Scaling and Memory Effect in Volatility Return Interval of the Chinese Stock Market

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Abstract

Abstract: We investigate the probability distribution of the volatility return intervals τ for the Chinese stock market. We rescale both the probability distribution $P_q(\tau)$ and the volatility return intervals τ as $P_q(\tau) = 1/\overline{\tau} f(\tau/\overline{\tau})$ to obtain a uniform scaling curve for different threshold value q. The scaling curve can be well fitted by the stretched exponential function $f(x) \sim e^{-\alpha x^\gamma}$, which suggests memory exists in τ. To demonstrate the memory effect, we investigate the conditional probability distribution $P_q(\tau|\tau_0)$, the mean conditional interval $\langle\tau|\tau_0\rangle$ and the cumulative probability distribution of the cluster size of τ. The results show clear clustering effect. We further investigate the persistence probability distribution $P_\pm(t)$ and find that $P_-(t)$ decays by a power law with the exponent far different from the value 0.5 for the random walk, which further confirms long memory exists in τ. The scaling and long memory effect of τ for the Chinese stock market are similar to those obtained from the United States and the Japanese financial markets.

Key words: Econophysics; Stock markets; Volatility return intervals
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1 Introduction

In recent years, physicists have paid much attention on the dynamics of financial markets. Scaling behavior is discovered in the financial system by analyzing the indices and the stock prices $y(t')$, such as the 'fat tail' of the probability distribution

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$P(Z,t)$of the two point price change return $Z(t') = ln\, y(t') - ln\, y(t'-1)$[1, 2]. The physical origin of the scaling behavior is often related to the long range correlation. It is interested to find, in spite of the absence of the return correlation, the volatility $|Z(t')|$ is long range correlated [3, 4].

Recently, the volatility return intervals τ, which is defined as the return intervals that the volatility is above a certain threshold q, is investigated for the United States and the Japanese financial markets [5, 6, 7, 8, 9, 10]. Scaling behavior of the probability distribution in the volatility return intervals τ is discovered, and long-range autocorrelation is demonstrated for τ. The scaling and the long-range autocorrelation are rather robust independent of the stock markets and the foreign exchange markets for the developed countries. However, it is known that the emerging markets may behave differently [11, 12, 13]. Especially, the Chinese stock market is newly set up in 1990 and shares a transiting social and political system. Due to the special background of the Chinese stock market, it may share similar properties as the mature financial markets [12], however, it may also exhibits special features far different from the mature financial markets in some aspects [11, 12, 13], such as the leverage effect reported in ref. [11, 12]. It is important to investigate the financial dynamics for the Chinese stock market to achieve more comprehensive understanding of the financial markets.

In this paper, to broaden the understanding of the scaling and memory effect of the volatility return intervals τ for the emerging markets, we investigate the probability distribution and the memory effect of τ for the Chinese stock market. In the next section, we present the data set we analyzed, In section 3, we show the probability distribution of τ. In section 4, we investigate the clustering phenomena by analyzing the conditional probability distribution $P_q(\tau|\tau_0)$, the mean conditional interval $\langle\tau|\tau_0\rangle$ and the cumulative distribution of the cluster size of τ. In section 5, we investigate the persistence probability distribution $P_\pm(t)$. Finally comes the conclusion.

2 Data Analyzed

The data we analyzed is based on the trade-by-trade data from the stocks of the Shanghai Stock Exchange market(SHSE) and the Shenzhen Stock Exchange market(SZSE). The SHSE was established on November 26, 1990 and put into operation on December 19, 1990. Shortly after, the SZSE was established on December 1, 1990 and put into operation on July 3, 1991. Most A-shares and B-shares are traded in the SHSE and SZSE.

