One-loop quantum gravity repulsion in the early Universe*

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Abstract

Perturbative quantum gravity is used to compute the lowest order corrections to the classical spatially flat cosmological FLRW solution (for the radiation). The presented approach is analogous to the approach used to compute quantum corrections to the Coulomb potential in electrodynamics, or rather to the approach used to compute quantum corrections to the Schwarzschild solution in gravity. In the framework of the standard perturbative quantum gravity, it is shown that the corrections to the classical deceleration, coming from the one-loop graviton vacuum polarization (self-energy), have (UV cutoff free) opposite to the classical repulsive properties which are not negligible in the very early Universe. The repulsive "quantum forces" resemble those known from loop quantum cosmology.

Keywords: one-loop graviton vacuum polarization; one-loop graviton self-energy; quantum corrections to classical gravitational fields; early Universe; quantum cosmology.

PACS (2010) numbers: 04.60.Gw Covariant and sum-over-histories quantization; 98.80.Es Observational cosmology (including Hubble constant, distance scale, cosmological constant, early Universe, etc): 04.60.Bc Phenomenology of quantum gravity; 04.60.Pp Loop quantum gravity, quantum geometry, spin foams.

*Dedicated to Jakub Rembieliński on the occasion of his 65th birthday.

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Introduction. The aim of our work it to explicitly show “repulsive forces” of quantum origin in the very early Universe. As fundamental guiding references to our work we would like to point out the publications perturbatively calculating quantum corrections to classical electromagnetic (the Uehling potential) and gravitational fields. We would mean, for example, the papers calculating one-loop quantum corrections to the Coulomb potential in electrodynamics (see, e.g. § 114 in [1]), or rather the lowest order quantum corrections to the Schwarzschild solution in gravity (see, [2], and also, e.g. [3]). Actually, we apply the method successfully used in the case of the Schwarzschild solution in [2] to the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) solution (for the radiation). Fortunately, it appears that the cosmological FLRW case is only a little bit more complicated than the Schwarzschildean one, and our results conform the present knowledge. Namely, the lowest order quantum corrections coming from the fluctuating graviton vacuum yield “repulsive forces” resembling the well-known situation in loop quantum cosmology. The phenomenon is negligible in our epoch, but it is not in the very early Universe. Moreover, it appears, the result is UV cutoff free, despite the fact that the cutoff has been primarily imposed (see, [3]). One should stress that our derivation is a lowest order approximation—the graviton vacuum polarization (self-energy) is taken in one-loop approximation, and the approach assumes the validity of the weak-field regime.

Quantum corrections. Our starting point is a general spatially flat FLRW metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\boldsymbol{dr}^2, \tag{1}$$

with the cosmic scale factor $a(t)$. To satisfy the condition of the weakness of the (perturbative) gravitational field $\kappa h_{\mu\nu}$ near our reference time $t = t_0$ in the expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{2}$$

($\kappa = \sqrt{32\pi G_N}$, with G_N—the Newton gravitational constant), the metric is rescaled such a way that it is exactly Minkowskian for $t = t_0$, i.e.

$$a^2(t) = 1 - \kappa h(t), \qquad h(t_0) = 0. \tag{3}$$

Then

$$h_{\mu\nu}(t, \boldsymbol{r}) = h(t)\mathcal{I}_{\mu\nu} \quad \text{and} \quad \mathcal{I}_{\mu\nu} \equiv \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} \end{pmatrix}. \tag{4}$$

In view of the harmonic gauge condition (see, the second eq. in (8)) to be imposed in a moment, we perform the following gauge transformation:

$$\kappa h_{\mu\nu} \to \kappa h_{\mu\nu}^{'} = \kappa h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \quad \text{with} \quad \xi_\mu\,(t) = \left(-\frac{3\kappa}{2} \int_0^t h(t')\, dt',\, 0,\, 0,\, 0 \right). \tag{5}$$

