}(1-\frac{r_{\rm s}}{r})^2$ i.e.

$$(1-\frac{r_{\rm s}}{r})\Big(\frac{\partial\phi}{dt}\Big)^2 - (1-\frac{r_{\rm s}}{r})^{-1}\Big(\frac{\partial\phi}{dr}\Big)^2 = \frac{1}{2c_{\rm s}^2}(1-\frac{r_{\rm s}}{r})^2 \quad (10)$$

Assume the solutions to be of the form $\phi_{\rm s}(r,t) = \phi_{1s}(r) + \phi_{2s}(t)$ Then (10) reduces to

$$\Big(\frac{\partial\phi_{2s}}{dt}\Big)^2 = \frac{1}{c_{\rm s}^2}(1-\frac{r_{\rm s}}{r}) + (1-\frac{r_{\rm s}}{r})^{-2}\Big(\frac{\partial\phi_{1s}}{dr}\Big)^2 \quad (11)$$

The *l.h.s.* of (11) is a function of t only and the *r.h.s.* a function of r only. So both sides must separately be equal to a constant k i.e.

$$\Big(\frac{\partial\phi_{2s}}{dt}\Big)^2 = \frac{1}{2c_{\rm s}^2}(1-\frac{r_{\rm s}}{r}) + (1-\frac{r_{\rm s}}{r})^{-2}\Big(\frac{\partial\phi_{1s}}{dr}\Big)^2 = k \quad (12)$$

Note that $c_{\rm s}^2$ is always positive and for convenience we take $k = 1$. Therefore $\phi_{2s}(t) = \sqrt{k}t \equiv t = t$, taking all integration constants to be zero. The spatial equation to be solved now is ($b = \frac{1}{2c_{\rm s}^2}$):

$$\Big(\frac{\partial\phi_{1s}}{dr}\Big) = \Big[(1-\frac{r_{\rm s}}{r})^2 - b(1-\frac{r_{\rm s}}{r})^3\Big]^{\frac{1}{2}} \quad (13)$$

The solution is

$$\phi_{1s}(r) = (r+2r_{\rm s})\sqrt{1-b+\frac{r_{\rm s}b}{r}} + \frac{r_{\rm s}}{\sqrt{1-b}}(3b-2)ln\Big[2\sqrt{1-b}\sqrt{r} + \sqrt{r}\sqrt{1-b+\frac{r_{\rm s}b}{r}}\Big] \quad (14)$$

Hence the full solution is

$$\phi_{\rm s}(r,t) = t + (r+2r_{\rm s})\sqrt{1-b+\frac{r_{\rm s}b}{r}} + \frac{r_{\rm s}}{\sqrt{1-b}}(3b-2)ln\Big[2\sqrt{1-b}\sqrt{r} + \sqrt{r}\sqrt{1-b+\frac{r_{\rm s}b}{r}}\Big] \quad (15)$$

When does this field configuration also satisfy the equation of motion (4)? It is seen that with our choice for L , the last two terms on the *r.h.s.* of (4) are zero and the temporal part of the equation *viz.* is $G^{00}(\partial_0^2\phi_{2s}) = 0$ for all times t while the spatial part i.e. $G^{11}(\partial_r^2\phi_{1s})$ vanishes for

$$r = r_{\rm s} \quad ; \quad r = \frac{3b}{3b-2} r_{\rm s} \tag{16}$$

As $b = \frac{1}{2c_{\rm s}^2}$ is always positive the equations of motion of emergent gravity for the scalar field are always satisfied at the Schwarzschild event horizon $r_{\rm s}$ and beyond in units of $r_{\rm s}$ for all times t. For obvious reasons we are interested only in solutions in regions that are accessible to an external observer i.e. in regions for which $r \geq r_s$.

It can also be verified that the terms $L_{XX}\big(\frac{\partial\phi}{dt}\big)^2$ and $L_{XX}\big(\frac{\partial\phi}{dr}\big)^2$ in (7) and (8) respectively remain well behaved with this field configuration $\phi_s(r,t)$ for $r \to 0$.