The Chinese stock market is an order-driven market and is based on the so called continuous double auction mechanism. In the trading day, there are 3 time periods. From 9:15 to 9:25 a.m., it is the opening call auction time, when the buy and sell orders are aggregated to match. From 9:25 to 9:30 a.m., it is the cool period, and then followed by the continuous double auction time. The time period for the continuous double auction is from 9:30 to 11:30 a.m. and from 13:00 to 15:00 p.m.. More Information about the development process and the current trading mechanism can be found in ref. [13, 14, 15, 16]. We study the transaction records for 3 whole year from 2003 to 2006. The number of the 3 whole year transactions is about 800,000 on average.

3 The Probability Distribution

Here we define the volatility return intervals $\tau(q)$ as the time intervals that volatility $|Z(t')|$ above a certain threshold q, where the sampling time interval for the volatility $|Z(t')|$ is 1 min. Therefore, $\tau(q)$ depends on the threshold q. Fig. 1 shows the volatility return intervals for $q = 0.50$, $q = 1.00$ and $q = 1.50$ in May 2003 of the Datang Telecom Co., Ltd(DTT). The big value of q corresponds to the large volatility that rarely occurs in the financial markets. We investigate the probability distribution function(PDF) $P_q(\tau)$ of the volatility return interval $\tau(q)$ with the threshold $q = 0.750$, 0.875, 1.000, 1.125, 1.375 and 1.500.

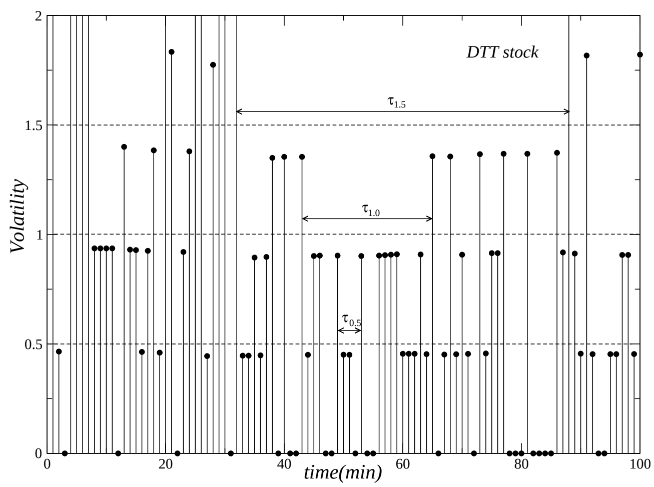

Fig. 1. Volatility return intervals τ for the DTT stock in May 2003 with the threshold values q =0.50, 1.00, 1.50 are displayed.

Fig. 2a shows the PDF $P_q(\tau)$ for the DTT stock and Fig. 2b shows the PDF $P_q(\tau)$ for the Chinese Minsheng Banking Co., Ltd(CMB). The seven curves are for q=0.750, 0.875, 1.000, 1.125, 1.375 and 1.500 respectively. The results show that the PDF $P_q(\tau)$ for large q decays slower than that for small q. We rescale $P_q(\tau)$ and τ as $P_q(\tau) = 1/\overline{\tau} f(\tau/\overline{\tau})$, which is mentioned in ref. [5, 6, 7, 8, 9, 10], to collapse the seven curves with different threshold q onto a single curve, where $\overline{\tau}$ is the average interval.

Fig. 3a and Fig. 3b shows the scaled PDF $P_q(\tau)\overline{\tau}$ as a function of the scaled volatil-

