Effective Theory in Spinfoam Cosmology: A First Order LQG-corrected FRW Cosmology and the Stiff Fluid

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We present an effective theory of a basic holomorphic spinfoam cosmology peaked on homogeneous isotropic metrics. The quantum-corrected Hamiltonian constraint of this effective theory is motivated by an operator equation $\hat{H}W = 0$, satisfied by the LQG transition amplitude W, reduced to a classical phase space symplectic structure. The analysis of this quantity shows that this effective model gives first order corrections to the classical FRW dynamical expressions in $\hbar$ resembling a universe with an ultralight irrotational stiff perfect fluid as matter-energy content. Such an exotic fluid can also be regarded as a massless real scalar field.

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I. INTRODUCTION

Recently, Bianchi, Rovelli and Vidotto [1] introduced a simple symmetry-reduced spinfoam model of quantum cosmology. They started their analysis with the full theory of LQG and computed the transition amplitude $W(z_i, z_f)$ between holomorphic coherent quantum states of spacetime, peaked on homogeneous isotropic metrics in a first order vertex expansion on a finite dimensional graph (dipole approximation) in the large volume limit, where the scale factor of the universe R is much larger than the Planck length $l_p \sim 10^{-35}$m. They also showed that the amplitude W is an element of the kernel of a differential operator $\hat{H}$, i.e., $\hat{H}W = 0$, whose classical limit is the Hamiltonian constraint of the FRW cosmology without extrinsic curvature ($k = 0$) in the absence of a mass content and a cosmological constant.

Here, we discuss an effective model of quantum cosmology that emerges from this operator equation disregarding the *full* transition to the classical limit. This leads to the classical FRW Hamiltonian constraint with a first order quantum gravity correction in $\hbar$. By solving this generalized Hamiltonian constraint and comparison to the classical Friedmann equations, we find the quantum correction to represent an ultralight irrotational stiff fluid matter-energy content filling the universe [2, 3]. This is of particular interest for the following reasons. First, the speed of sound of this exotic fluid is equal to the speed of light. Therefore, it is the limiting case in which causality still holds. Second, a correspondence between a stiff fluid and a scalar field as shown in [4] allows the interpretation of the quantum correction as a decelerating massless real scalar field.

II. EFFECTIVE SPINFOAM COSMOLOGY

A. Motivation

In order to give a proper motivation for the effective theory, we derive the quantum-corrected Hamiltonian constraint using the fact that the transition amplitude between different quantum states of spacetime satisfies a fundamental operator equation and by imposing a classical symplectic phase space structure.

The transition amplitude between two cosmological homogeneous isotropic coherent states reads in a holomorphic representation [1]

$$W(z_i, z_f) = N^2 z_i z_f \exp\left(-\frac{z_i^2 + z_f^2}{2t\hbar}\right). \tag{1}$$

The complex variables z_j, with $j \in \{i, f\}$, are given by

$$z_j = \alpha c_j + \mathtt{i}\beta p_j, \tag{2}$$

where $c_j = \gamma \dot{R}_j$ and $p_j = R_j^2$ are the LQC canonical variables as functions of the scale factor R_j and its canonical conjugated momentum $\dot{R}_j$ [5, 6] and α, β, γ are real-valued constants to be determined later. A detailed derivation of the multiplicative factor $N = -4i\pi^2 N_0/t^3\hbar$ can be found in [1, 7]. The transition amplitude (1) can be decomposed into the product

$$W(z_i, z_f) = W_i(z_i) W_f(z_f) \tag{3}$$

with the functions

$$W_j(z_j) = N z_j \exp\left(-\frac{z_j^2}{2t\hbar}\right). \tag{4}$$

These Gaussians fulfill the operator equation

$$\hat{H}W_j := \left(z_j^2 - t^2\hbar^2 \frac{\partial^2}{\partial z_j^2} - 3t\hbar\right) W_j = 0. \tag{5}$$

