Galactic rotation curves in noncommutative geometry

F. Rahaman[*], Peter K. F. Kuhfittig [†], K. Chakraborty [‡], A. A. Usmani [§] and Saibal Ray [¶]

[*] Department of Mathematics, Jadavpur University, Kolkata - 700032, India

[†] Department of Mathematics, Milwaukee School of Engineering, Milwaukee, Wisconsin 53202-3109, USA

[‡] Department of Physics, Government Training College, Hooghly - 712103, West Bengal, India

[§] Department of Physics, Aligarh Muslim University, Aligarh 202 002, Uttar Pradesh, India.

[¶] Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700 010, West Bengal, India

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Abstract

This paper discusses the observed flat rotation curves of galaxies in the context of noncommutative geometry. The energy density of such a geometry is diffused throughout a region due to the uncertainty encoded in the coordinate commutator. This intrinsic property appears to be sufficient for producing stable circular orbits, as well as attractive gravity, without the need for dark matter.

1 Introduction

The inability to account for stellar motions in the outer regions of galaxies has led to the hypothesis that galaxies and even clusters of galaxies are pervaded by dark matter [1, 2, 3, 4, 5]. This hypothesis has been confirmed by observing the flatness of galactic

[*] farook_rahaman@yahoo.com

[†] kuhfitti@msoe.edu

[‡] kchakraborty28@yahoo.com

[§] anisul@iucaa.ernet.in

[¶] saibal@iucaa.ernet.in

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rotation curves [6, 7, 8, 9, 10, 11]. The composition of such matter, if it exists, has remained unknown.

The dark-matter problem originated in the measurement of the tangential velocity v^ϕ of stable circular orbits of hydrogen clouds in the outer regions of the halo. To explain the observed constant velocity, it is assumed that the decrease in the energy density is proportional to r^{-2}, where r is the distance from the center of the galaxy.

A number of candidates for dark matter have been proposed. The most favored is the standard cold dark matter (SCDM) paradigm [12, 13]. Another strong possibility is the Λ-CDM model that is related to the accelerated expansion of the Universe [14, 15]. For a summary of some alternative theories, such as scalar-tensor and brane-world models, see Rahaman, et al. [16, 17, 18].

In this paper we study the dark-matter problem from a completely different perspective: an important outcome of string theory is the realization that coordinates may become noncommuting operators on a D-brane [19, 20]. The result is a discretization of spacetime due to the commutator $[\mathbf{x}^\mu, \mathbf{x}^\nu] = i\,\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is an antisymmetric matrix. Noncommutativity replaces point-like structures by smeared objects [21], suggesting the possibility of eliminating the divergences that normally appear in general relativity. The smearing effect is accomplished by using a Gaussian distribution of minimal length $\sqrt{\alpha}$ instead of the Dirac-delta function. More precisely, the energy density of the static and spherically symmetric smeared and particle-like gravitational source has the following form [22]:

$$\rho^* = \frac{M}{(4\pi\alpha)^{\frac{3}{2}}}\,\exp\left(-\frac{r^2}{4\alpha}\right), \tag{1}$$

where the mass M is diffused throughout a region of linear dimension $\sqrt{\alpha}$ due to the uncertainty. The noncommutativity is an intrinsic geometric property of the manifold, and not of its matter content, and can be taken into account by keeping the standard form of the Einstein tensor on the left-hand side of the field equations. The right-hand side is modified, however, by introducing a new energy-momentum tensor as a gravitational source.

Taking the flat rotation curves as input, this model predicts both stable circular orbits and attractive gravity in a typical galaxy.

2 The model

In this paper the metric for a static spherically symmetric spacetime is taken as

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{2}$$

where the functions of the radial coordinate r, $\nu(r)$ and $\lambda(r)$, are the metric potentials.