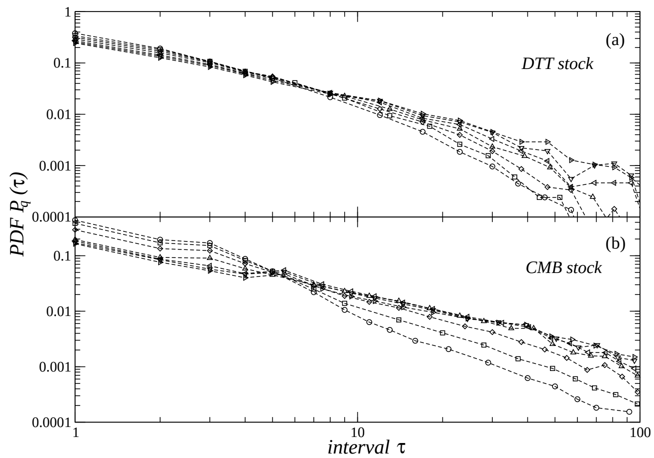

Fig. 2. (a) Probability distribution functions $P_q(\tau)$ for the DTT stock are displayed in log-log scale. The circles, squares, diamonds, triangle ups, triangle lefts, triangle downs and triangle rights are for the threshold values q from 0.75 to 1.50 (0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500) respectively. (b) Probability distribution functions $P_q(\tau)$ for the CMB stock are displayed in log-log scale. The circles, squares, diamonds, triangle ups, triangle lefts, triangle downs and triangle rights are for the threshold values q from 0.75 to 1.50 (0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500 respectively.

ity return intervals $\tau/\overline{\tau}$ for the DTT stock and the CMB stock. Fig. 3c shows the scaled PDF $P_q(\tau)\overline{\tau}$ for the DTT stock, the CMB stock, the CITIC Securities Co., Ltd. (CITIC) stock and the Bird Telecom Co., Ltd(BDT) stock.

The scaling behavior is observed for the normalized τ. The scaling function $f(\tau/\overline{\tau})$ does not directly depend on the threshold q but through $\overline{\tau} \equiv \overline{\tau}(q)$. The scaling behavior is similar to that obtained from the United States and the Japanese stock markets, i.e., the generality of the scaling is further confirmed for both the mature financial markets and the Chinese stock market. It helps us to overcome the difficulty to perform statistics for the rare event with big price fluctuation. If we know the $P_q(\tau)$ with a small q, the behavior of τ with large q then can be predicted by the scaling function. We fit the scaled PDF with a stretched exponential function form [17],

$$f(x) \sim e^{-\alpha x^{\gamma}}, \tag{1}$$

We find that the DTT stock and the CMB stock have similar exponent value with $(\gamma, \alpha) = (0.20 \pm 0.05, 4.0 \pm 0.5)$. The exponent is close to the value $(\gamma, \alpha) = (0.38 \pm 0.05, 3.9 \pm 0.5)$ of the United States stock market [6]. The function form is far different from the Poission distribution, which indicates there may exist correlation in τ.

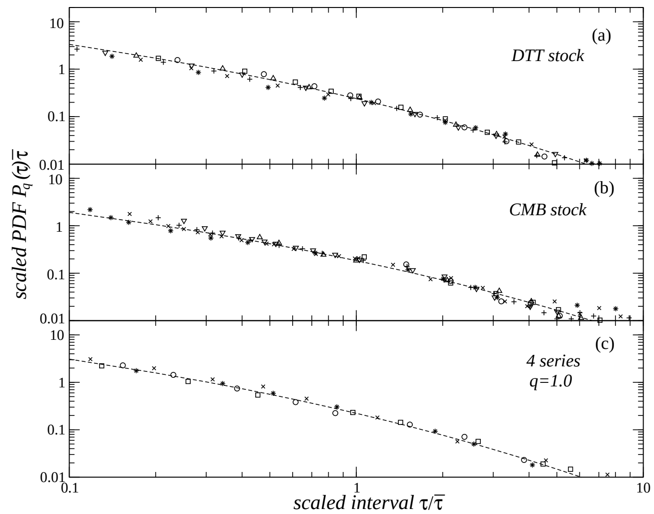

Fig. 3. (a) Scaling of the volatility return intervals for the DTT stock with the threshold values q ranging from 0.75 to 1.50 (0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500) are displayed with circles, squares, triangle ups, triangle downs, pluses, crosses and stars in log-log scale. (b) Scaling of the volatility return intervals for the CMB stock with the threshold values q ranging from 0.75 to 1.50 (0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500) are displayed with circles, squares, triangle ups, triangle downs, pluses, crosses and stars in log-log scale. (c) Scaling of the volatility return intervals for the DTT stock, the CMB stock, the CITIC stock and the BDT stock with the threshold value $q = 1.0$ are displayed with circles, squares, crosses and stars in log-log scale.

