Supersymmetric Cosmological FRW Model and Dark Energy

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In this work we consider a flat cosmological model with a set of fluids in the framework of supersymmetric cosmology. The obtained supersymmetric algebra allowed us to take quantum solutions. It is shown that only in the case of a cosmological constant we have a condition between the density of dark energy ρ_Λ and density energy of matter ρ_M, $\rho_\Lambda > 2\rho_M$.

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Minisuperspace models are useful toy models for canonical quantum gravity, because they capture many of the essential features of general relativity and are at the same time free of technical difficulties associated with the presence of an infinite number of degrees of freedom. As it is well known, the equation that governs the quantum behavior of these models is the Wheeler-DeWitt equation, which results in a quadratic Hamiltonian leading to an equation of the Klein-Gordon type. Introduction of supersymmetric minisuperspace models in which the Grassmann variables are not identified as the supersymmetric partners of the cosmological bosonic variables has led to the definition and study of linear "square root" equations defining the quantum evolution of the universe $[1-11]$.

Recently, we have used the superfield formulation to investigate supersymmetric cosmological models $[12-15]$. In previous works [14, 16] it was shown that the spatially homogeneous part of the fields in the supergravity theory

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preserves the invariance under the local time $n=2$ supersymmetry. This supersymmetry is a subgroup of the four-dimensional spacetime supersymmetry of the supergravity theory. This local supersymmetry procedure has the advantage that, by defining the superfields on superspace, all the component fields in a supermultiplet can be manipulated simultaneously in a manner that automatically preserves supersymmetry. Besides, the Grassmann variables are obtained in a clear manner as the supersymmetric partners of the cosmological bosonic variables. At the quantum level the Grassmann variables are elements of the Clifford algebra. Using superfield formulation the canonical quantization procedure for a closed FRW cosmological model, filled with pressureless matter (dust) content and the corresponding superpartner, was reported [16].

In the present work we have constructed the $n=2$ supersymmetric action for the spatially homogeneous isotropic flat, $(k=0)$ Friedmann-Robertson-Walker including a mixture of fluids with a constant equation of state parameters γ_i, $p_i = \gamma_i \rho_i$.

Let us start with the action [17, 18]

$$S = \int \Big[-\frac{R}{2N\tilde{G}} \Big(\frac{dR}{dt}\Big)^2 + \frac{N\Lambda}{6\tilde{G}} R^3 + N\Big(\sum_i M_{\gamma_i}^{1/2} R^{-3\gamma_i/2}\Big)^2 \Big] dt, \qquad (1)$$

with $\tilde{G} = \frac{8\pi G}{6}$, where G is the Newtonian gravitational constant and Λ is the cosmological constant; $N(t), R(t)$ are the lapse function and the scale factor, respectively; M_{γ_i} is the mass by unit $(\text{length})^{3\gamma_i - 1}$. Summation over i includes all types of fluids. In this work we have used units in which $c = \hbar = 1$.

The action (1) is invariant under the time reparametrization $t' \to t + a(t)$, if the transformations of $R(t)$ and $N(t)$ are defined as $\delta R = a\dot{R}$, and $\delta N = (aN)^{\cdot}$. The variation with respect to $R(t)$ and $N(t)$ leads to the classical equation for the scale factor $R(t)$ and the constraint, which generates the local reparametrization of $R(t)$ and $N(t)$. This constraint leads to the Wheeler-DeWitt equation in quantum cosmology.

In order to obtain the corresponding supersymmetric action for (1), we follow the superfield approach. Thus, we extend the transformation of time reparametrization to the $n=2$ local supersymmetry of time $(t, \eta, \bar{\eta})$. Then, we have the following local supersymmetric transformation

$$\begin{aligned} \delta t &= a(t) + \frac{i}{2}[\eta\bar{\beta}'(t) + \bar{\eta}\beta'(t)], \\ \delta\eta &= \frac{1}{2}\beta'(t) + \frac{1}{2}[\dot{a}(t) + ib(t)]\eta + \frac{i}{2}\dot{\beta}'(t)\eta\bar{\eta}, \\ \delta\bar{\eta} &= \frac{1}{2}\bar{\beta}'(t) + \frac{1}{2}[\dot{a}(t) - ib(t)]\bar{\eta} - \frac{i}{2}\dot{\bar{\beta}}'(t)\eta\bar{\eta}, \end{aligned} \qquad (2)$$