3. The Reissner-Nordstrom black hole

For a static charged black hole with charge Q the metric is the Reissner-Nordstrom metric :

$$ds^2 = (1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2})dt^2 - (1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2})^{-1}dr^2 \\ - r^2(d\theta^2 + sin^2\theta d\Phi^2) \tag{17}$$

with $r_{\rm Q}^2 = GQ^2/4\pi\epsilon_0 c^4 \equiv GQ^2/4\pi\epsilon_0$ taking $c = 1$. The emergent gravity metric components are related to their Reissner-Nordstrom counterparts by

$$\tilde{G}^{00} = \left(1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2}\right)L_{\rm X} + L_{XX}\left(\frac{\partial\phi}{dt}\right)^2 \tag{18}$$

$$\tilde{G}^{11} = \left(1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2}\right)^{-1}L_{\rm X} + L_{XX}\left(\frac{\partial\phi}{dr}\right)^2 \tag{19}$$

We now carry out a similar analysis as before for the same lagrangian $L = \sqrt{X}$. The equation to be solved now is $L_{\rm X}(1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2}) = c_{rn}$ i.e.

$$(1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2})\left(\frac{\partial\phi}{dt}\right)^2 - (1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2})^{-1}\left(\frac{\partial\phi}{dr}\right)^2 \\ = \frac{1}{2c_{\rm rn}^2}(1 - \frac{r_{\rm s}}{r} + \frac{r_{\rm Q}^2}{r^2})^2 \tag{20}$$

Again assuming solutions to be like $\phi_{\rm rn}(r,t) = \phi_{1rns}(r) + \phi_{2rn}(t)$ and proceeding exactly as before , the full solution for the field is obtained as

$$\phi_{\rm rn}(r,t) = t + \sqrt{r_{\rm s}r - r_{\rm Q}^2}\left[2 + \frac{5r_{\rm s}}{4r} - \frac{r_{\rm Q}^2}{2r^2}\right] \\ - \left(\frac{3r_{\rm s}^2}{4r_{\rm Q}} + 2r_{\rm Q}\right)tan^{-1}\left(\frac{\sqrt{r_{\rm s}r - r_{\rm Q}^2}}{r_{\rm Q}}\right) \tag{21}$$

This solution will also be a solution of the equation of motion (4) for all times t and for the following values of the radial coordinate

$$r_1 = 2r_{\rm Q}^2/r_{\rm s} \quad ; \quad r_2 = \frac{1}{2}[3r_s \pm \sqrt{9r_s^2 - 12r_{\rm Q}^2}] \\ r_3 \equiv r_\pm = \frac{1}{2}[r_s \pm \sqrt{r_s^2 - 4r_{\rm Q}^2}] \tag{22}$$

The solutions should be on or outside the event horizon for obvious regions. Thus r_2 and r_3 are valid solutions whereas r_1 is unphysical. (r_3 denotes the two event horizons while $r_2 > r_3$ and we are ruling out extremal black holes). Again, the terms $L_{XX}\big(\frac{\partial\phi}{dt}\big)^2$ and $L_{XX}\big(\frac{\partial\phi}{dr}\big)^2$ in (18) and (19) respectively remain well behaved with this field configuration $\phi_{rn}(r,t)$ for $r \to 0$.

4.Conclusion

In this work we have shown that for observers whose world is in an emergent gravity metric $\tilde{G}^{\mu\nu}$, singularities in the gravitational metric $g^{\mu\nu}$ can remain masked for certain $k-$essence field configurations which are also solutions of the equations of motion. This means that observers travelling with the perturbations of $k-$essence fields that are solutions of the equations of motion will never be aware of the physical singularities of the gravitational metric as this is not conformally equivalent to the emergent gravity metric. This has been shown for the Schwarzschild and the Reissner-Nordstrom metrics.

The scenario works for certain specific values of the radial coordinate r and for *all* values of the temporal coordinates. In the Schwarzschild case the radial coordinate $r \geq r_{\rm s}$ where $r_{\rm s}$ is the event horizon, while in the Reissner-Nordstrom case the solutions are again the event horizons $r_\pm = \frac{1}{2}[r_s \pm \sqrt{r_s^2 - 4r_{\rm Q}^2}]$ and two other values greater than $r_\pm$.

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