4 Clustering Phenomena of the Volatility Return Intervals

If clustering effect occurs in time serials, it suggests memory exists in those serials. To demonstrate the memory in the volatility return intervals τ, we investigate the clustering effect by study the conditional probability distribution $P_q(\tau|\tau_0)$, the mean conditional interval $\langle\tau|\tau_0\rangle$ and the cumulative probability distribution of the cluster size of τ.

4.1 Conditional Probability Distribution

To investigate the memory effect of the volatility return intervals τ, we analyze the conditional PDF $P_q(\tau|\tau_0)$. The $P_q(\tau|\tau_0)$ is denoted as the probability distribution function of the τ that immediately follow a given volatility return interval τ_0 [5, 6, 7, 8, 9, 10]. If memory exists in τ, the $P_q(\tau|\tau_0)$ should depend on the preceding volatility return interval τ_0. To achieve good statistics with more data points, we sort the volatility return intervals in increasing direction and divide it into two subsets. Fig. 4 shows the scaled conditional PDF $P_q(\tau|\tau_0)/\overline{\tau}$ for the DTT stock and the CMB stock with q=0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500. The $P_q(\tau|\tau_0)/\overline{\tau}$ for different threshold q are collapsed onto a single curve. It is observed that the $P_q(\tau|\tau_0)/\overline{\tau}$ for the lower subset is higher for the small τ_0 while the $P_q(\tau|\tau_0)/\overline{\tau}$ for the larger subset is higher for the large τ_0, i.e., small τ follows small τ_0 and large τ follows large τ_0. It implies that the intervals with the similar size form clusters, i.e., there exists memory in τ. The clustering effect has been investigated in ref. [5, 6, 7, 8, 9, 10], we obtain the similar result for the Chinese stock market. To guide the eye, we fit the data with the stretched exponential function $f(x) \sim e^{-\alpha x^{\gamma}}$. For the DTT stock, the exponent is measured to be $(\gamma, \alpha) = (0.30, 4.00)$ for the Lower 1/2 subset and (0.20,4.00) for the larger 1/2 subset. For the CMB stock, the exponent is measured to be $(\gamma, \alpha) = (0.30, 4.0)$ for the Lower 1/2 subset and (0.20,3.50) for the larger 1/2 subset.