Skipping the prime for simplicity, we have now got, after the gauge transformation,

$$h_{\mu\nu}(t, \boldsymbol{r}) = h(t) \begin{pmatrix} -3 & 0 \\ 0 & \delta_{ij} \end{pmatrix} \quad \text{and} \quad h^\lambda_\lambda(t) = -6h(t). \tag{6}$$

Switching from $h_{\mu\nu}$ to standard ("better") perturbative gravitational variables, namely to the "barred" field $\bar{h}_{\mu\nu}$ defined by

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu} h^\lambda_\lambda, \tag{7}$$

we get

$$\bar{h}_{\mu\nu}(t, \boldsymbol{r}) = -2h(t)\mathcal{I}_{\mu\nu} \quad \text{with} \quad \partial^\mu \bar{h}_{\mu\nu} = 0. \tag{8}$$

Its Fourier transform is given by

$$\tilde{\bar{h}}_{\mu\nu}(p) = -2\tilde{h}(E)\,(2\pi)^3\,\delta^3(\boldsymbol{p})\mathcal{I}_{\mu\nu}. \tag{9}$$

The lowest order quantum corrections $\tilde{\bar{h}}^{\rm q}_{\mu\nu}$ to the classical gravitational field $\tilde{\bar{h}}^{\rm c}_{\mu\nu}$ are given, in the momentum representation, by the formula (see, e.g. [2], or § 114 in [1] for an electrodynamic version—the Uehling potential)

$$\tilde{\bar{h}}^{\rm q}_{\mu\nu}(p) = \left(D\Pi\tilde{\bar{h}}^{\rm c} \right)_{\mu\nu}(p), \tag{10}$$

where

$$D^{\alpha\beta}_{\mu\nu}(p) = \frac{i}{p^2}\mathbb{D}^{\alpha\beta}_{\mu\nu} \tag{11}$$

is the free graviton propagator in the harmonic gauge with the auxiliary (constant) tensor $\mathbb{D}$ defined in Eq.(12) below, and $\Pi^{\alpha\beta}_{\mu\nu}(p)$ is the (one-loop) graviton vacuum polarization (self-energy) tensor operator. Now, we are defining the following useful auxiliary tensors:

$$\mathbb{D} \equiv \mathbb{E} - 2\mathbb{P}, \quad \text{where} \quad \mathbb{E}^{\alpha\beta}_{\mu\nu} \equiv \tfrac{1}{2}\left(\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu \right) \quad \text{and} \quad \mathbb{P}^{\alpha\beta}_{\mu\nu} \equiv \tfrac{1}{4}\eta^{\alpha\beta}\eta_{\mu\nu}; \tag{12}$$

which satisfy the following obvious identities:

$$\mathbb{E}^2 = \mathbb{E}, \quad \mathbb{P}^2 = \mathbb{P}, \quad \mathbb{E}\mathbb{P} = \mathbb{P}\mathbb{E} = \mathbb{P} \quad \text{and} \quad \mathbb{D}^2 = \mathbb{E}. \tag{13}$$

By virtue of the definition (7), we observe that

$$\bar{h}_{\mu\nu} = (\mathbb{D}h)_{\mu\nu}\,. \tag{14}$$

Multiplying Eq.(10) from the left by $\mathbb{D}$, we obtain (using (11), (14), and the last identity in the series (13))

$$\tilde{h^{\mathrm{q}}}_{\mu\nu}(p) = \frac{i}{p^2}\left(\Pi\tilde{\bar{h}}^{\mathrm{c}}\right)_{\mu\nu}(p). \tag{15}$$

