

# $R^2$ Dark Matter<sup>1</sup>

Jose A. R. Cembranos

*William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, 55455, USA*  
*School of Physics and Astronomy, University of Minnesota, Minneapolis, 55455, USA*

**Abstract.** There is a non-trivial four-derivative extension of the gravitational spectrum that is free of ghosts and phenomenologically viable. It is the so called  $R^2$ -gravity since it is defined by the only addition of a term proportional to the square of the scalar curvature. Just the presence of this term does not improve the ultraviolet behaviour of Einstein gravity but introduces one additional scalar degree of freedom that can account for the dark matter of our Universe.

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## INTRODUCTION

The non-unitarity and non-renormalizability of the gravitational interaction described by the Einstein-Hilbert action (EHA) demands its modification at high energies. It has been pointed out that this correction cannot be accomplished without the introduction of new states [1]; these states typically interact with SM fields through Planck scale suppressed couplings and potentially work as dark matter (DM).

In spite of many and continuous efforts, the ultraviolet (UV) completion of the gravitational interaction is still an open question. In these conditions, it is difficult to make general statements about its phenomenology although different types of new scalar fields are commonly predicted [2, 3]. We can adopt a conservative and minimal approach in order to capture the fundamental physics of this fact [1]. The simplest correction to the EHA at high energies is provided by the inclusion of four-derivative terms in the metric that preserve general covariance. The most general four-derivative action supports, in addition to the usual massless spin-two graviton, a massive spin-two and a massive scalar mode, with a total of eight degrees of freedom (in the *physical* or *transverse* gauge [4, 5]). Indeed, four-derivative gravity is renormalizable, although the massive spin-two gravitons are ghost-like particles that generate new unitarity violations, breaking of causality, and inadmissible instabilities [6].

## $R^2$ GRAVITY

However, we can work with  $R^2$ -gravity, that is defined by the only addition of a term proportional to the square of the scalar curvature to the EHA. It illustrates the idea in

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a consistent and minimal way since it only introduces one additional scalar degree of freedom, whose mass  $m_0$  is given by the corresponding new constant in the action:

$$S_G = \int \sqrt{g} \left\{ -\Lambda^4 - \frac{M_{\text{P}}^2}{2} R + \frac{M_{\text{P}}^2}{12 m_0^2} R^2 + \dots \right\} \quad (1)$$

where  $M_{\text{P}} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$  GeV,  $\Lambda \simeq 2.3 \times 10^{-3}$  eV, and the dots refer to higher energy corrections that must be present in the model to complete the UV limit. In [1], it has been shown that just the Action (1) can explain the late time cosmology since the first term can account for the dark energy (DE) content, while the third term is able to explain the DM one.

$R^2$ -gravity modifies Einstein's Equations (EEs) as [7, 8] (following notation from [9]):

$$\left[ 1 - \frac{1}{3 m_0^2} R \right] R_{\mu\nu} - \frac{1}{2} \left[ R - \frac{1}{6 m_0^2} R^2 \right] g_{\mu\nu} - \mathcal{I}_{\alpha\beta\mu\nu} \nabla^\alpha \nabla^\beta \left[ \frac{1}{3 m_0^2} R \right] = \frac{T_{\mu\nu}}{M_{\text{P}}^2}, \quad (2)$$

where  $\mathcal{I}_{\alpha\beta\mu\nu} \equiv (g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\mu} g_{\beta\nu})$ . The new terms do not modify the standard EEs at low energies except for the mentioned introduction of a new mode. It is straight forward to check that the metric  $g_{\mu\nu} = [1 + c_1 \sin(m_0 t)] \eta_{\mu\nu}$  is solution of the linearized Eq. (2), i.e. for  $c_1 \ll 1$ , without any kind of energy source. It has been argue that the energy stored in such oscillations behaves exactly as cold DM and can explain the missing matter problem of the Universe [1].

## SCALAR GRAVITON COUPLINGS

The phenomenology of the new scalar can be computed inside the *Jordan* or the *Einstein frame* by expanding the metric perturbatively:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \frac{2}{M_{\text{P}}} h_{\mu\nu} - \sqrt{\frac{2}{3}} \frac{1}{M_{\text{P}}} \phi \hat{g}_{\mu\nu}, \quad (3)$$

where  $\hat{g}_{\mu\nu}$  is its classical background solution,  $h_{\mu\nu}$  takes into account the standard two degrees of freedom associated with the spin-two (traceless) graviton, and  $\phi$  corresponds to the new mode, which owns a standard kinetic term.