Now we consider the model with a maximally localized source of energy. Here the Einstein equations can be written as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{3}$$

(We assume that $c = 1$.) The most general energy momentum tensor compatible with static spherical symmetry is

$$T^{\mu}_{\nu} = \text{diag}\,(-\rho, p_r, p_t, p_t). \tag{4}$$

For the metric (2), the Einstein field equations are

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi G\rho, \tag{5}$$

$$e^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = 8\pi G p_r, \tag{6}$$

$$\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = 8\pi G p_t. \tag{7}$$

3 The solutions

Using the observed flat rotation curves as a starting point, it is well known [17, 23] that this condition gives the solution

$$e^{\nu} = B_0 r^l, \tag{8}$$

where l is given by $l = 2v^{2\phi}$ and B_0 is an integration constant. (For a derivation, see Ref. [18].) According to Matos, Guzman, and Lopez [24], the observed rotation curve profile in the presumed dark matter dominated region is such that the rotational velocity v^{ϕ} becomes approximately constant with $v^{\phi} \sim 300$ km/s ($\sim 10^{-3}$) for a typical galaxy. So $l = 0.000001$, as shown by Nandi et al. [25], and we likewise assume large distances measured in kpc from the galactic center.

Using equation (1), equation (5) yields

$$e^{-\lambda} = 1 - \frac{2m^*(r)}{r}, \tag{9}$$

where

$$m^*(r) = \frac{2M}{\sqrt{\pi}}\,\gamma\left(\frac{3}{2}, \frac{r^2}{4\alpha}\right) = \frac{2M}{\sqrt{\pi}}\int_0^{r^2/4\alpha} \sqrt{t}\; e^{-t}dt \tag{10}$$

and

$$\gamma\left(\frac{3}{2}, \frac{r^2}{4\alpha}\right) = \int_0^{r^2/4\alpha} \sqrt{t}\; e^{-t}dt \tag{11}$$

is the lower incomplete gamma function [22]. The classical Schwarzschild mass is recovered in the limit as $r/\sqrt{\alpha} \to \infty$. Furthermore, new physics can only be expected if $r \approx \sqrt{\alpha}$.

The mass M could be a diffused centralized object such as a wormhole [26] or a gravastar [27]. Since we are interested in rotation curves at some fixed distance $r = R_0$ from the center, we need to consider instead a spherical shell of radius $r = R_0$. So

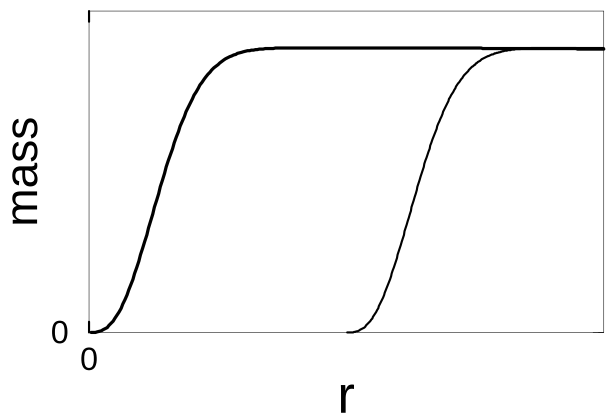

Figure 1: The graphs of $m^*(r)$ (left) and $m(r)$.

instead of a smeared particle-like object, we get a smeared shell. We will therefore need to translate the above curves as follows:

$$\rho = \frac{M}{(4\pi\alpha)^{\frac{3}{2}}} \exp\left(-\frac{(r-R_0)^2}{4\alpha}\right) \tag{12}$$

and

$$m(r) = \frac{2M}{\sqrt{\pi}}\,\gamma\left(\frac{3}{2}, \frac{(r-R_0)^2}{4\alpha}\right). \tag{13}$$

These replace ρ^* and $m^*(r)$. As before,

$$\gamma\left(\frac{3}{2}, \frac{(r-R_0)^2}{4\alpha}\right) = \int_0^{(r-R_0)^2/4\alpha} \sqrt{t}\, e^{-t} dt \tag{14}$$

is the lower incomplete gamma function. It is important to note that Eq. (14) consists of a pure translation of Eq. (11) and therefore remains an appropriate model (see Fig. 1). Here M denotes the mass of a spherical shell of radius $r = R_0$. Once again, the classical Schwarzschild limit is recovered as $(r - R_0)/\sqrt{\alpha} \to \infty$, while new physics can only be expected if $r - R_0 \approx \sqrt{\alpha}$.