where η is a complex Grassmann coordinate, $\beta'(t) = N^{-1/2}\beta(t)$ is the Grassmann complex parameter of the local "small" $n=2$ supersymmetry (SUSY) transformation, and $b(t)$ is the parameter of local $U(1)$ rotations of the complex η. The superfield generalization of the action (1), invariant under supersymmetric transformation (2) has the form

$$S_{susy} = \int \Big[-\frac{1}{2\tilde{G}} I\!N^{-1} I\!R D_{\bar{\eta}} I\!R D_\eta I\!R + \frac{\Lambda^{1/2}}{3\sqrt{3}\tilde{G}} I\!R^3 - \\ - \frac{2\sqrt{2}}{\tilde{G}^{1/2}} \sum_i \frac{M_{\gamma_i}^{1/2}}{(3-3\gamma_i)} I\!R^{\frac{3-3\gamma_i}{2}} \Big] d\eta d\bar{\eta} dt, \tag{3}$$

where

$$D_\eta = \frac{\partial}{\partial\eta} + i\bar{\eta}\frac{\partial}{\partial t}, \qquad D_{\bar{\eta}} = -\frac{\partial}{\partial\bar{\eta}} - i\eta\frac{\partial}{\partial t}, \tag{4}$$

are the supercovariant derivatives of the global "small" supersymmetry of the generalized parameter corresponding to t. The local supercovariant derivatives have the form $\tilde{D}_\eta = I\!N^{-1/2} D_\eta$, $\tilde{D}_{\bar{\eta}} = I\!N^{-1/2} D_{\bar{\eta}}$, and $I\!R(t,\eta,\bar{\eta}), I\!N(t,\eta,\bar{\eta})$ are superfields. The Taylor series expansion for the superfields $I\!N(t,\eta,\bar{\eta})$ and $I\!R(t,\eta,\bar{\eta})$ is the following

$$I\!N(t,\eta,\bar{\eta}) = N(t) + i\eta\bar{\psi}'(t) + i\bar{\eta}\psi'(t) + V'(t)\eta\bar{\eta}, \tag{5}$$

$$I\!R(t,\eta,\bar{\eta}) = R(t) + i\eta\bar{\lambda}'(t) + i\bar{\eta}\lambda'(t) + B'(t)\eta\bar{\eta}. \tag{6}$$

In these expressions we have introduced the redefinitions $\psi'(t) = N^{1/2}\psi(t)$, $V' = N(t)V(t) + \bar{\psi}(t)\psi(t)$, $\lambda' = \frac{\tilde{G}^{1/2}N^{1/2}}{R^{1/2}}\lambda$ and $B' = \tilde{G}^{1/2}NB + \frac{\tilde{G}^{1/2}}{2R^{1/2}}(\bar{\psi}\lambda - \psi\bar{\lambda})$. The components of the superfield $I\!N(t,\eta,\bar{\eta})$ are gauge fields of the one-dimensional $n=2$ extended supergravity. $N(t)$ is the einbein, $\psi(t), \bar{\psi}(t)$ are the complex gravitino fields, and $V(t)$ is the $U(1)$ gauge field. The component $B(t)$ in (6) is an auxiliary degree of freedom (non-dynamical variable), and $\lambda, \bar{\lambda}$ are the fermion partners of the scale factor $R(t)$.

After integration over the Grassmann coordinates $\eta, \bar{\eta}$ and eliminating the auxiliary variable B, by means of their equation of motion, the action (3) acquires its component form

$$S_{susy} = \int \Bigg\{ -\frac{R(DR)^2}{2N\tilde{G}} + \frac{N\Lambda R^3}{6\tilde{G}} + N\Big(\sum_i M_{\gamma_i}^{1/2} R^{-3/2\gamma_i}\Big)^2 - \\ -\frac{\sqrt{2}\Lambda^{1/2}}{\sqrt{3}\tilde{G}^{1/2}} \sum_i M_{\gamma_i}^{1/2} R^{\frac{3-3\gamma_i}{2}} + \frac{i}{2}(\bar{\lambda}D\lambda - D\bar{\lambda}\lambda) + \frac{\sqrt{3}}{2}\Lambda^{1/2} N\bar{\lambda}\lambda \\ + \frac{N\sqrt{2}}{\tilde{G}^{1/2}} \sum_i (-1+6\gamma_i) M_{\gamma_i}^{1/2} R^{\frac{-3-3\gamma_i}{2}} \bar{\lambda}\lambda \\ + \frac{\Lambda^{1/2}}{2\sqrt{3}\tilde{G}^{1/2}} R^{3/2}(\bar{\psi}\lambda - \psi\bar{\lambda}) - \frac{\sqrt{2}}{2}\sum_i M_{\gamma_i}^{1/2} R^{-\frac{3\gamma_i}{2}}(\bar{\psi}\lambda - \psi\bar{\lambda}) \Bigg\} dt, \tag{7}$$