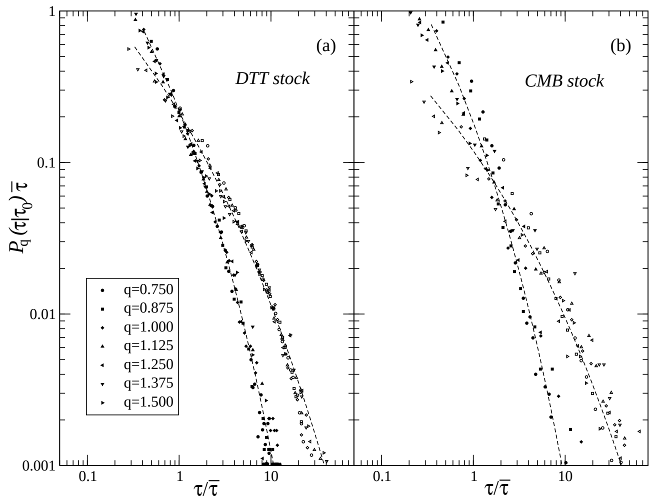

Fig. 4. (a) The scaled conditional probability distribution $P_q(\tau|\tau_0)/\overline{\tau}$ vs $\tau/\overline{\tau}$ for the DTT stock with the threshold values q ranging from 0.75 to 1.50 (0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500) are displayed with circles, squares, diamonds, triangle ups, triangle lefts, triangle downs and triangle rights. The closed symbols are for the lower 1/2 subset, and the open symbols are for the larger 1/2 subset. The dashed lines are for guiding the eyes and with a stretched exponential form $f(x) \sim e^{-\alpha x^{\gamma}}$, where $(\gamma, \alpha) = (0.30, 4.00)$ for the lower 1/2 subset and (0.20,4.00) for the larger 1/2 subset respectively. (b) The scaled conditional probability distribution $P_q(\tau|\tau_0)/\overline{\tau}$ vs $\tau/\overline{\tau}$ for the CMB stock with the threshold values q ranging from 0.75 to 1.50 (0.750, 0.875, 1.000, 1.125, 1.250, 1.375 and 1.500) are displayed with circles, squares, diamonds, triangle ups, triangle lefts, triangle downs and triangle rights. The closed symbols are for the lower 1/2 subset, and the open symbols are for the larger 1/2 subset. The dashed lines are for guiding the eyes and with a stretched exponential form $f(x) \sim e^{-\alpha x^{\gamma}}$, where $(\gamma, \alpha) = (0.30, 4.00)$ for the lower 1/2 subset and (0.20,3.50) for the larger 1/2 subset respectively.

4.2 Mean Conditional Interval

To further demonstrate the memory effect of the volatility return intervals, we investigate the mean conditional return interval $\langle\tau|\tau_0\rangle$, which is defined as the mean of the volatility return intervals τ that immediately follow a given τ_0 subset. Fig. 5

shows the scaled mean conditional return interval $\langle\tau|\tau_0\rangle/\overline{\tau}$ for the DTT stock and the CMB stock with q=0.75, 1.00, 1.25. The closed symbols are for the volatility return intervals and the open symbols are for the shuffled data respectively. It is found that the shuffled data $\langle\tau|\tau_0\rangle/\overline{\tau}$ almost keeps a constant, i.e., there is no correlation in the shuffled data. However, the clustering phenomena that small τ follows small τ_0 while large τ follows large τ_0 are observed for the volatility return intervals τ. It further demonstrates the clustering effect of the intervals for the Chinese stock market. Therefore, it implies autocorrelation in τ. The result is similar to that of the United States and the Japanese stock markets [5, 6, 7, 8].

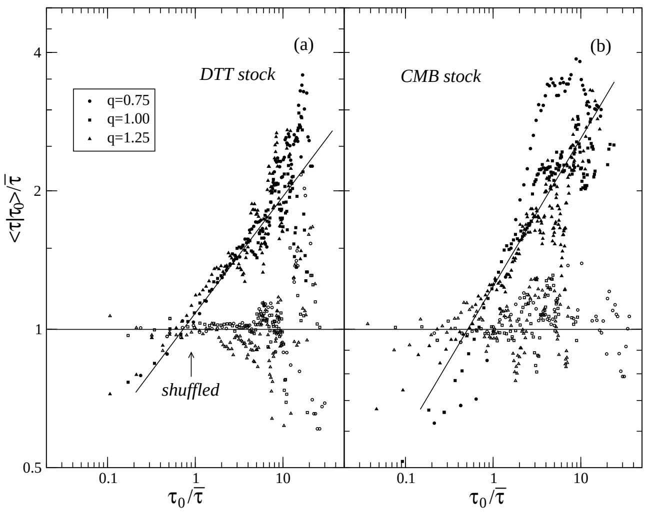

Fig. 5. (a) The scaled mean conditional return interval $\langle\tau|\tau_0\rangle/\overline{\tau}$ vs $\tau_0/\overline{\tau}$ for the DTT stock with the threshold values q=0.75, 1.00, 1.25 are displayed with circles, squares and triangles. The closed symbols are for the volatility return intervals τ and the open symbols are for the shuffled data. (b) The scaled mean conditional return interval $\langle\tau|\tau_0\rangle/\overline{\tau}$ vs $\tau_0/\overline{\tau}$ for the CMB stock with the threshold values q=0.75, 1.00, 1.25 are displayed with circles, squares and triangles. The closed symbols are for the volatility return intervals τ and the open symbols are for the shuffled data.