The couplings of this scalar graviton with the SM fields have been computed in [1] by supposing that gravity is minimally coupled to matter (in the Jordan frame). In such a case, there is a linear coupling to matter through the trace of the standard energy-momentum tensor:

$$\mathcal{L}_{\phi-T_{\mu\nu}} = \frac{1}{M_{\text{P}} \sqrt{6}} \phi T_{\mu}^{\mu}. \quad (4)$$

Therefore, the couplings with the massive SM particles -Higgs boson ( $\Phi$ ), (Dirac) fermions ( $\psi$ ), and electroweak gauge bosons- are:

$$\mathcal{L}_{\phi-SM}^{\text{tree-level}} = \frac{1}{M_{\text{P}} \sqrt{6}} \phi \{ 2 m_{\Phi}^2 \Phi^2 - \nabla_{\mu} \Phi \nabla^{\mu} \Phi \} \quad (5)$$

$$+ \sum_{\psi} m_{\psi} \bar{\psi} \psi - 2m_W^2 W_{\mu}^+ W^{-\mu} - m_Z^2 Z_{\mu} Z^{\mu} \}.$$

In addition, this scalar graviton couples to photons and gluons due to the *conformal anomaly* [1]:

$$\mathcal{L}_{\phi-SM}^{\text{one-loop}} = \frac{1}{M_{\text{P}}\sqrt{6}} \phi \left\{ \frac{\alpha_{EM} c_{EM}}{8\pi} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha_s c_G}{8\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\}. \quad (6)$$

The particular value of the couplings ( $c_{EM}$  and  $c_G$ ) depends on the energy and possible heavy particles, charged with respect to these gauge interactions, that may extend the SM at high energies.

## SCALAR GRAVITON ABUNDANCE

The thermal abundance that this field can achieve depends on the UV completion of the theory. However, there is no reason to expect that the initial value of the scalar field ( $\phi_1$ ) should coincide with the minimum of its potential ( $\phi = 0$ ) if  $H(T) \gg m_0$ . It implies that the scalar graviton may have associated big abundances through the so called *misalignment mechanism*. Below the temperature  $T_1$  for which  $3H(T_1) \simeq m_0$ ,  $\phi$  behaves as a standard scalar. It oscillates around the minimum. These oscillations correspond to a zero-momentum condensate, whose initial number density:  $n_{\phi} \sim m_0 \phi_1^2 / 2$  (where  $\phi_1 = \sqrt{\langle \phi(T_1)^2 \rangle}$ ), will evolve as the typical one associated to standard non-relativistic matter. The abundance of this particle has been computed in [1]:

$$\Omega_{\phi} h^2 \simeq 0.86 \left[ \frac{m_0}{1 \text{ eV}} \right]^{\frac{1}{2}} \left[ \frac{\phi_1}{10^{12} \text{ GeV}} \right]^2 \left[ \frac{100 g_{e1}^3}{(\gamma_{s1} g_{s1})^4} \right]^{\frac{1}{4}}, \quad (7)$$

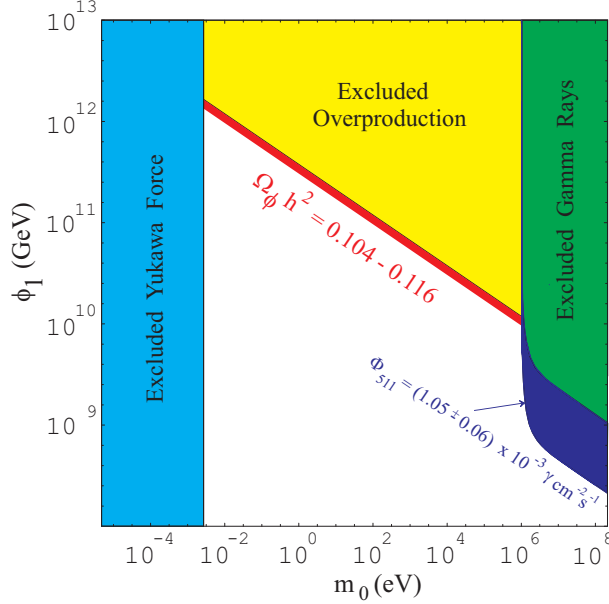
where  $g_{e1}$  ( $g_{s1}$ ) are the effective energy (entropy) number of relativistic degrees of freedom at  $T_1$ ,  $h \simeq 0.70$  is the Hubble parameter, and  $\gamma_{s1}$  is the factor that this entropy has increased in a comoving volume since the onset of scalar oscillations. We see that initial conditions of order of  $\phi_1 \sim 10^{12}$  GeV can lead to the non-baryonic DM (NBDM) abundance depending on the rest of parameters and the early physics of the Universe (see Fig. 1).

