On the Unruh effect in de Sitter space

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Abstract

We give an interpretation of the temperature in de Sitter universe in terms of a dynamical Unruh effect associated with the Hubble sphere. As with the quantum noise perceived by a uniformly accelerated observer in static space-times, observers endowed with a proper motion can in principle detect the effect. In particular, we study a "Kodama observer" as a two-field Unruh detector for which we show the effect is approximately thermal. We also estimate the backreaction of the emitted radiation and find trajectories associated with the Kodama vector fields are stable.

1 Introduction

In Ref. [1], Kodama introduced a vector field which generalizes the notion of time-like Killing vectors to space-times with dynamical horizons [1]. Recently many authors have focused their attention on the Kodama vector field and found interesting results related to the measurement of the Hawking radiation emitted by different kinds of horizons [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] or timelike naked singularities [13].

We here present a more "physical" process of measurement in which the detector has finite mass and extension [14], and interacts with a massless scalar field like in Ref. [15]. The background is a de Sitter universe and the horizon is provided by its exponential expansion. The Kodama observer does not follow a cosmic fluid geodesics, but remains at constant proper distance from the horizon, and is thus "accelerated" with respect to locally inertial observers. Consequently, it perceives the de Sitter vacuum as a thermal bath with a temperature associated with the surface gravity of the Hubble sphere, much like a uniformly accelerated detector in Minkowski space-time measures a temperature associated with its acceleration. As usual, such result is obtained from the transition probability for the detector to emit a (scalar) radiation quantum and increase its energy. Further, assuming finite detector's size and mass, we show that the Kodama trajectory remains stable during this process.

We shall use the metric signature $(+ - - -)$ and implement units where $c = G = k_B = 1$.

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[1]For a clarification of the differences between Kodama and Killing vectors, see Ref. [2]. For the ability of Kodama vectors to define a local time flow, see Ref. [3]

2 Kodama observer in de Sitter

For an observer comoving with the cosmic fluid, the de Sitter metric reads

$$\mathrm{d}s^2 = \mathrm{d}t^2 - e^{2\,H\,t}\left[\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega\right] \ , \tag{2.1}$$

and the Kodama vector is given by

$$k^\mu = (1, -H\,r) \ , \tag{2.2}$$

which is related to the surface gravity $\mathcal{K}$ of the de Sitter horizon, namely

$$k^\alpha \nabla_{[\alpha} k_{\beta]} = -\mathcal{K}\,k_\beta = \left.\frac{1}{K}\right|_{K=1/H} = H \ . \tag{2.3}$$

We are interested in a detector moving along the Kodama trajectory,

$$r\,e^{Ht} = K \ , \tag{2.4}$$

where K is constant, and its four-velocity is then given by

$$u^\mu = \frac{k^\mu}{\sqrt{1 - H^2 K^2}} \ . \tag{2.5}$$

Analogously, the four-acceleration has components

$$\begin{cases} a^t = \dfrac{H^3\,K^2}{1 - H^2\,K^2} \\[2ex] a^r = -\dfrac{H^2\,K\,e^{-H\,t}}{1 - H^2 K^2} \ , \end{cases} \tag{2.6}$$

and

$$A^2 \equiv a^\mu\,a_\mu = -\frac{H^4\,K^2}{1 - H^2\,K^2} \ . \tag{2.7}$$

A very simple interpretation of the Kodama observers can be found if we go on to the static representation of de Sitter space,

$$\mathrm{d}s^2 = \left(1 - H^2\,\bar{r}^2\right)\mathrm{d}t^2 - \left(1 - H^2\,\bar{r}^2\right)^{-1}\mathrm{d}\bar{r}^2 - \bar{r}^2\,\mathrm{d}\Omega^2 \ . \tag{2.8}$$

The Kodama trajectory then corresponds to $\bar{r} = K$ with fixed angles, that is to observers, if we wish, which are at rest relative to the static coordinate system.