4.3 Cluster Size Distribution of the Volatility Return Intervals

To investigate the clustering phenomena in a more direct way, we study the cumulative probability distribution of the cluster size of τ. The cluster size is obtained by calculating the successive intervals with similar size [6, 7, 8, 9]. We separate the data into two sets by the median data of τ. The data which are above (below) the median data is signed by $'+'$ ($'-'$). Accordingly, n consecutive $'+'$ or $'-'$ intervals form a cluster and the corresponding cumulative probability distribution is denoted as $p_{n+}(\tau)$ ($p_{n-}(\tau)$). Fig. 6 shows the cumulative probability distribution $p_{n\pm}(\tau)$ for the DTT stock and the CMB stock with q=0.75, 1.00 and 1.25. The open symbols are for the $p_{n+}(\tau)$ and the closed symbols are for the $p_{n-}(\tau)$. We find that the $p_{n-}(\tau)$ presents longer tail persisting up to about $n = 25$ than the $p_{n+}(\tau)$ does, i.e., the relative small value of τ may form big clusters. Similar clusters have been found in the United States and the Japanese stock markets [6, 7, 8, 9]. It also indicates long memory exists in τ.

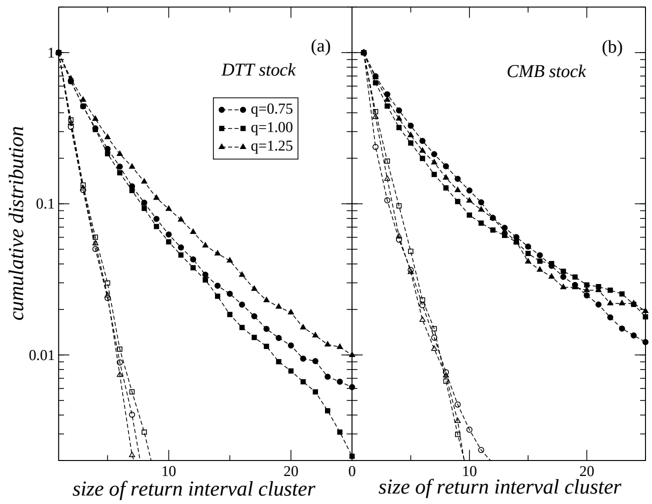

Fig. 6. (a) Cumulative distribution of the cluster size of τ for the DTT stock with the threshold values q=0.75, 1.00, 1.25 are displayed with circles, squares and triangles. The open (closed) symbols are for the consecutive volatility return intervals that are all above (below) the median interval records. (b) Cumulative distribution of the cluster size of τ for the CMB stock with the threshold values q=0.75, 1.00, 1.25 are displayed with circles, squares and triangles. The open (closed) symbols are for the consecutive volatility return intervals that are all above (below) the median interval records.

5 Persistence Probability

To achieve deeper understanding of the memory effect of the volatility return intervals, we investigate the persistence probability, which has been systematically studied in nonequilibrium dynamics such as phase ordering dynamics and critical dynamics [18, 19, 20, 21, 22]. The idea of persistence is closely related to the first passage time which has been widely studied in physics, biology and engineering [23, 24, 25, 26, 27]. In general, the persistence probability provides additional information to the autocorrelation.

The persistence probability $P_+(t)$ ($P_-(t)$) is defined as the probability that $\tau(t'+\widetilde{t})$ has always been above (below) $\tau(t')$ in time t, i.e., $\tau(t'+\widetilde{t}) > \tau(t')$ ($\tau(t'+\widetilde{t}) < \tau(t')$) for all $\widetilde{t} < t$. The average is taken over t'. In Fig. 7, the persistence probability distribution of τ is plotted in log-log scale with q=0.75, 1.00 and 1.25. It is found that $P_+(t)$ decays much faster than $P_-(\tau)$ does. The $P_-(t)$ is observed to decay by a power-law $P_-(t) \sim t^{-\beta}$. The high-low asymmetry of the persistence probability is similar to that of the volatility [19, 20, 21, 22]. The persistence exponents measured from the slopes of $P_-(t)$ with q=0.75, 1.00 and 1.25 are close and estimated to be $\beta = 0.25 \pm 0.05$ for the DTT stock, and $\beta = 0.86 \pm 0.05$ for the CMB stock. The exponents are far different from those of the random walk, where both $P_+(t)$ and $P_-(t)$ show a power law behavior with a persistence exponent $\beta = 0.50$. It further supports that long-range correlation exists in τ.