In order to make the transition to a classical phase space symplectic formulation with the symplectic 2-form

$$\omega = \frac{\mathtt{i}}{t} dz \wedge d\bar{z} \tag{6}$$

and the classical phase space variables z and $\bar{z}$, we introduce the multiplicative operator $\hat{z}_j = z_j$ and the derivative operator $\hat{\bar{z}}_j = t\hbar \frac{\partial}{\partial z_j}$. Then, Eq.(5) yields

$$\left(\hat{z}_j^2 - \hat{\bar{z}}_j^2 - 3t\hbar\right) W_j = 0. \tag{7}$$

We obtain the classical limit of this equation by replacing the operators $\hat{z}_j$ and $\hat{\bar{z}}_j$ with the corresponding classical phase space variables z and $\bar{z}$ and have

$$z^2 - \bar{z}^2 - 3t\hbar = 0. \tag{8}$$

In terms of the scale factor $R(T)$ and its canonical conjugated momentum $\dot{R}(T)$ this equation reads

$$4\mathtt{i}\alpha\beta\gamma R(T)^2\dot{R}(T) - 3t\hbar = 0. \tag{9}$$

Here, the variable T refers to the cosmological time. Multiplication with $3\mathtt{i}\dot{R}/(32\pi G\alpha\beta\gamma R)$ leads to a first order quantum geometry corrected complex FRW Hamiltonian constraint

$$H_{FRW}^{QG} = -\frac{3}{8\pi G}\left(R\dot{R}^2 + \frac{3\mathtt{i}t\hbar}{4\alpha\beta\gamma}\,\frac{\dot{R}}{R}\right) = 0. \tag{10}$$

The constants α and β are given by [1]

$$\alpha = 2\pi \;\text{ and }\; \beta = \frac{3t}{32\pi^2 G\gamma}, \tag{11}$$

where γ is the Immirzi parameter. Using the modified Planck length $l_{P,mod.} := 2\sqrt{\pi}l_P = 2\sqrt{\pi G\hbar}$, the Hamiltonian constraint leads to the non-linear homogeneous ordinary differential equation

$$\dot{R} + \frac{\mathtt{i}l_{P,mod.}^2}{R^2} = 0. \tag{12}$$

B. Solution to the Quantum Hamiltonian Constraint

Integrating Eq.(12) with respect to the cosmological time T, the solution for the scale factor is given by

$$R(T) = \left(-3\mathtt{i}l_{P,mod.}^2 T + \lambda_0\right)^{1/3}, \tag{13}$$

where λ_0 denotes an undetermined constant. The transition amplitude (1) and, hence, the Hamiltonian constraint (10) are computed in the framework of *Euclidean* spinfoam cosmology. Therefore, in order to include the scale factor (13) into an actual cosmological context, we have to place it in the background of the Lorentzian theory. The transition from the Euclidean theory to the Lorentzian theory can be accomplished by a simple transformation of the cosmological time T as follows. The FRW metric of the Euclidean theory with signature $+1$ can be written in the form

$$ds^2 = dT_{Eucl.}^2 + R(T_{Eucl.})^2 d\omega_3^2, \tag{14}$$

while the FRW metric of the Lorentzian theory with signature -1 reads

$$ds^2 = -dT_{Lor.}^2 + R(T_{Lor.})^2 d\omega_3^2. \tag{15}$$

The transition condition between the two is, thus, given by

$$T_{Eucl.} = \mathtt{i}T_{Lor.}. \tag{16}$$

Now, we obtain for the scale factor of the Lorentzian model a real-valued quantity

$$R(T) = \left(3l_{P,mod.}^2 T + \lambda_0\right)^{1/3}. \tag{17}$$

In order to interpret this quantum geometry corrected scale factor, we study the Lorentzian Friedmann equations for a flat universe ($k = 0$) with the possibility of considering a *massless* matter-energy distribution but without the presence of a cosmological constant ($\Lambda = 0$) as assumed in [1]. The Friedmann equations generally quantify the cosmological evolution of perfect fluids. Hence, for their derivation, the line element (15) as well as the energy-momentum tensor for a perfect fluid

$$T^{\mu\nu} = \left(\rho + p\right)u^\mu u^\nu + pg^{\mu\nu} \tag{18}$$

are considered. The physical quantity ρ is the energy density, p is the isotropic pressure and $(u^\mu) = (1, 0, 0, 0)$ is the fluid's 4-velocity in a comoving inertial frame. Then, the Friedmann equations of energy and acceleration are

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho \tag{19}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{20}$$

In contrast to several vacuum solutions of the Einstein field equations, isotropic expansion or contraction of space described by the FRW metric in cosmology is possible because of the presence of a matter-energy content.

