

Cosmological evolutions of completely degenerated Fermi-system with the scalar interparticles interactions

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Cosmological solutions of Einstein's equations for equilibrium statistical systems of particles with scalar interaction are investigated. It is shown that the scalar field can effectively change the state equation of a statistical system, that leads to the possibility of secondary acceleration of the cosmological expansion.

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I. INTRODUCTION

To the moment it has been published many papers devoted to the secondary acceleration of the Universe. In order to solve the problem of the secondary acceleration of the Universe it is often proposed in many papers to sufficiently change the fundamental principles of physics. However, recently there are indications that complicated multi-component, classical physically systems can also bring to the secondary acceleration of the Universe[6]. In this case there is no need to revise the fundamental principles of physics. Some indications of the possibility of such behavior of the systems with scalar interaction were given in the paper [1]. Scalar fields were introduced in general relativistic statistics by one of the Authors in the early 1980 years [2], [3]. In these papers, based on the kinetic theory, a system of particles with scalar interaction is obtained.

II. MACROSCOPIC DENSITIES

The full degeneration condition supposes:

$$\frac{\mu}{\theta} \rightarrow \infty. \quad (1)$$

(μ - chemical potentials, θ - temperature) local equilibrium distribution function of Fermi-system is a step function (see for example [4]). In this case, the local equilibrium distribution function has the form [3]:

$$f^0(x, P) = \chi_+(\mu - \sqrt{m_*^2 + p^2}), \quad (2)$$

where $\chi_+(z)$ is a Heaviside function;

$$m_* = |m + q\Phi|, \quad (3)$$

are the particles effective masses, q is a scalar charge of the fermions, Φ is a scalar field potential, m is a vacuum mass of the fermions. Therefore the integration of macroscopic densities are representable in elementary functions [3]:

$$\varepsilon_f = \frac{m_*^4}{8\pi^2} [\psi\sqrt{1+\psi^2}(1+2\psi^2) - \ln(\psi + \sqrt{1+\psi^2})]; \quad (4)$$

$$p_f = \frac{m_*^4}{24\pi^2} [\psi\sqrt{1+\psi^2}(2\psi^2-3) + 3\ln(\psi + \sqrt{1+\psi^2})]; \quad (5)$$

$$T_f = \varepsilon_f - 3p_f = \frac{m_*^4}{2\pi^2} [\psi\sqrt{1+\psi^2} - \ln(\psi + \sqrt{1+\psi^2})]; \quad (6)$$

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$$\varepsilon_f + p_f = \frac{m_*^4}{3\pi^2} \psi^3 \sqrt{1 + \psi^2}; \quad (7)$$

$$\begin{aligned} \sigma &= \frac{q}{m_*} T_f = \\ &= \frac{q \cdot m_*^3}{2\pi^2} \left[\psi \sqrt{1 + \psi^2} - \ln(\psi + \sqrt{1 + \psi^2}) \right]; \end{aligned} \quad (8)$$

where

$$\psi = p_F / |m_*| \quad (9)$$

is a ratio of the Fermi momentum to effective mass, $\varepsilon_f, p_f, \sigma$ is an energy density, pressure and scalar charge density of fermions, respectively[7]. In this case the self-consistent equation of massive scalar field with a mass of fermions μ [3]:

$$+ \mu^2 \Phi = -4\pi\sigma. \quad (10)$$

The fermion number density is connected with the Fermi momentum by relation [4]:

$$n(x) = \frac{p_f^3}{3\pi^2} \Rightarrow p_f = (3\pi^2 n(x))^{\frac{1}{3}}. \quad (11)$$

Thus, the variable ψ can be expressed in terms of two scalars - the particles number density, $n(x)$, in the own frame of reference and the scalar potential, $\Phi(x)$:

$$\psi = \frac{(3\pi^2 n(x))^{\frac{1}{3}}}{|m + q\Phi|}. \quad (12)$$

III. SELF-CONSISTENT SYSTEM OF EQUATIONS FOR A FLAT FRIEDMANN MODEL

Let's consider a cosmological situation where matter is represented by only a degenerate Fermi system with scalar interaction of particles and the associated scalar field, described by the equation (10) (see [1]). As the metric we choose the Spatially Flat Friedmann Model [5]:

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (13)$$

in which the independent Einstein equations have the form:

$$3 \frac{\dot{a}^2}{a^2} = 8\pi\varepsilon; \quad (14)$$

$$3 \frac{\dot{a}}{a} = - \frac{\dot{\varepsilon}}{\varepsilon + P}. \quad (15)$$

In this metric the conservation law for particles takes form: [8]:

$$\partial_t \sqrt{-g} n = 0,$$

from where, taking into account expression (11) we obtain a momentum integral instead of the energy integral [1]:

$$ap_F = \text{Const}. \quad (16)$$

Supposing further $\Phi = \Phi(t)$, we obtain $\varepsilon = \varepsilon(t)$ $p = p(t)$ and the structure of the total scalar field EMT in the form of the EMT of a perfect fluid with the macroscopic velocity $v^i = \delta_4^i$, energy density $\varepsilon_s(t)$ and pressure $p_s(t)$:

$$\varepsilon_s = \frac{1}{8\pi} (\dot{\Phi}^2 + \mu_s^2 \Phi^2); \quad p_s = \frac{1}{8\pi} \left(\frac{1}{3} \dot{\Phi}^2 - \mu_s^2 \Phi^2 \right). \quad (17)$$

Differentiating (17) and substituting the results in (15), it can be seen, that this equation is a consequence of field equations (10), which in metric (13) takes form:

$$\frac{1}{a^3} \frac{d}{dt} \left(a^3 \frac{d\Phi}{dt} \right) + \mu^2 \Phi = -4\pi\sigma. \quad (18)$$

Thus, for an independent equations one can chose the field equation (18) and one of Einstein's equations (14), which with the condition ($\dot{a} > 0$) can be written as:

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3}\varepsilon}, \quad (19)$$

where

$$\varepsilon = \varepsilon_f + \varepsilon_s$$

is a total energy density of fermions and a scalar field.

Let us carry out an analysis of possible cosmological evolution of the degenerate scalar charged fermions system. From the expressions for the macroscopic densities (4), (5) we obtain:

$$p_f = \frac{1}{3}\varepsilon_f - \frac{m_*^4}{24\pi^2} \left[4\psi\sqrt{1+\psi^2} - 3\ln(\psi + \sqrt{1+\psi^2}) \right]. \quad (20)$$

We can show, that the expression in square brackets on the right side (20) is a nonnegative for nonnegative $\psi \geq 0$. Therefore the strict inequality is satisfied:

$$0 \leq p_f \leq \frac{1}{3}\varepsilon_f. \quad (21)$$

Similarly from (17), we find the relation for the scalar field:

$$p_s = \frac{1}{3}\varepsilon_s - \frac{1}{6\pi}\mu^2\Phi^2, \quad (22)$$

Also for the scalar field the strict inequality is satisfied:

$$p_s \leq \frac{1}{3}\varepsilon_s. \quad (23)$$

Now we find *coefficient of barotropic*, as a coefficient κ in the linear relation between total pressure and energy density of matter:

$$p = \kappa\varepsilon. \quad (24)$$

As a result we find:

$$\kappa(t) = \frac{1}{3} - \frac{m_*^4 G(\psi) + 4\pi\mu^2\Phi^2}{24\pi^2(\varepsilon_s + \varepsilon_f)}, \quad (25)$$

where:

$$G(\psi) = 4\psi\sqrt{1+\psi^2} - 3\ln(\psi + \sqrt{1+\psi^2}).$$

So, we can show, that for the concerned system the following relation is always fulfilled:

$$\Rightarrow -1 \leq \kappa(t) \leq \frac{1}{3}. \quad (26)$$

Thus, for the cosmological acceleration Universe, Ω , that consists of degenerate fermions and scalar field,

$$\Omega = \frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{2}(1 + 3\kappa), \quad (27)$$

we obtain the restriction:

$$-1 \leq \Omega \leq 1. \quad (28)$$

IV. NUMERICAL SOLUTION OF EINSTEIN'S EQUATIONS FOR A DEGENERATE PLASMA WITH SCALAR FIELD

Attempts of direct numerical integration of Einstein and Klein-Gordon equations (14), (10) in most cases don't give results owing to nonlinear character of these equations and ambiguity of radical functions and logarithms in the right side of equations. Therefore, for the numerical solution of Einstein's equations the expressions for the macroscopic densities were extrapolated by means of elementary functions. As it can be seen from (4) - (8), all these expressions, taking into account the integral (16) up to multiplication by a conformal factor $1/a^4$, -

$$m_*^4 = \left(\frac{a_0 p_F^0}{a\psi} \right)^4. \quad (29)$$

are elementary functions of a single dimensionless function ψ . This fact allows us to find the interpolation expression for the corresponding conformal densities:

$$\tilde{p}_f = a^4 P_f; \quad \tilde{\varepsilon}_f = a^4 \varepsilon_f; \quad \tilde{\sigma} = a^4 \sigma. \quad (30)$$