with $DR = \dot{R} - \frac{i\tilde{G}^{1/2}}{2R^{1/2}}(\psi\bar{\lambda} + \bar{\psi}\lambda)$ and $D\lambda = \dot{\lambda} - \frac{1}{2}V\lambda$, $D\bar{\lambda} = \dot{\bar{\lambda}} + \frac{1}{2}V\bar{\lambda}$. Proceeding with canonical quantization, the classical canonical Hamiltonian is calculated in the usual way for systems with constraints. It has the form

$$H_c = NH + \frac{1}{2}\bar{\psi}S - \frac{1}{2}\psi\bar{S} + \frac{1}{2}VF, \tag{8}$$

where H is the Hamiltonian of the system, S and $\bar{S}$ are the supercharges and F is the $U(1)$ rotation generator. The canonical Hamiltonian form (8) explains the fact that $N, \psi, \bar{\psi}$ and V are Lagrangian multipliers, which only enforce the first-class constraints $H = 0, S = 0, \bar{S} = 0$ and $F = 0$, and express the invariance under the $n = 2$ supersymmetric transformations. The first-class constraints may be obtained from the action (7) rewriting it in first order form varying $N(t), \psi(t), \bar{\psi}(t)$ and $V(t)$, respectively. The first-class constraints are

$$\begin{aligned} H &= -\frac{\tilde{G}}{2R}\pi_R^2 - \frac{\Lambda R^3}{6\tilde{G}} - \Big(\sum_i M_{\gamma_i}^{1/2} R^{-3/2\gamma_i}\Big)^2 + \frac{\sqrt{2}\Lambda^{1/2}}{\sqrt{3}\tilde{G}^{1/2}} \sum_i M_{\gamma_i}^{1/2} R^{\frac{3-3\gamma_i}{2}} \\ &- \frac{\sqrt{3}}{2}\Lambda^{1/2}\bar{\lambda}\lambda - \frac{1}{\sqrt{2}\tilde{G}^{1/2}} \sum_i (6\gamma_i - 1) M_{\gamma_i}^{1/2} R^{\frac{-3-3\gamma_i}{2}} \bar{\lambda}\lambda, \end{aligned} \tag{9}$$

$$S = \Big(\frac{i\tilde{G}^{1/2}}{R^{1/2}}\pi_R - \frac{\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}\sum_i M_{\gamma_i}^{1/2} R^{-\frac{3\gamma_i}{2}}\Big)\lambda, \tag{10}$$

$$\bar{S} = \Big(-\frac{i\tilde{G}^{1/2}}{R^{1/2}}\pi_R - \frac{\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}\sum_i M_{\gamma_i}^{1/2} R^{-\frac{3\gamma_i}{2}}\Big)\bar{\lambda}, \tag{11}$$

$$F = -\bar{\lambda}\lambda, \tag{12}$$

where $\pi_R = -\frac{R}{\tilde{G}N}\dot{R} + \frac{iR^{1/2}}{2N\tilde{G}^{1/2}}(\bar{\psi}\lambda + \psi\bar{\lambda})$ is the canonical momentum associated to R with Poisson brackets

$$\{R, \pi_R\} = 1. \tag{13}$$

As usually with Grassmann variables, we have second-class constraints, which can be eliminated by Dirac procedure. As a result, we only have the following non-zero Dirac brackets

$$\{\lambda, \bar{\lambda}\} = i. \tag{14}$$

With respect to these brackets the super-algebra for the generators $H, S, \bar{S}$ and F becomes

$$\{S, \bar{S}\} = -2iH, \quad \{S, H\} = \{\bar{S}, H\} = 0, \quad \{F, S\} = iS. \tag{15}$$

In a quantum theory the brackets (13) and (14) must be replaced by commutator and anticommutator; they can be considered as generators of the Clifford algebra

$$[R, \pi_R] = i \qquad \text{with} \qquad \pi_R = -i\frac{\partial}{\partial R}, \qquad \{\lambda, \bar{\lambda}\} = -1. \tag{16}$$