## SCALAR GRAVITON SIGNATURES

On the other hand, Eqs. (4,5) imply that the new scalar graviton mediates an attractive Yukawa force between two non-relativistic particles of masses  $M_a$  and  $M_b$ :

$$V_{ab} = -\frac{1}{24\pi M_{\text{P}}^2} \frac{M_a M_b}{r} e^{-m_0 r}. \quad (8)$$

Since it has not been observed, torsion-balance measurements are able to constraint the



**FIGURE 1.** Different regions of the parameter space of  $R^2$  gravity:  $m_0$  is the mass of the scalar graviton and  $\phi_1$  is its misalignment when  $3H \sim m_0$  (we assume  $g_{e1} = g_{s1} \simeq 106.75$ , and  $\gamma_{s1} \simeq 1$ ). The left side is excluded by modifications of Newton's law. The right one is excluded by cosmic ray observations. In the limit of this region,  $R^2$ -gravity can account for the positron production in order to explain the 511 keV line coming from the GC confirmed by INTEGRAL [10] (up to  $m_0 \sim 10$  MeV). The upper area is ruled out by DM overproduction. The diagonal line corresponds to the NBDM abundance fitted with WMAP data (Figure taken from [1]).

scalar mass [1]:

$$m_0 \geq 2.7 \times 10^{-3} \text{eV} \quad \text{at 95\% c.l.} \quad (9)$$

This is the most important lower bound on this mass, and it is independent of its abundance. Depending on its abundance,  $m_0$  is constrained from above. The decay in  $e^+e^-$  is the most constraining if  $\phi$  constitutes the total NBDM. From (5), it is possible to calculate the  $\phi$  decay rate into a generic pair fermion anti-fermion [1]. Restrictions are set by the observations of the SPI spectrometer on the INTEGRAL (International Gamma-ray Astrophysics Laboratory) satellite, which has measured a 511 keV line emission of  $1.05 \pm 0.06 \times 10^{-3}$  photons  $\text{cm}^{-2} \text{s}^{-1}$  from the Galactic center (GC) [10], confirming previous measurements. This 511 keV line flux is fully consistent with an  $e^+e^-$  annihilation spectrum although the source of positrons is unknown.

If  $m_0 \geq 1.2$  MeV, the scalar mode cannot constitute the total local DM since we should observe a bigger excess of the 511 line coming from the GC. On the other hand, decaying DM (DDM) has been already proposed in different works [11, 12] as a possible source of the inferred positrons if its mass is lighter than  $M_{\text{DDM}} \lesssim 10$  MeV [13] and its decay rate in  $e^+e^-$  verifies [1]:

$$\frac{\Omega_{\text{DDM}} h^2 \Gamma_{\text{DDM}}}{M_{\text{DDM}}} \simeq [(0.2 - 4) \times 10^{27} \text{s MeV}]^{-1}. \quad (10)$$

The most important uncertainty for this interval comes from the dark halo profile, although a cuspy density is definitely needed (with a inner slope  $\gamma \gtrsim 1.5$  [12]). If  $m_0$  is tuned to  $2m_e$  with an accuracy of 5-10%, the line could be explained by  $R^2$ -gravity. The same gravitational DM can explain the 511 line with a less tuned mass (up to  $m_0 \sim 10$  MeV) if  $\phi_1 \sim 10^9$  GeV, i.e. with a lower abundance (See Fig. 1).

If  $m_0 < 2m_e$ , the only decay channel that may be observable is in two photons. If  $m_0 \lesssim 1$  MeV, it is difficult to detect these gravitational decays in the isotropic diffuse photon background (iDPB) [14, 12]. However, a more promising analysis is associated with the search of photon lines at  $E_\gamma = m_0/2$  from localized sources. The iDPB is continuum since it suffers the cosmological redshift, but the mono-energetic photons originated by local sources may give a clear signal of  $R^2$ -gravity in future experiments if the scalar graviton is inside the heavier allowed region of the model [14].

## CONCLUSIONS

Although there are other possibilities [15], DM is usually assumed to be in the form of stable Weakly-interacting massive particles (WIMPs) that naturally freeze-out with the right thermal abundance. One of the most interesting features of WIMPs, is that they emerge in well-motivated particle physics scenarios as in R-parity conserving supersymmetry (SUSY) models [16, 17], universal extra dimensions (UED) [18, 19], or brane-worlds [20, 21, 22]. In this analysis, we have studied the possibility that the DM origin resides in UV modifications of gravity. We have focused on  $R^2$ -gravity, but the low energy phenomenology of the studied scalar mode is present in the same well-motivated frameworks such as string theory, supersymmetry or extra dimensional models (in form of dilatons, graviscalars or radions). Another interesting property of WIMPs, it is that they can be tested with high energy experiments as the new generation of colliders [23]. This possibility seems remote for the type of gravitational DM discussed in this work. However, indirect observations as modifications of Newton's law or cosmic rays can provide the first signatures of this type of DM.

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