We can also introduce coordinates associated with the Kodama detector, namely

$$K = r\,e^{H\,t} \ , \quad T = t \ , \tag{2.9}$$

so that the metric reads [2]

$$\mathrm{d}s^2 = \left(1 - H^2\,K^2\right)\mathrm{d}T^2 + 2\,H\,K\,\mathrm{d}T\,\mathrm{d}K - \mathrm{d}K^2 - K^2\,\mathrm{d}\Omega^2 \ . \tag{2.10}$$

The four-velocity of the detector in Kodama coordinates $\{T, K\}$ is thus

$$\begin{cases} u^T = u^t \\[2ex] u^K = 0 \ , \end{cases} \tag{2.11}$$

and the four-acceleration

$$\begin{cases} a^T = a^t \\[2ex] a^K = -H^2\,K \ . \end{cases} \tag{2.12}$$

[2]Note that, in this case, the Kodama vector $\partial/\partial T$ is also a Killing vector.

3 Particles detectors

A key concepts of Physics – no matter if classical or quantum – is the concept of *particle*. The question "What is a particle?" is so important in our atomistic viewpoint that even in Quantum Field Theory (QFT) we feel necessary to find an answer.

As far as gravitation is not considered (an approximation which turns out to be often natural since gravity is the weakest known interaction), the answer runs straight without many accidents: in this case, in fact, the space-time where quantum fields propagate is Minkowskian. It is well known [16] that Minkowski space-time possesses a rectangular coordinate system (t, x, y, z) naturally associated with the Poincaré group, whose action leaves the Minkowski line element unchanged. The vector field ∂_t is a Killing vector of Minkowski space-time orthogonal to the space-like hypersurfaces $t = \text{constant}$ and the wave modes $u_{\mathbf{k}} \propto \exp(i\,\omega_{\mathbf{k}}\,t - \mathbf{k}\cdot\mathbf{x})$ are eigenfunctions of this Killing vector with eigenvalues $i\,\omega$. Thus, a particle mode solution in Minkowski space is one which is positive frequency $(\omega > 0)$ with respect to the time coordinate t. Under Poincaré transformations, positive frequency solutions transform to positive frequency solutions and the concept of particle is the same for every inertial observer, in the sense that all inertial observers agree on the number of particles present. Further, the Minkowski vacuum state, defined as the state with no particles present, is invariant under the Poincaré group.

Problems arise as one turns gravity on. In curved space-time, in fact, the Poincaré group is no longer a symmetry group. In general, there are no Killing vectors with which to define positive frequency modes and no coordinate choice is available to make the field decomposition in some modes more natural than others. This, of course, is not just an accident but is rooted in the same guiding principle of general relativity: that coordinate systems are physically irrelevant. As a possible way out, DeWitt and others suggested an operational definition of a particle: "a particle is something that can be detected by a particle detector". (That this definition may look as a tautology will not concern us further.) The particle detector proposed by Unruh [17] and later, in simplified version, by DeWitt [18] can be described as a quantum mechanical particle with many energy levels linearly coupled to a massless scalar field via a monopole moment operator. This is the construction we will employ in the following.

3.1 Point-like detectors

Let us consider a massless scalar field ϕ with Hamiltonian $\hat{H}_\phi$ obeying the massless Klein-Gordon equation. The free field operator is expanded in terms of a complete orthonormal set of solutions to the field equation as

$$\hat{\phi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d\mathbf{k}}{\sqrt{2\omega_{\mathbf{k}}}} \left(\hat{a}_{\mathbf{k}} e^{-i(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\mathbf{x})} + \hat{a}^\dagger_{\mathbf{k}} e^{i(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\mathbf{x})} \right) \ , \tag{3.1}$$

where, in the massless case, $\omega_{\mathbf{k}} = |\mathbf{k}|$. Field quantization is realized by imposing the usual commutation relations on the creation and annihilation operators,

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = \delta^3(\mathbf{k} - \mathbf{k}') \ , \qquad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = 0 = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] \ . \tag{3.2}$$

The Minkowski vacuum is the state $|\,0\,\rangle$ annihilated by $\hat{a}_{\mathbf{k}}$, for all $\mathbf{k}$.