As we can see from the Hamiltonian (9), the energy of scalar factor R is negative. This is reflected in the fact that the anticommutator value (16) of superpartner λ and $\bar{\lambda}$ of the scalar factor is negative. This anticommutator relation may be satisfied under condition

$$\begin{aligned} \bar{\lambda} &= \xi^{-1}\lambda^{\dagger}\xi = -\lambda^{\dagger}, \qquad \{\lambda, \lambda^{\dagger}\} = 1, \\ \lambda^{\dagger}\xi &= \xi\lambda^{\dagger} \qquad \text{and} \qquad \xi^{\dagger} = \xi. \end{aligned} \tag{17}$$

So, for the supercharge operator $\bar{S}$ we have $\bar{S} = \xi^{-1} S^\dagger \xi$. The quantum generators $H, S, \bar{S}$ and F form a closed super-algebra

$$\{S, \bar{S}\} = 2H, \quad [S, H] = [\bar{S}, H] = 0, \quad [F, S] = -S, \quad S^2 = 0. \tag{18}$$

In the case of standard supersymmetric quantum mechanics, we could have $\bar{\lambda} = \lambda^\dagger$, $\bar{S} = S^\dagger$ and the Hamiltonian would be positive. We can see, that the anticommutator of supercharges S and their conjugated $\bar{S}$ under our conjugated operation (17) has the form $\overline{\{S, \bar{S}\}} = \{S, \bar{S}\}$ and the Hamiltonian operator is self-conjugated $\bar{H} = H$ under the operation $\bar{H} = \xi^{-1} H^\dagger \xi$. We can fulfill them on the Fock space representation with $\bar{\lambda}$ as a creation and λ as annihilation vacuum operators on the Fock space, then the general quantum state can be written as vectors depending on R in the corresponding Fock space [14].

We can choose the matrix representation for the fermionic parameters $\lambda, \bar{\lambda}$ and ξ as $\lambda = \sigma_-, \quad \bar{\lambda} = -\sigma_+, \quad \xi = \sigma_3$, with $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$, where σ_1, σ_2 and σ_3 are the Pauli matrices. The supercharges S, $\bar{S}$ have the following structures

$$S = A\lambda, \qquad S^\dagger = A^\dagger \lambda^\dagger, \tag{19}$$

where

$$A = i\tilde{G}^{1/2} R^{-1/2} \pi_R - \frac{\Lambda^{1/2} R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2} \sum_i M_{\gamma_i}^{1/2} R^{-\frac{3\gamma_i}{2}}. \tag{20}$$

An ambiguity exist in the factor ordering of these operators, such ambiguities always arise when the operator expression contains the product of non-commuting operators R and π_R, as in our case. It is then necessary to find some criteria to know which factor ordering should be selected. We propose the following; to integrate with measure $R^{1/2} dR$ in the inner product of two states [15, 16]. In this measure the conjugate momentum π_R is non-Hermitian with $\pi_R^\dagger = R^{-1/2} \pi_R R^{1/2}$. However, the combination $(R^{-1/2} \pi_R)^\dagger = R^{-1/2} \pi_R$ is Hermitian one, and $(R^{-1/2} \pi_R R^{-1/2} \pi_R)^\dagger = R^{-1/2} \pi_R R^{-1/2} \pi_R$ is Hermitian too.

In the quantum theory, the first-class constraints become conditions on the wave function $\Psi(R)$. Furthermore, any physical state must be satisfy the quantum constraints

$$H\Psi(R) = 0, \quad S\Psi(R) = 0, \quad \bar{S}\Psi(R) = 0, \quad F\Psi(R) = 0, \tag{21}$$

where the first equation is the Wheeler-DeWitt equation for the minisuperspace model. The eigenstates of the Hamiltonian have two components in the matrix representation

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \tag{22}$$

However, the supersymmetric physical states are obtained applying the supercharges operators $S\Psi = 0, \bar{S}\Psi = 0$. Using the algebra given by (18), these are rewritten in the following form

$$(\lambda \bar{S} - \bar{\lambda} S)\Psi = 0. \tag{23}$$

Using the matrix representation for λ and $\bar{\lambda}$ we obtain the following differential equations for $\Psi_1(R)$ and $\Psi_2(R)$ components

$$\Big(\tilde{G}^{1/2}R^{-1/2}\frac{\partial}{\partial R} - \frac{\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}\sum_i M_{\gamma_i}^{1/2}R^{-\frac{3\gamma_i}{2}}\Big)\Psi_1(R) = 0, \tag{24}$$

$$\Big(\tilde{G}^{1/2}R^{-1/2}\frac{\partial}{\partial R} + \frac{\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} - \sqrt{2}\sum_i M_{\gamma_i}^{1/2}R^{-\frac{3\gamma_i}{2}}\Big)\Psi_2(R) = 0. \tag{25}$$