Using solutions (8) and (9) in the Einstein field equations, one can readily get the following expressions for p_r and p_t:

$$p_r = \frac{1}{8\pi G}\frac{l}{r^2}\left\{\left[1 - \frac{4M}{\sqrt{\pi}r}\,\gamma\left(\frac{3}{2}, \frac{(r-R_0)^2}{4\alpha}\right)\right](1+l) - 1\right\} \tag{15}$$

(since $\nu' = l/r$) and

$$p_t = \frac{1}{8\pi G}\left[\frac{1}{2}\left(1 - \frac{2m(r)}{r}\right)\left(\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right)\right]. \tag{16}$$

Moreover, since

$$\lambda' = \frac{2}{r}\,\frac{m'r - m}{r - 2m} \quad \text{and} \quad m' = \frac{M(r-R_0)^2}{2\sqrt{\pi}\;\alpha^{\frac{3}{2}}}\,\exp\left(-\frac{(r-R_0)^2}{4\alpha}\right),$$

we arrive at

$$p_t = \frac{1}{8\pi G}\frac{1}{4r^3}\left\{l^2\left[r - \frac{4M}{\sqrt{\pi}}\;\gamma\left(\frac{3}{2},\frac{(r-R_0)^2}{4\alpha}\right)\right]\right.$$
$$\left. -2(l+2)\left[\frac{Mr(r-R_0)^2}{2\sqrt{\pi}\;\alpha^{3/2}}\;\exp\left(-\frac{(r-R_0)^2}{4\alpha}\right) - \frac{2M}{\sqrt{\pi}}\;\gamma\left(\frac{3}{2}.\frac{(r-R_0)^2}{4\alpha}\right)\right]\right\}. \quad (17)$$

As a final comment, the spacetime is not asymptotically flat. If this spacetime were to be joined to an exterior Schwarzschild spacetime at the boundary of the halo, then the pressure anisotropy would be an advantage [25].

4 Stability of circular orbits

Given the four-velocity $U^\alpha = \frac{dx^\sigma}{d\tau}$ of a test particle moving solely in the subspace of the halo and restricting ourselves to $\theta = \pi/2$, the equation $g_{\nu\sigma}U^\nu U^\sigma = -m_0^2$ can be cast in a Newtonian form

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 + V(r), \quad (18)$$

which gives

$$V(r) = -\left[E^2\left(1 - \frac{r^{-l}e^{-\lambda}}{B_0}\right) + e^{-\lambda}\left(1 + \frac{L^2}{r^2}\right)\right]. \quad (19)$$

The constants

$$E = \frac{U_0}{m_0} \quad \text{and} \quad L = \frac{U_3}{m_0} \quad (20)$$

are, respectively, the conserved relativistic energy and angular momentum per unit rest mass of the test particle [25]. Circular orbits are defined by $r = R_0$, a constant, so that $\frac{dR_0}{d\tau} = 0$ and, additionally, $\frac{dV}{dr}\;|_{r=R_0} = 0$. From these two conditions follow the conserved parameters:

$$L = \pm\sqrt{\frac{l}{2-l}}R_0 \quad (21)$$

and, using L in $V(R) = -E^2$, we get

$$E = \pm\sqrt{\frac{2B_0}{2-l}}R_0^{l/2}. \quad (22)$$

The orbits will be stable if $\frac{d^2V}{dr^2}\mid_{r=R_0}< 0$ and unstable if $\frac{d^2V}{dr^2}\mid_{r=R_0}> 0$. We first determine $\frac{d^2V}{dr^2}$ at $r = R_0$ by substituting the expressions for L and E:

$$\frac{d^2V}{dr^2}\bigg|_{r=R_0} = A + B + C + D, \quad \text{where}$$

$$A = \left[1 - \frac{4M}{r\sqrt{\pi}}\int_0^{(r-R_0)^2/4\alpha}\sqrt{t}e^{-t}dt\right]\left[\frac{2}{2-l}l(l+1)\frac{R_0}{r^{2+l}} - \frac{6l}{2-l}\frac{R_0^2}{r^4}\right],$$

$$B = -\frac{2M}{r^3}\left[\frac{r^2(r-R_0)}{\sqrt{\pi}\alpha^{3/2}}e^{-(r-R_0)^2/4\alpha} - \frac{r^2(r-R_0)^3}{4\sqrt{\pi}\alpha^{5/2}}e^{-(r-R_0)^2/4\alpha}\right.$$
$$\left.-\frac{r(r-R_0)^2}{\sqrt{\pi}\alpha^{3/2}}e^{-(r-R_0)^2/4\alpha}\right]\left(\frac{2}{2-l}\frac{R_0}{r^l} - 1 - \frac{l}{2-l}\frac{R_0^2}{r^2}\right),$$

$$C = -\frac{4M}{r^2}\frac{r(r-R_0)^2}{2\sqrt{\pi}\alpha^{3/2}}e^{-(r-R_0)^2/4\alpha}\frac{2}{2-l}l\left(-\frac{R_0}{r^{1+1}} + \frac{R_0^2}{r^3}\right),$$

$$D = \frac{8M}{\sqrt{\pi}r^2}\left[\int_0^{(r-R_0)^2/4\alpha}\sqrt{t}e^{-t}dt\right]\frac{2}{2-l}l\left(-\frac{R_0}{r^{1+l}} + \frac{R_0^2}{r^3}\right)$$
$$-\frac{8M}{\sqrt{\pi}r^3}\int_0^{(r-R_0)^2/4\alpha}\sqrt{t}e^{-t}dt\left(\frac{2}{2-l}\frac{R_0}{r^l} - 1 - \frac{l}{2-l}\frac{R_0^2}{r^2}\right). \quad (23)$$

In Eqs. (1) and (10), r is the distance in the radial outward direction. We will therefore assume that $r > R_0$, which does not result in a loss of generality. Our aim is to show that under certain conditions, $V''(R_0)$ is negative, resulting in a stable orbit.

Since α controls the "width" of the Gaussian curve, we study the smearing effect by assuming that $r - R_0 \approx \sqrt{\alpha}$, the fundamental condition discussed in Sec. 3. Returning to Eq. (23), the mass of the shell, which we are treating as the analogue of a smeared central object, has a relatively small value when measured in kpc (and we must therefore assume that $G = 1$). In conjunction with the condition $r - R_0 \approx \sqrt{\alpha}$, the first term A is less than l/R_0. Next, factoring B, we have

$$\left[r - \frac{r(r-R_0)^2}{4\alpha} - (r-R_0)\right]\left(-\frac{2M}{r^2}\frac{r-R_0}{\sqrt{\pi}\alpha^{3/2}}e^{-(r-R_0)^2/4\alpha}\right)$$
$$\times\left(\frac{2}{2-l}\frac{R_0}{r^l} - 1 - \frac{l}{2-l}\frac{R_0^2}{r^2}\right).$$

Since $(r - R_0)^2 \approx \alpha$, the first factor is approximately equal to $R_0 - r/4$, which is positive for r not too large. Observe that B exceeds C in absolute value. Because of the α in the denominator, $|B|$ easily overtakes A as well, even for moderately small α. Finally, since D is negative, $V''(R_0)$ is also negative, provided, of course, that $R_0 - r/4$ is positive. The last condition limits the size of the variable r.