The detector is a quantum mechanical system with a set of energy eigenstates $\{|\,0\,\rangle_d, |\,E_i\,\rangle\}$ which moves along a prescribed classical trajectory $t = t(\tau),\ \mathbf{x} = \mathbf{x}(\tau)$, where τ is the detector's proper time. The detector is coupled to the scalar field ϕ via the interaction Hamiltonian

$$\hat{H}_{int} = \lambda\,\hat{M}(\tau)\hat{\phi}(\tau) \ . \tag{3.3}$$

λ is treated here as a small parameter while $\hat{M}(\tau)$ is the detector's monopole moment operator whose evolution is provided by

$$\hat{M}(\tau) = e^{i\hat{H}_d\tau}\,\hat{M}(0)\,e^{-i\hat{H}_d\tau} \ , \tag{3.4}$$

$\hat{H}_d$ being the detector's Hamiltonian. This model is also known as the *point-like detector* since the interaction takes place at a point along the given trajectory at any given time.

Suppose that at time τ_0 the detector and the field are in the product state $|\,0, E_0\,\rangle = |\,0\,\rangle |\,E_0\,\rangle$, where $|\,E_0\,\rangle$ is a detector state with energy E_0 [19]. We want to know the probability that at a later time $\tau_1 > \tau_0$ the detector is found in the state $|\,E_1\,\rangle$ with energy $E_1 \gtrless E_0$, no matter what is the final state of the field ϕ. The answer is provided by the so-called interaction picture where we let both operators and states evolve in time according to the following understandings: operators evolution is governed by free Hamiltonians; states evolution is governed by the Schrödinger equation depending on the interaction Hamiltonian, that is

$$i\,\frac{d}{d\tau}|\,\varphi(\tau)\,\rangle = \hat{H}_{int}|\,\varphi(\tau)\,\rangle\ . \tag{3.5}$$

The amplitude for the transition from the state $|\,0, E_0\,\rangle$ at time $\tau = \tau_0$ to the state $|\,\varphi, E_1\,\rangle$ at time $\tau = \tau_1$ is then provided by

$$\langle\,\varphi, E_1\,|\,0, E_0\,\rangle = \langle\,\varphi, E_1\,|\hat{T}\exp\left(-i\int_{\tau_0}^{\tau_1} d\tau\,\hat{H}_{int}(\tau)\right)|\,0, E_0\,\rangle\ , \tag{3.6}$$

where $\hat{T}$ is the time-ordering operator. To first order in perturbation theory, we get

$$\begin{aligned}\langle\,\varphi, E_1\,|\,0, E_0\,\rangle &= \langle\,\varphi, E_1\,|\hat{\mathbb{I}}|\,0, E_0\,\rangle - i\,\lambda\,\langle\,\varphi, E_1\,|\int_{\tau_0}^{\tau_1} d\tau\,e^{i\hat{H}_d\tau}\,\hat{M}(0)\,e^{-i\hat{H}_d\tau}\hat{\phi}(\tau)|\,0, E_0\,\rangle + \ldots \\ &= -i\,\lambda\,\langle\,E_1\,|\hat{M}(0)|\,E_0\,\rangle\int_{\tau_0}^{\tau_1} d\tau\,e^{i\tau(E_1-E_0)}\langle\,\varphi\,|\hat{\phi}(\tau)|\,0\,\rangle + \ldots \end{aligned} \tag{3.7}$$

The transition probability to all possible finale states of the field ϕ is given by squaring (3.7) and summing over the complete set $\{|\,\varphi\,\rangle\}$ of final unobserved field states,

$$\sum_\varphi |\langle\,\varphi, E_1\,|\,0, E_0\,\rangle|^2 = \lambda^2|\langle\,E_1\,|\hat{M}(0)|\,E_0\,\rangle|^2\int_{\tau_0}^{\tau_1} d\tau\int_{\tau_0}^{\tau_1} d\tau'\,e^{-i(E_1-E_0)(\tau-\tau')}\langle\,0\,|\hat{\phi}(\tau)\,\hat{\phi}(\tau')|\,0\,\rangle\ . \tag{3.8}$$