Interpolation of analytic functions for the energy density, pressure and charge density on the range under consideration can be written as:

$$\tilde{p}_f = \frac{8}{15} e^{-2\psi} \psi + \frac{2\psi^2}{3(1+\psi^2)} \quad (31)$$

$$\tilde{\varepsilon}_f = \frac{\left(1 - e^{-2\psi - 2\psi^2}\right) \left(\frac{38}{15} + \frac{4}{3\psi^2} + \frac{8\psi^2}{15}\right)}{1 + \frac{4\psi^2}{15}} \quad (32)$$

$$\begin{aligned} \tilde{T}_f = \frac{\left(1 - e^{-2\psi - 2\psi^2}\right) \left(\frac{38}{15} + \frac{4}{3\psi^2} + \frac{8\psi^2}{15}\right)}{1 + \frac{4\psi^2}{15}} - \\ - \frac{8}{5} e^{-2\psi} \psi - \frac{2\psi^2}{1 + \psi^2}. \end{aligned} \quad (33)$$

It is obvious that due to the large number of essential parameters of the model, (4 parameters, p_f, m, μ, q), and initial conditions, (2 independent conditions, $\Phi(0), \dot{\Phi}(0)$), a considered cosmological model is extreme rich in types of behavior. Therefore let us consider the main ones. Here are the results of numerical solutions of Einstein and Klein - Gordon equations for the degenerated Fermi system in math package Mathematica v7.

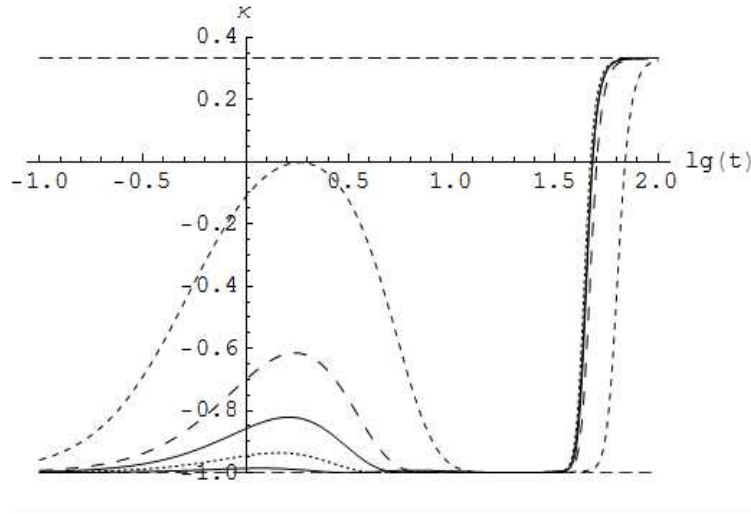


Fig. 1. Evolution of the barotropic coefficient, \varkappa , depending on mass of scalar bosons. Frequent dotted line: $\mu = 0, 3$; dotted line: $\mu = 0, 1$; rare dotted line - $\mu = 0, 2$; normal line: $\mu = 0, 25$; thick line: $\mu = 0, 35$. Everywhere: $p_F = 0, 01$; $m = 1$; $q = 0, 3$; $\Phi(0) = 1$; $\dot{\Phi}(0) = 0$.