4.1 The effect of the noncommutative geometry

It is shown by Nicolini, Smailagic, and Spallucci [22] that at large distances one would expect only a minimal deviation from the standard vacuum Schwarzschild geometry. Referring to Eqs. (1) and (10), new physics can only be expected if $r \approx \sqrt{\alpha}$. In our model, this condition corresponds to $r - R_0 \approx \sqrt{\alpha}$. As we just saw, if r is too large, then $V''(R_0)$ is no longer negative. This case is an example of a large distance that produces the appearance of a Schwarzschild geometry since the smearing, although very much present, is no longer apparent. So, in a sense, the unseen dark matter is replaced by the unseen noncommutative geometry. (Regarding large-scale structures, it is worth noting that a Gaussian source has also been used in Ref. [28] to model phantom-energy supported wormholes, as well as in Ref. [29] to model the physical effects of short-distance fluctuations of noncommutative coordinates in the study of black holes.)

4.2 Negative radial pressure

It is emphasized in Ref. [22] that a negative radial pressure is needed to retain the smearing effect near the origin, referring now to Eqs. (1) and (10). The negative pressure counteracts the inward gravitational pull, thereby preventing a collapse to a matter point.

Returning to Eq. (15), if $r - R_0 \approx \sqrt{\alpha}$, then p_r is indeed negative if M, the mass of the shell, is sufficiently large (since the shell need not be arbitrarily thin). But as long as $r - R_0 \approx \sqrt{\alpha}$, we have $V''(R_0) < 0$, so that the negative pressure is attributable to the noncommutative geometry, much like traversable wormholes sustained in this manner [26]. We will return to this observation in Sec. 6.

5 Attraction and total gravitational energy

Our next step is to consider the question of attractive gravity by studying the geodesic equation for a test particle that is moving along a circular path of radius $r = R$:

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\gamma}\frac{dx^\mu}{d\tau}\frac{dx^\gamma}{d\tau} = 0. \tag{24}$$

This equation implies that

$$\frac{d^2r}{d\tau^2} = -\frac{1}{2}e^{-\lambda}\left[\frac{d}{dr}e^{\lambda}\left(\frac{dr}{d\tau}\right)^2 + \frac{d}{dr}e^{\nu}\left(\frac{dt}{d\tau}\right)^2\right], \tag{25}$$

making use of Eq. (8). As before, as long as $\frac{dr}{d\tau} = 0$, we get

$$\frac{d^2r}{d\tau^2} = -\frac{1}{2}e^{-\lambda}B_0 l r^{l-1}\left(\frac{dt}{d\tau}\right)^2 < 0. \tag{26}$$

We conclude that objects are attracted toward the center.

According to Lyndell - Bell et al. [30], we can determine the total gravitational energy E_G between two fixed radii r_1 and r_2 by means of the following formula:

$$E_G = M_N - E_M = 4\pi\int_{r_1}^{r_2}[1 - \sqrt{e^{\lambda(r)}}]\rho r^2 dr, \tag{27}$$

where

$$M_N = 4\pi\int_{r_1}^{r_2}\rho r^2 dr \tag{28}$$

is the Newtonian mass given by

$$M_N = 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \frac{M}{(4\pi)^{1/2}(\alpha)^{\frac{3}{2}}} \int_{r_1}^{r_2} \exp\left(-\frac{(r-R_0)^2}{4\alpha}\right) r^2 dr. \tag{29}$$

So the total gravitational energy is

$$E_G = \frac{M}{(4\pi)^{1/2}(\alpha)^{\frac{3}{2}}} \int_{r_1}^{r_2} \left[1 - \left[1 - \frac{2m(r)}{r}\right]^{-\frac{1}{2}}\right] \exp\left(-\frac{(r-R_0)^2}{4\alpha}\right) r^2 dr, \tag{30}$$

where $m(r)$ is given in Eq. (13). Because of the small M, the integrand, and hence E_G, are negative, showing that gravity in the halo is indeed attractive.