This expression has two parts: the pre-factor $\lambda^2|\langle\,E_1\,|\hat{M}(0)|\,E_0\,\rangle|^2$ which depends only on the peculiar details of the detector and the *response function*

$$R_{\tau_0,\tau_1}(\Delta E) = \int_{\tau_0}^{\tau_1} d\tau\int_{\tau_0}^{\tau_1} d\tau'\,e^{-i\Delta E(\tau-\tau')}\langle\,0\,|\hat{\phi}(\tau)\,\hat{\phi}(\tau')|\,0\,\rangle\ , \tag{3.9}$$

which is insensitive to the internal structure of the detector and is thus the same for all possible detectors. Here, we have set the energy gap $\Delta E \equiv E_1 - E_0 \gtrless 0$ for excitations or decay, respectively. From now on, we will only consider the model-independent response function.

Introducing new coordinates $u := \tau$, $s := \tau - \tau'$ for $\tau > \tau'$ and $u := \tau'$, $s := \tau' - \tau$ for $\tau' > \tau$, the response function can be re-written as

$$R_{\tau_0,\tau_1}(\Delta E) = 2\int_{\tau_0}^{\tau_1} du\int_0^{u-\tau_0} ds\ \mathrm{Re}\left(e^{-i\Delta E s}\langle\,0\,|\hat{\phi}(u)\,\hat{\phi}(u-s)|\,0\,\rangle\right)\ , \tag{3.10}$$

having used $\langle\,0\,|\hat{\phi}(\tau')\hat{\phi}(\tau)|\,0\,\rangle = \langle\,0\,|\hat{\phi}(\tau)\hat{\phi}(\tau')|\,0\,\rangle^*$, since $\hat{\phi}$ is a self-adjoint operator. Eq. (3.10) can be differentiated with respect to τ_1 in order to obtain the *transition rate*

$$\dot{R}_{\tau_0,\tau}(\Delta E) = 2\int_0^{\tau-\tau_0} ds\ \mathrm{Re}\left(e^{-i\Delta E s}\langle\,0\,|\hat{\phi}(\tau)\,\hat{\phi}(\tau-s)|\,0\,\rangle\right)\ , \tag{3.11}$$

where we set $\tau_1 \equiv \tau$. If the correlation function $\langle\,0\,|\hat{\phi}(\tau)\hat{\phi}(\tau-s)|\,0\,\rangle$ is invariant under τ-translations, (3.11) can be further simplified,

$$\dot{R}_{\tau_0,\tau}(\Delta E) = \int_{-(\tau-\tau_0)}^{\tau-\tau_0} ds\ e^{-i\Delta E s}\langle\,0\,|\hat{\phi}(s)\,\hat{\phi}(0)|\,0\,\rangle\ . \tag{3.12}$$

The correlation function $\langle\, 0\, |\hat{\phi}(x)\hat{\phi}(x')|\, 0\,\rangle$ which appears in these expressions is the positive frequency Wightman function that can be obtained from (3.1),

$$\langle\, 0\, |\hat{\phi}(x)\, \hat{\phi}(x')|\, 0\,\rangle = \frac{1}{(2\pi)^3}\int \frac{d\mathbf{k}}{2\omega_{\mathbf{k}}} e^{-i\omega_{\mathbf{k}}(t-t')+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\ . \tag{3.13}$$

The integral in $|\mathbf{k}|$ contains UV divergences and can be regularized [16] by introducing the exponential cut-off $e^{-\epsilon|\mathbf{k}|}$, with $\epsilon > 0$ and small, in the high frequency modes. The resulting expression is

$$\langle\, 0\, |\hat{\phi}(x(\tau))\, \hat{\phi}(x(\tau'))|\, 0\,\rangle = \frac{1/4\pi^2}{|\mathbf{x}(\tau)-\mathbf{x}(\tau')|^2 - [t(\tau)-t(\tau')-i\epsilon]^2} \equiv W_\epsilon(x(\tau), x(\tau'))\ , \tag{3.14}$$

so that, we finally have

$$\dot{R}_{\tau_0,\tau}(\Delta E) = \lim_{\epsilon\to 0^+}\int_{-(\tau-\tau_0)}^{\tau-\tau_0} ds\ e^{-i\Delta E s}\, W_\epsilon(x(s), x(0))\ . \tag{3.15}$$