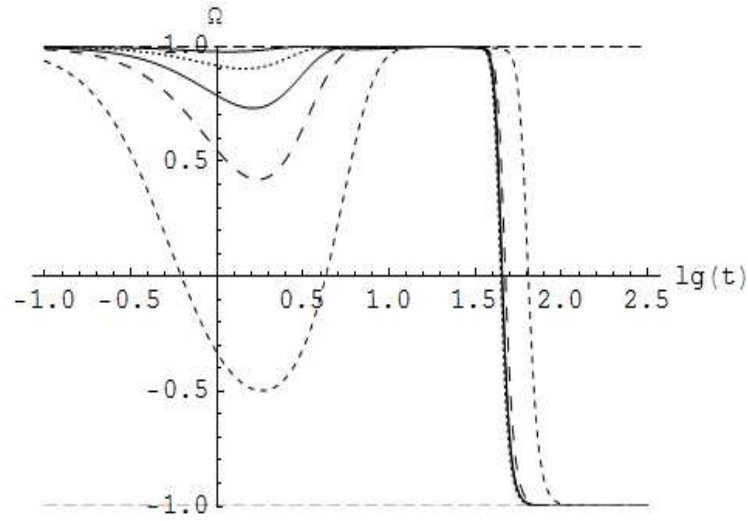


Fig. 2. Evolution of the cosmological acceleration, Ω , depending on mass of scalar bosons. Frequent dotted line: $\mu = 0,3$; dotted line: $\mu = 0,1$; rare dotted line: $\mu = 0,2$; normal line: $\mu = 0,25$; thick line: $\mu = 0,35$. Everywhere: $p_F = 0,01$; $m = 1$; $q = 0,3$; $\Phi(0) = 1$; $\dot{\Phi}(0) = 0$.

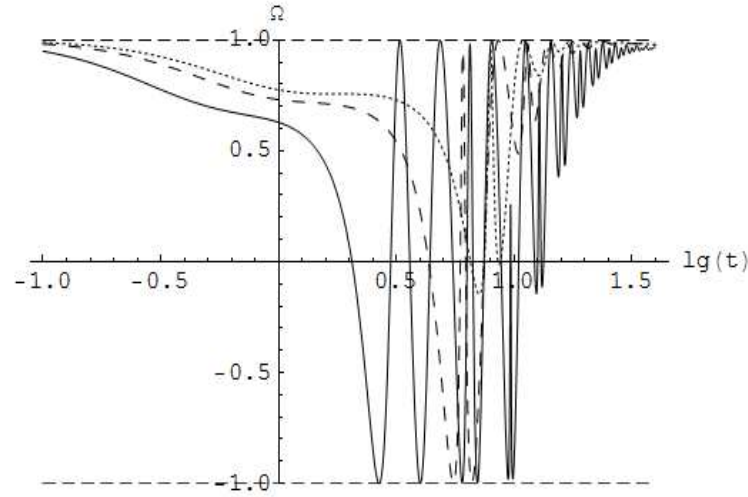


Fig. 3. Evolution of the cosmological acceleration, Ω , depending on mass of scalar bosons. Frequent dotted line: $\mu = 1$; dotted line: $\mu = 1,2$; normal line: $\mu = 2$; thick line: $\mu = 3$. Everywhere: $p_F = 0,01$; $m = 1$; $q = 0,3$; $\Phi(0) = 1$; $\dot{\Phi}(0) = 0$.

Let us present the solutions which transgress to the accelerated cosmological phase with maximum acceleration. These solutions correspond to very small values of the fundamental charge.

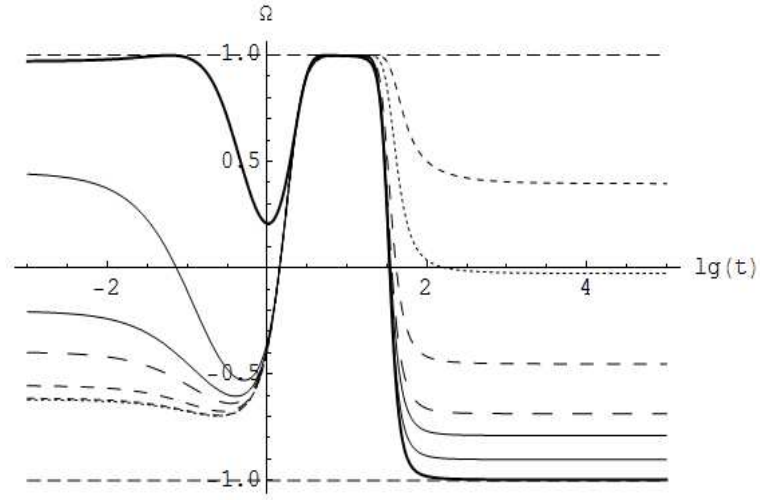


Fig. 4. Evolution of the cosmological acceleration, Ω , depending on mass of fermions. Frequent dotted line: $m = 0$; dotted line: $m = 0,001$; rare dotted line: $m = 0,01$; very rare dotted line: $m = 0,03$; thin line: $m = 0,05$; normal line: $m = 0,1$, thick line: $m = 1$. Everywhere: $p_F = 0,01$; $\mu = 0,3$; $q = 1$; $\Phi(0) = 1$; $\dot{\Phi}(0) = 0$.