C. Interpretation of the Generalized Scale Factor: The Ultralight Irrotational Stiff Fluid

Studying classical FRW cosmologies with $\rho \neq 0$, $\Lambda = 0$ and $k = 0$, it is possible to regard the quantum scale factor (17) as a consequence of the presence of a stable ultralight irrotational stiff perfect fluid as matter-energy content of the universe [2, 3]. In this special case the general perfect fluid equation of state

$$p = w\rho \tag{21}$$

becomes

$$p = \rho \tag{22}$$

with the equation of state parameter $w = 1$. This is the largest value consistent with causality, e.g., the speed of sound of the fluid equals the speed of light. Then, the variation of the energy density ρ with the scale factor as implied by the Friedmann equations (19) and (20) for such a fluid yields

$$\rho \propto \frac{1}{R(T)^6}. \tag{23}$$

Also, an irrotational stiff fluid can be considered as a massless real scalar field decelerating the expansion in the evolution of the universe [4]. With the equation of state (22), we obtain for the energy-momentum tensor (18)

$$T^{\mu\nu} = \rho\big(2u^\mu u^\nu + g^{\mu\nu}\big). \tag{24}$$

If the 4-velocity is irrotational

$$\nabla_{[\mu} u_{\nu]} = 0, \tag{25}$$

it can be written in terms of a future-pointing timelike gradient of a scalar field

$$\nabla_\mu \psi = \sqrt{2\rho}\, u_\mu. \tag{26}$$

The energy-momentum tensor (24) then becomes

$$T^{\mu\nu} = \nabla^\mu \psi \nabla^\nu \psi - \frac{1}{2} g^{\mu\nu} \nabla_\sigma \psi \nabla^\sigma \psi. \tag{27}$$

Meeting the energy-momentum conservation condition $\nabla_\mu T^{\mu\nu} = 0$, the scalar field ψ satisfies a massless minimally coupled wave equation

$$\nabla_\mu \nabla^\mu \psi = 0. \tag{28}$$

The mathematical equivalence presented in this analysis points out the local correspondence between the irrotational stiff fluid and the massless scalar field with timelike gradient. For a further and more detailed discussion of this subject see [4] and references therein. The equivalence between those two models, strengthens the opinion that scalar fields can be interpreted as classical approximations, in the sense of an effective theory, to some more general quantum fields. This LQG effective theory seemingly provides a natural link to a hypothetical epoch in the evolutionary phase of the universe where $w = 1$, i.e., in this epoch there exists a scalar field that is slowing down the expansion rate of the universe.

III. DISCUSSION & CONCLUSIONS

Following the formalism of the latest paper on spinfoam cosmology by Bianchi, Rovelli and Vidotto [1], we constructed an effective theory of quantum cosmology in adding a first order LQG correction in $\hbar$ to the classical flat FRW Hamiltonian constraint. We showed that this additional contribution leads to a scale factor that can be interpreted to be caused by an irrotational stiff perfect fluid matter distribution that equals a massless real scalar field. This result stands for an exiguous deceleration (of the order of l_P^2) of the expansion in the evolution of the universe with a deceleration parameter $q = 2$.

The next step in this study will be the inclusion of matter in the calculation of the transition amplitude (1) and, thus, in the effective model. Hopefully, it will then be possible to set theoretical limits to a realistic Hubble parameter for comparison with experiments and to explain the concept of dark energy in terms of loop quantum gravity modifications.

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