3.2 Unruh effect in De Sitter universe

Let us apply the above construction to Kodama detectors in de Sitter space-time. We first recall that de Sitter metric (2.1) and (2.8) in the cosmological global system reads

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - H^{-2}\cosh^2(H\tau)\, \mathrm{d}\Omega_3^2\ . \tag{3.16}$$

An easy calculation then gives the equivalent definitions of the Kodama trajectory (2.4)

$$r = K\, e^{-H\, t} \quad\Longleftrightarrow\quad \bar{r} = K \quad\Longleftrightarrow\quad \sin\chi = \frac{KH}{\cosh(H\tau)}\ . \tag{3.17}$$

As we mentioned before, the relevant equality is the second one: it means that the Kodama observers are just the stationary de Sitter observers at constant distance from their cosmological horizon. We know that these observers will perceive a thermal bath at de Sitter temperature $T = H/2\pi$, so this must be true in the inflationary patch as well.

We can confirm this expectation by rewriting Eq. (3.14) with the following relations for de Sitter space

$$\eta = -H^{-1}e^{-H\, t},\quad r = K\, e^{-Ht} = -K\, H\, \eta,\quad \tau = t\, \sqrt{1-K^2\, H^2}\ . \tag{3.18}$$

Then, provided $1 - K^2H^2 > 0$, the Wightman function (3.14) becomes

$$W_\epsilon(x, x') = -\frac{1}{4\pi^2}\frac{H^2}{e^{H(t+t')}}\frac{1}{(1-K^2H^2)(e^{-Ht}-e^{-Ht'}-i\epsilon)^2}\ . \tag{3.19}$$

and, in the limit of $\tau_0 \to -\infty$, the detector transition rate (3.15) becomes

$$\dot{R}(\Delta E) = -\frac{H^2}{4\pi^2}\lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty} ds\ \frac{\exp(-i\Delta E\, \sqrt{1-K^2H^2}\, s)}{(e^{Hs/2}-e^{-Hs/2}-i\epsilon)^2}\ . \tag{3.20}$$

As long as $H > 0$ (expanding universe) we can write the denominator as

$$\left(e^{H\, s/2} - e^{-H\, s/2} - i\epsilon\right)^2 = 4\, \sinh^2\left[H\, (s - i\epsilon)/2\right]\ . \tag{3.21}$$

This function has infinitely many double poles in the complex s-plane, namely for $s = s_j$ with

$$s_j = \frac{2\,\pi\, i}{H} j\ ,\quad j\in\mathbb{Z}\ . \tag{3.22}$$

Since we are interested in the case of $\Delta E > 0$, we can close the contour of integration in the lower half plane, summing over the residues of all the double poles in the lower complex s-plane, with the exception of the $s = 0$ pole which has been slightly displaced above the integration path by the $i\epsilon$-prescription. The well known result turns out to be confirmed, namely

$$\dot{R}(\Delta E) = \frac{\Delta E\,\sqrt{1-K^2H^2}\,\exp\left(\frac{2\,\pi}{H}\,\Delta E\,\sqrt{1-K^2\,H^2}\right)}{\exp\left(\frac{2\,\pi\,\Delta E\,\sqrt{1-K^2\,H^2}}{H}\right)-1}\ , \tag{3.23}$$

showing the presence of a cosmological Unruh effect with de Sitter temperature

$$T_H = \frac{H}{2\,\pi}\ , \tag{3.24}$$

red-shifted by the Tolman factor $\sqrt{1-K^2\,H^2}$, here appearing as a Doppler shift due to the proper motion of the detector. In fact, we remarked after Eq (3.17) that K is also the value of the static coordinate $\bar{r}$ of the detector relative to the static patch.