Solving these equations and using the relation $M_{\gamma_i} = \frac{1}{2}\rho_{\gamma_i}R^{3(\gamma_i+1)}$, we have the following solutions

$$\Psi_1(R) = C_1 \exp\Big[\frac{1}{\sqrt{6\pi}}\Big(\frac{\rho_\Lambda}{\rho_{pl}}\Big)^{1/2}\Big(\frac{R}{l_{pl}}\Big)^3 - \frac{\sqrt{18}}{\sqrt{6\pi}}\frac{1}{\rho_{pl}^{1/2}}\Big(\frac{R}{l_{pl}}\Big)^3\sum_i\frac{\rho_{\gamma_i}^{1/2}}{(3-3\gamma_i)}\Big], \tag{26}$$

$$\Psi_2(R) = C_2 \exp\Big[-\frac{1}{\sqrt{6\pi}}\Big(\frac{\rho_\Lambda}{\rho_{pl}}\Big)^{1/2}\Big(\frac{R}{l_{pl}}\Big)^3 + \frac{\sqrt{18}}{\sqrt{6\pi}}\frac{1}{\rho_{pl}^{1/2}}\Big(\frac{R}{l_{pl}}\Big)^3\sum_i\frac{\rho_{\gamma_i}^{1/2}}{(3-3\gamma_i)}\Big], \tag{27}$$

where $\rho_{pl} = G^{-2}$ is the Planck density and $l_{pl} = G^{1/2}$ is the Planck length. We can see, that the function Ψ_1 in (26) has good behavior when $R \to \infty$ under the condition $\rho_\Lambda < 18\Big(\sum_i \frac{\rho_{\gamma_i}^{1/2}}{(3-3\gamma_i)}\Big)^2$, while Ψ_2 does not. On the other hand, the wave function Ψ_2 in (27) has good behavior when $R \to \infty$ under the condition

$$\rho_\Lambda > 18\Big(\sum_i \frac{\rho_{\gamma_i}^{1/2}}{(3-3\gamma_i)}\Big)^2, \tag{28}$$

because the main contribution comes from the first term of the exponent, while Ψ_1 does not have good behavior. However, the wave function Ψ in the state with zero energy; $\Psi^T = (0, \Psi_2)$ is normalizable in the measure $R^{1/2}dR$ under the condition (28), such as $H\Psi = S\Psi = \bar{S}\Psi = F\Psi = 0$ with $F = \sigma_+\sigma_-$.

If $H\Psi = E\Psi$, the eigenstate Ψ_1 (26) of the quantum Hamiltonian (9) for $E = 0$ is non-normalizable. But for non-zero eigenvalues of the Hamiltonian (9) it is known that there exist two normalizable components (wave functions).

The condition (28) does not contradict the astrophysical observation at $\rho_\Lambda \approx (2 \sim 3)\rho_M$, due to the fact that the dust-like matter introduces the main contribution to the total energy density of matter $\rho_M = \sum_i \rho_{\gamma_i} \approx \rho_{\gamma=0}$. We have from (28) the pressureless fluid ($\gamma = 0$) contents barionic and cold dark matter ($\rho_\Lambda > 2\rho_M$).

On the other hand, according to recent astrophysical data, our universe is dominated by a mysterious form of dark energy [19], which counts up to about 70 per cent of the total energy density. As a result, the universe expansion is accelerating [20, 21]. Vacuum energy density $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ is a concrete example of dark energy [22].

The recent cosmological data give us the following range for the dark energy state parameter $-1.14 < \gamma < -0.88$. However, in the literature we can find different theoretical models [23, 24] for the dark energy with state parameter $\gamma > -1$ and $\gamma < -1$. In the case $\Lambda = 0$ we see from (26), that the wave function Ψ; $\Psi^T = (\Psi_1, 0)$ is normalizable. Moreover, if we assume that the universe may enter into phantom phase ($\gamma < -1$) or quintessence phase ($\gamma > -1$) we don't have conditions between density of dark energy $\rho_{ph}(\gamma < -1)$, $\rho_q(-1 < \gamma)$ and density of energy matter ρ_M.

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