6 The observed equation of state

The equation of state of the halo fluid can be obtained from a combination of rotation curves and lensing measurements. To this end, let us rewrite the metric, Eq. (2), in the following form:

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{1}{[1 - \frac{2m(r)}{r}]} dr^2 + r^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{31}$$

where

$$\Phi(r) = \frac{1}{2}\left[\ln B_0 + l \ln r\right] \tag{32}$$

and $m(r)$ is given in Eq. (13). As discussed in Ref. [25], the functions are determined indirectly from certain lensing measurements defined by

$$\Phi_{\text{lens}} = \frac{\Phi(r)}{2} + \frac{1}{2}\int \frac{m(r)}{r^2} dr = \frac{\ln B_0}{4} + \frac{\ln rl}{4} + \int \frac{M\ \gamma\left(3/2, (r-R_0)^2/4\alpha\right)}{\sqrt{\pi} r^2} dr \tag{33}$$

and

$$m_{\text{lens}} = \frac{1}{2} r^2 \Phi'(r) + \frac{1}{2} m(r) = \frac{lr}{4} + \frac{M\ \gamma\left(3/2, (r-R_0)^2/4\alpha\right)}{\sqrt{\pi}}. \tag{34}$$

The observed equation of state depends on the dimensionless quantity

$$\omega(r) = \frac{p_r + 2p_t}{3\rho} \approx \frac{2}{3} \frac{m'_{\text{RC}} - m'_{\text{lens}}}{2m'_{\text{lens}} - m'_{\text{RC}}}, \tag{35}$$

due to Faber and Visser [31]. The subscript RC refers to the rotation curve, i. e.,

$$\phi_{RC} = \Phi(r) = \frac{1}{2}\left[\ln B_0 + l \ln r\right] \tag{36}$$

and

$$m_{RC} = r^2 \Phi'(r) = \frac{lr}{2}. \tag{37}$$

The prime denotes the derivative with respect to r. The result is

$$\omega(r) = \frac{2}{3}\frac{m'_{\rm RC} - m'_{\rm lens}}{2m'_{\rm lens} - m'_{\rm RC}} = \frac{l\sqrt{\pi}\alpha^{\frac{3}{2}} - M(r-R_0)^2 \exp\left[-(r-R_0)^2/4\alpha\right]}{3M(r-R_0)^2 \exp\left[-(r-R_0)^2/4\alpha\right]}. \tag{38}$$

At first glance, $\omega(r) > 0$ due to the small value of M, just as in the case of ordinary matter. But if α is very small and M sufficiently large, a situation we encountered in Sec. 4, then $\omega(r) < 0$. This also follows from the fact that $p_t < 0$ for a sufficiently small α [Eq. (17)], since we already know that p_r is negative. So the "quintessence-like" condition is due entirely to the noncommutative geometry. Observe, however, that as α gets very small, $\omega(r)$ approached $-1/3$ from the right, so that $\omega(r)$ is not in the actual quintessence range.

As a final comment, at large distances, where the pressures are no longer negative, we have $\omega(r) > 0$. This may be interpreted to mean that the smearing, although still present, is no longer seen at this distance, resulting in the appearance of ordinary matter.

7 Conclusion

In this paper we assume the existence of flat rotation curves in galaxies and discuss such curves in the context of noncommutative geometry. The energy density of this geometry is a smeared gravitational source, where the mass is diffused throughout a region of linear dimension $\sqrt{\alpha}$. This diffusion is due to the uncertainty attributable to the commutator and is applicable to other large-scale structures. It is shown that this intrinsic property of the noncommutative geometry is able to account for the stable circular orbits in remote regions of the halo, as well as for attractive gravity. Sufficiently far from the orbit, the smearing is no longer observed, which may be interpreted to mean that stable orbits are due to the unseen noncommutative geometry instead of the unseen dark matter.

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