Another intriguing formula can be written if we recall the value (2.7) of the acceleration: one easily sees that

$$\frac{T_H}{\sqrt{1-H^2\,K^2}} = \frac{\sqrt{A^2+H^2}}{2\pi}\ . \tag{3.25}$$

One can interpret this formula by saying that the temperature is actually due to a mixing of a pure Unruh effect (the acceleration term) plus a cosmological expansion term (the H term), and is the de Sitter version [20] of a formula discovered by Deser & Levine for detectors in anti-de Sitter space [21].

Alternatively, we can understand this effect as the transition from cosmological energy ΔE, conjugated to cosmic de Sitter time t, to the energy $\Delta E\,\sqrt{1-K^2\,H^2}$ as measured locally by Kodama's observers (with proper time T).

3.3 Extended detector and backreaction

We now repeat the previous analysis assuming the detector's size is not *a priori* negligible. In order to simplify the computation, we still assume spherical symmetry and the detector therefore only extends along one dimension. We will not display all the details but focus on the main differences.

It is easy to see that a detector moving along a Kodama trajectory (2.4) in an expanding de Sitter universe, has the same dynamics of a particle which moves along the separatrix in the potential of an inverted harmonic oscillator. From the equation of motion (2.4), we can therefore introduce the effective Lagrangian

$$\mathcal{L}_{\rm iho} = m\left(\frac{{\rm d}^2 r}{{\rm d}t^2} + H^2\,r^2\right)\ , \tag{3.26}$$

where m is a parameter with mass dimension whose relation with physical quantities will be clarified later. Our detector is initially (at $t = 0$) represented by a Gaussian wave packet of size b peaked around $r = K$,

$$\psi(t,r) = \frac{\exp\left[-i\dfrac{H\,K\,m\,r}{\hbar} - \dfrac{(r-K)^2}{2\,b^2}\right]}{\sqrt{b\,\sqrt{\pi}}}\ , \tag{3.27}$$

which is then propagated to later times by the propagator obtained from the Lagrangian (3.26) (see Ref. [14])

$$G(t,r;0,r') = \sqrt{\frac{i\,H\,m}{2\,\pi\,\hbar\,\sinh(H\,t)}}\,\exp\left[i\,\frac{H\,m\left[(r^2-r'^2)\,\cosh(H\,t) - 2\,r\,r'\right]}{2\,\hbar\,\sinh(H\,t)}\right]\ . \tag{3.28}$$

The complete expression of the detector's propagated wavefunction is rather cumbersome, however we notice that its square modulus yields

$$|\psi(t,r)|^2 \simeq \exp\left[-\frac{2\,b^2\,H^2\,m^2\,(r-K\,e^{-H\,t})^2}{b^4\,H^2\,m^2-\hbar^2+(\hbar^2+b^4\,H^2\,m^2)\,\cosh(H\,t)}\right] \ , \tag{3.29}$$

and the classical behaviour is properly recovered in the limit $\hbar \to 0$ followed by $b \to 0$ [14], in which the detector's wavefunction $\psi(r,t)$ reproduces the usual Dirac δ-function peaked on the classical trajectory employed in Sections 3.1-3.2.

However, in order to study the probability for the detector to absorb a scalar quantum and make a transition between two different trajectories (parameterized by different m_i and K_i), one needs to compute the transition amplitude for finite b and $\hbar$ (otherwise the result would automatically vanish). The detector now interacts with the quantized scalar field $\varphi = \varphi(t,r)$ according to

$$\mathcal{L}_{\rm int} = \frac{1}{2}\,Q\,(\psi_2^*\,\psi_1+\psi_2\,\psi_1^*)\,\varphi \ , \tag{3.30}$$

where Q is a coupling constant and $\psi_i = \psi_i(t,r)$ two possible states of the detector corresponding to different trajectories $r_i = K_i\,e^{-H\,t}$ and mass parameters m_i [3]. We assume the difference between the two states is small,