Actually, a simplification takes place, namely,

$$\tilde{h^{\mathrm{q}}}_{\mu\nu}(p) = \frac{i}{p^2}\left(\Pi'\tilde{\bar{h}}^{\mathrm{c}}\right)_{\mu\nu}(p), \tag{16}$$

where $\Pi'(p)$ is an "essential" part of the full (in one-loop approximation) graviton polarization operator $\Pi(p)$. The "essential" part Π' of the full graviton vacuum polarization Π is obtained from Π by simply skipping all the terms with the momenta p with free indices (e.g. α, β, μ, or ν). Such simplifying possibility follows from the gauge freedom the $\tilde{h^{\mathrm{q}}}_{\mu\nu}$ enjoys, and from the harmonic gauge condition the $\tilde{\bar{h}}^{\mathrm{c}}_{\alpha\beta}$ fulfills. In other words (see, e.g. [4], for details),

$$\Pi(p) = \underbrace{\Pi'(p)}_{\text{2 terms}} + \underbrace{\cdots\ p\ \cdots}_{\text{3 skipped terms}}\ , \tag{17}$$

where since the momenta p in the ellipses posses free indices, they can be ignored, and only the terms with dummy indices (p^2) survive in $\Pi'(p)$. Thus,

$$\Pi'(p) = \kappa^2 p^4 I(p^2)(2\alpha_1\mathbb{E} + 4\alpha_2\mathbb{P}), \tag{18}$$

where the numerical values of the coefficients α_1 and α_2 depend on the kind of the virtual field circulating in the loop, and the (scalar) loop integral $I(p^2)$ with the UV cutoff M is asymptotically of the form (see, e.g., Chapt. 9.4.2 in [5])

$$I(p^2) = -\frac{i}{(4\pi)^2}\log\left(-\frac{p^2}{M^2}\right) + \cdots\quad , \tag{19}$$

where the dots mean terms $\mathcal{O}\left(p^2/M^2\right)$. Finally, we obtain

$$\begin{aligned}\tilde{h^{\mathrm{q}}}_{\mu\nu}(p) &= \frac{i}{p^2}\kappa^2 p^4\left[-\frac{i}{(4\pi)^2}\log\left(-\frac{p^2}{M^2}\right)\right]\left[-2\tilde{h^{\mathrm{c}}}(E)\,(2\pi)^3\,\delta^3(\boldsymbol{p})\right][(2\alpha_1\mathbb{E}+4\alpha_2\mathbb{P})\mathcal{I}]_{\mu\nu} \\ &= -2\pi\kappa^2E^2\log\left|\frac{E}{M}\right|\tilde{h^{\mathrm{c}}}(E)\delta^3(\boldsymbol{p})\begin{pmatrix}-3\alpha_2 & 0\\ 0 & (2\alpha_1+3\alpha_2)\,\delta_{ij}\end{pmatrix}.\end{aligned} \tag{20}$$

The unnecessary modulus sign in Eq.(20) is only to remind the fact that there is also an imaginary contribution to the metric due to creation processes which are ignored in our further analysis.

Radiation source. Now, we should specify our input classical metric. For definiteness, we choose the radiation as a source (the early Universe), but it is not crucial, and assume

$$a^2(t) = \theta(t)\left(\frac{t}{t_0}\right). \tag{21}$$

Then

$$\kappa h^{\rm c}(t_0) = 0, \qquad \kappa\dot{h}^{\rm c}(t_0) = -\frac{1}{t_0} \qquad \text{and} \qquad \kappa\ddot{h}^{\rm c}(t_0) = 0. \tag{22}$$

By virtue of the definition of the deceleration parameter q, expressed by

$$q(t_0) \equiv -\frac{a\ddot{a}}{(\dot{a})^2}(t_0) = 1 + 2\left[1 - \kappa h(t_0)\right]\frac{\kappa\ddot{h}(t_0)}{\left(\kappa\dot{h}(t_0)\right)^2}, \tag{23}$$

we immediately get the classical result

$$q^{\rm c}(t_0) = 1. \tag{24}$$

According to (3) and (21) the Fourier transform of $h^{\rm c}(t)$ is

$$\tilde{h}^{\rm c}(E) = \frac{1}{\kappa t_0}\left(\frac{1}{E^2} + \cdots\right), \tag{25}$$

where the dots mean terms (vanishing in the next formula) proportional to the Dirac delta and its first derivative. Hence

$$\tilde{h}^{\rm q}{}_{\mu\nu}(p) = -\frac{2\pi\alpha\kappa}{t_0}\log\left|\frac{E}{M}\right|\delta^3(\boldsymbol{p})\mathcal{I}_{\mu\nu}, \tag{26}$$