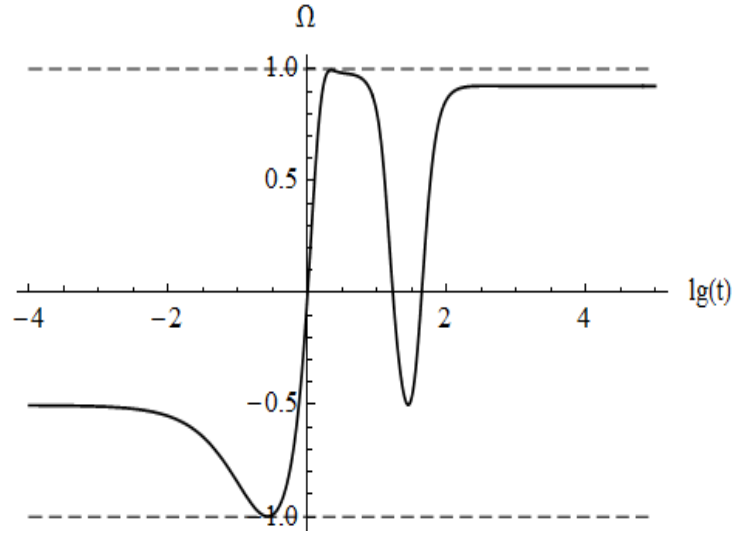


Fig. 5. Evolution of the cosmological acceleration, Ω where $p_F = 0$; $m = 1$; $\mu = 0,001$; $q \rightarrow 0$.

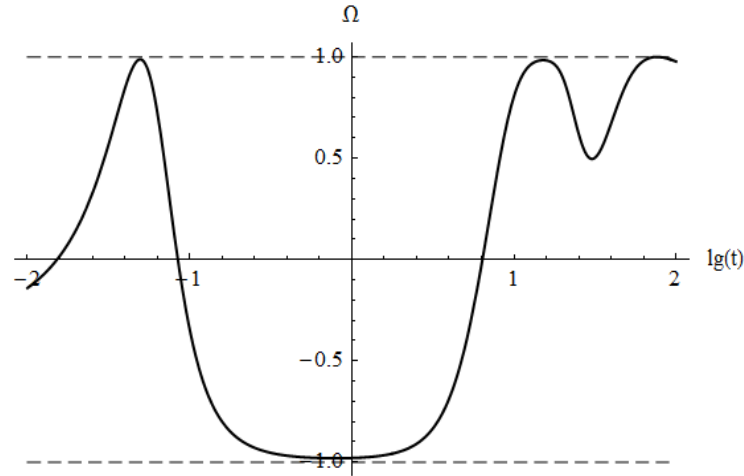


Fig. 6. Evolution of the cosmological acceleration, Ω where $p_F = 0$; $m = 1$; $\mu = 0, 1$; $q \rightarrow 0$.

V. CONCLUSION

Thus, we can resume:

1. Evolution of the cosmological model depends heavily on the fundamental constants q, μ and m has many types of behavior;
 2. There are ranges of fundamental constants and initial conditions, in which the cosmological models come out later on stable acceleration;
 3. Access to the phase later acceleration can be smooth or accompanying by the acceleration oscillations;
 4. Final acceleration is constant and can vary in the range $-1 \leq \Omega \leq 1$.
- Therefore, constant acceleration (deceleration) is of a general nature.

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 - [5] L.D. Landau, Ye.M. Lifshits. *Field's theory*. M, Nauka, 1973 (in Russia)
 - [6] In particular, such arguments were represented by Dmitry V. Gal'tsov, as well as present paper's Authors at Gracos-2009
 - [7] The specifications see in [3].
 - [8] Everywhere $G = \hbar = c = 1$, time and mass are measured in Planck units.