$$\begin{cases} K_1 = K - \frac{1}{2}\,\delta K \\ \\ K_2 = K + \frac{1}{2}\,\delta K \end{cases} \qquad \begin{cases} m_1 = m - \frac{1}{2}\,\delta m \\ \\ m_2 = m + \frac{1}{2}\,\delta m \ , \end{cases} \tag{3.31}$$

and expand to leading order in δK and δm and, subsequently, for short times ($H\,t \sim H\,t' \ll 1$), keeping $\hbar$ and b finite. In particular, one obtains

$$\psi_2^*\,\psi_1(t)\,\psi_1^*\,\psi_2(t') \simeq \exp\left[-i\,\frac{H^2\,K^2}{\hbar}\,\delta m\,(t-t')+O(b)\right] \ , \tag{3.32}$$

in which we have evaluated the phase for r along the average trajectory between r_1 and r_2. Upon comparing with the result obtained for the point-like case, we immediately recognize that

$$H^2\,K^2\,m = E\,\sqrt{1-H^2\,K^2} \ , \tag{3.33}$$

where E is the detector's proper energy and

$$\psi_2^*\,\psi_1(t)\,\psi_1^*\,\psi_2(t') \simeq \exp\left[-\frac{i}{\hbar}\,\delta E\,\sqrt{1-H^2\,K^2}\,(t-t')+\frac{i}{\hbar}\,\frac{2-3H^2\,K^2}{K\sqrt{1-K^2\,H^2}}\,E\,\delta K\,(t-t')+O(b)\right] \ . \tag{3.34}$$

We can now take the limit $b \to 0$, as part of the point-like limit in which one would not expect the second term in the above exponential. In Ref. [15] we required the analogue of the second term above vanished and obtained the equation of motion for a uniformly accelerated detector in Minkowski space-time, namely $m\,a = f$ and constant. Following the same line of reasoning, we now obtain the equation of motion

$$\delta K = 0 \ . \tag{3.35}$$

This can be interpreted as meaning the Kodama trajectory is stable against thermal emission of scalar quanta in the de Sitter background.

The transition probability per unit de Sitter time can finally be computed by taking the classical limit, in which one recovers the same result in Eq. (3.23) with $\Delta E = \delta E$.

[3] A fundamental difference with respect to the Unruh effect analyzed in Ref. [15] is that the acceleration parameter H is not varied here, since it is a property of the background space-time. A change δK implies a change in the detector's acceleration according to Eq. (2.7).

4 Conclusions

We considered an expanding de Sitter universe that is inflating exponentially and studied the dynamics of an observer as an object which is collapsing towards the Hubble sphere. This is the "Kodama observer" placed at $K = r\,e^{H\,t}$ constant, and its radial velocity (2.5) is in fact negative which means that the observer is moving towards decreasing radii. In particular, the observer's motion is described by an inverted harmonic potential and involves the negative radial acceleration in Eq. (2.6). We can picture our observer/detector in spherical coordinates as a "shell" with a "density" profile $|\psi|^2$ peaked on the average radius r [see Eq. (3.29)] which sees the universe becoming less dense (in time) but in a homogenous way (in space). Of course, the detector's proper time differs from the cosmological time t for an observer comoving with the cosmic fluid.

The detector interacts with a scalar field so that it can absorb or emit quanta and change its proper mass (energy). Indeed, the Unruh effect is given by the simultaneous emission of a scalar quantum and detector's excitation. We therefore considered the transition between two states of the detector corresponding to two different trajectories associated with different energies and values of K and obtained the transition rate (3.23) in the point-like limit. Our result shows the expected thermal behaviour (3.24) associated with the trajectory's stability (3.35).

In summary, we studied the response of a Kodama detector moving along trajectories at constant distance from the Hubble sphere of the de Sitter universe and found that it perceives a thermal noise associated with the emission of scalar quanta. We also estimated the backreaction of the emitted radiation and showed trajectories associated with the Kodama vector fields are stable. This represents a novel semiclassical property of the (classically defined) Kodama vector.

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