where $\alpha \equiv 2\alpha_1 + 3\alpha_2$, and performing the gauge transformation in the spirit of (5), we have removed the purely time component of $h^{\rm q}{}_{\mu\nu}$, i.e. $h^{\rm q}{}_{00} \to h^{\rm q}{}'_{00} = 0$. The inverse Fourier transform yields

$$h^{\rm q}{}_{\mu\nu}(t) = \frac{2\pi^2\alpha\kappa}{(2\pi)^4 t_0}\left(|t|^{-1} + \cdots\right)\mathcal{I}_{\mu\nu}, \tag{27}$$

where this time the dots mean a term (vanishing for $t > 0$) proportional to the Dirac delta. Therefore, for $t > 0$ we have

$$\kappa h^{\rm q}(t) = \frac{\alpha\kappa^2}{8\pi^2 t_0}t^{-1} = -\frac{G}{4\pi t_0}t^{-1}, \tag{28}$$

where according to Table I only the graviton field contributes with $\alpha = -\frac{1}{16}$. Now,

$$\kappa h^{\rm q}(t_0) = -\frac{G}{4\pi t_0^2}, \qquad \kappa\dot{h}^{\rm q}(t_0) = \frac{G}{4\pi t_0^3} \qquad \text{and} \qquad \kappa\ddot{h}^{\rm q}(t_0) = -\frac{G}{2\pi t_0^4}. \tag{29}$$

spin	α_1	α_2	α
0	$\frac{1}{480}$	$-\frac{1}{720}$	0
$\frac{1}{2}$	$\frac{1}{160}$	$-\frac{1}{240}$	0
1	$\frac{1}{40}$	$-\frac{1}{60}$	0
2	$\frac{27}{80}$	-$\frac{59}{240}$	-$\frac{1}{16}$

Table I: Coefficients α_1 and α_2 entering the one-loop graviton vacuum polarization (self-energy) tensor operator (18) (taken from [4, 6–9]); $\alpha \equiv 2\alpha_1 + 3\alpha_2$.

The total graviton field $\kappa h = \kappa h^{\mathrm{c}} + \kappa h^{\mathrm{q}}$, and its derivatives at the time t_0, expressed in the dimensionless (Planck's) time unit

$$\tau \equiv \frac{1}{\sqrt{G}} t_0, \tag{30}$$

are

$$\kappa h(\tau) = -\frac{1}{4\pi\tau^2}, \qquad \kappa \dot{h}(\tau) = -\frac{1}{t_0}\left(1 - \frac{1}{4\pi\tau^2}\right) \qquad \text{and} \qquad \kappa \ddot{h}(\tau) = -\frac{1}{t_0^2}\left(\frac{1}{2\pi\tau^2}\right). \tag{31}$$

Finally, by virtue of (23), we obtain the total deceleration parameter of the form

$$q(\tau) = 1 - \frac{1}{\pi\tau^2} + \mathcal{O}\left(\tau^{-4}\right). \tag{32}$$

Final remarks. In the framework of the standard (one-loop) perturbative quantum gravity, we have derived the formula (32) expressing the value of the total (effective) deceleration parameter $q(\tau)$. The quantum contribution, $\delta q(\tau) = q(\tau) - q^{\mathrm{c}}(\tau) \approx -\frac{1}{\pi\tau^2}$, is negligible in our epoch, but certainly it could play a role in a very early evolution of the Universe. Perturbative nature of the approach imposes bounds on the applicability of the result, but nevertheless one can observe its distinctive features: actually, it is an independent perturbative confirmation of the existence of strong repulsive (singularity resolving) forces typically being attributed to the realm of loop quantum cosmology (cosmological bounce); inputs and outputs are consequently given in terms of the metric tensor; only pure gravity contributes to our result (see α in Table I); and finally, no trace of the UV cutoff is present anywhere.

Supported by the University of Łódź grant.

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