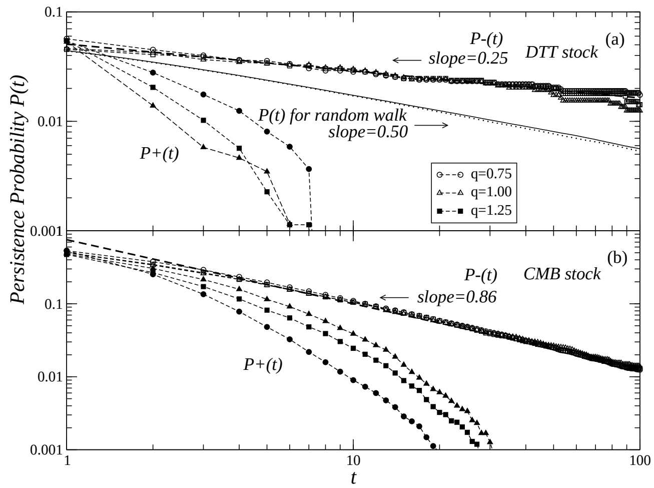

Fig. 7. (a) Persistence probability of the volatility return intervals τ for the DTT stock with the threshold values q=0.75, 1.00, 1.25 are displayed with circles, squares and triangles in log-log scale. The open (closed) symbols are for $p_-(t)$ ($p_+(t)$). The solid line(the dotted line) is the $p_-(t)$ ($p_+(t)$) for the random walk. (b) Persistence probability of the volatility return interval τ for the CMB stock with the threshold values q=0.75, 1.00, 1.25 are displayed with circles, squares and triangles in log-log scale. The open (closed) symbols are for $p_-(t)$ ($p_+(t)$).

6 Conclusion

In summary, we have investigated the probability distribution function $P_q(\tau)$ of the volatility return intervals τ for the Chinese stock market. Scaling behavior is observed after $P_q(\tau)$ and τ are rescaled as $P_q(\tau) = 1/\overline{\tau} f(\tau/\overline{\tau})$. The scaling curve can be fitted by a stretched exponential function $f(x) \sim e^{-\alpha x^\gamma}$ with $\alpha = 0.39$ and $\gamma = 4.00$, which is far different from the Poission distribution. It suggests there exists memory in τ. We then study the conditional probability distribution $P_q(\tau|\tau_0)$ and the mean conditional return interval $\langle\tau|\tau_0\rangle$. The results show that both the $P_q(\tau|\tau_0)$ and the $\langle\tau|\tau_0\rangle$ depend on the previous volatility return intervals τ_0. To obtain the clustering phenomena in a more direct way, we investigate the cumulative probability distribution of the cluster size of τ. Clear clustering effect is observed, especially for the relative small value of τ. We further investigate the persistence probability distribution of τ. It is found that $P_-(t)$ decays by a power law, with the exponent $\beta = 0.25 \pm 0.05$ for the DTT stock and $\beta = 0.86 \pm 0.05$ for the CMB stock, which is far different from the value 0.5 for the random walk. The $P_q(\tau|\tau_0)$, the $\langle\tau|\tau_0\rangle$ and the cumulative probability distribution of the cluster size of τ for the Chinese stock market are similar to those obtained from the United States and the Japanese stock markets. The persistence probability further confirms the long memory of τ for the Chinese stock market. Compared with the mature financial markets, we find that, as a emerging market, the Chinese stock market may have some unique features, however, it shares the similar scaling and long memory properties for the volatility return intervals as the United States and the Japanese stock markets.

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