Dynamical effects of General Relativity on the satellite-to-satellite range and range-rate in the GRACE mission

L. Iorio*

*Ministero dell'Istruzione, dell'Università e della Ricerca (M.I.U.R.)
Fellow of the Royal Astronomical Society (F.R.A.S.)
Permanent address: Viale Unità di Italia 68 70125 Bari (BA), Italy.
e-mail: lorenzo.iorio@libero.it

Abstract

We numerically investigate the impact of the General Theory of Relativity (GTR) on the satellite-to-satellite range ρ and range-rate $\dot{\rho}$ of the twin GRACE A/B spacecrafts through their dynamical equations of motion. The present-day accuracies in measuring such observables are $\sigma_\rho \sim 1 - 10~\mu$m, $\sigma_{\dot{\rho}} \sim 0.1 - 1~\mu$m s^{-1}. Studies for a follow-on of such a mission points toward a range-rate accuracy of the order of $\sigma_{\dot{\rho}} \sim 1$ nm s^{-1} or better. We also compute the dynamical range and range-rate perturbations caused by the first six zonal harmonic coefficients $J_\ell, \ell = 2, 3, 4, 5, 6, 7$ of the classical multipolar expansion of the terrestrial gravitational potential in order to evaluate their aliasing impact on the relativistic effects. Conversely, we also quantitatively assessed the possible a-priori "imprinting" of GTR itself, not solved-for in all the GRACE-based Earth's gravity models produced so far, on the estimated values of the low degree zonals of the geopotential. The present analysis can also be extended, in principle, to different orbital configurations in order to design a suitable dedicated mission able to accurately measure the relativistic effects considered. Moreover, also other non-classical dynamical features of motion, caused by, e.g., modified models of gravity, can be considered in further studies.

1   Introduction

The Gravity Recovery and Climate Experiment (GRACE) mission [1, 2, 3], jointly launched in March 2002 by NASA and the German Space Agency (DLR) to map the terrestrial gravitational field with an unprecedented accuracy, consists of a tandem of two spacecrafts moving along low-altitude, nearly polar orbits (see Table 1 and Table 2) continuously linked by an inter-satellite microwave K-band ranging (KBR) system accurate to better than 10 μm (biased range ρ) [4] and 1 μm s^{-1} (range-rate) [4, 5]. Investigations concerning a follow-on of the GRACE mission are being currently performed [6]; by using an interferometric laser ranging system it would be possible to reach an accuracy level of $\sim$nm s^{-1} or better in measuring the range-rate [7].

Although GRACE was not specifically de-

signed to directly test the General Theory of Relativity (GTR), which, actually, has never been solved-for in the several global gravity field solutions[1] produced so far by different institutions from long data records from GRACE, the great accuracy in its KBR may, in fact, allow, in principle, to measure some consequences of GTR by exploiting such direct accurate observables. Concerning the Lense-Thirring effect [8], connected with the rotation of the source of the gravitational field, a similar idea was envisaged in Ref. [9]. Thus, it is important to investigate the impact of the dynamical[2] effects of GTR on both range and range-rate to check if it falls within the present-or future-measurability domain of GRACE-type missions. This will be the subject of Section 2. Concerning other, clock-related relativistic effects in GRACE, it must be recalled that dual-frequency carrier-phase Global Positioning System (GPS) receivers are flying on both satellites. They are used for precise orbit determination of both the GRACE A/B spacecrafts, and to time-tag the KBR system; the relativistic effects in the GRACE GPS data have been examined in Ref. [10].

It should be recalled that some of the Earth's gravity field solutions retrieved from GRACE data have been used as background reference models in the LAGEOS-based tests [11, 12] of the Lense-Thirring effect [8]; the even zonal harmonic coefficients $J_\ell, \ell = 2, 4, 6, ...$ of the multipolar expansion of the classical part of the terrestrial gravitational potential, estimated as solve-for parameters in the GRACE-based models, may retain an a-priori "imprinting" by GTR itself which, as already noted, has never been

explicitly solved-for so far in the GRACE data processing. This aspect will be treated in Section 3. Section 4 is devoted to the conclusions.

Finally, let us remark that the approach followed here in the specific case of GTR can well be extended to other dynamical effects predicted, e.g., by modified models of gravity. An investigation of the effects of a Yukawa-like extra-force on the orbit of GRACE-A has recently been performed in Ref. [13].

2 General relativistic effects in the satellite-to-satellite range and range-rate

The approach followed is as follows. We simultaneously integrated the equations of motion of both the GRACE spacecrafts in cartesian coordinates with and without the dynamical orbital perturbations of the Newtonian monopole which we are interested in. The time span of the integrations is $\Delta t = 30$ d. The method adopted, implemented with the software package MATHEMATICA, is the Runge-Kutta one. The initial conditions, common to all the numerical integrations, are in Table 1; they correspond to the Keplerian orbital elements listed in Table 2. It can be noted that the altitudes of the twin GRACE spacecrafts are of about 500 km with respect to the Earth's surface, and that their orbits are almost circular and polar. The resulting numerically integrated trajectories have, then, been used to compute the satellite-to-satellite range perturbation $\Delta\rho$ as the difference among the perturbed and the un-perturbed ranges. The range-rate perturbation $\Delta\dot\rho$ has been computed by numerically differentiating $\Delta\rho$. Our code was previously tested by numerically computing the time evolution of the node perturbation

---

[1]See http://icgem.gfz-potsdam.de/ICGEM/ on the WEB.

[2]Here we will not deal with the relativistic features of the propagation of electromagnetic waves.

Table 1: State vectors, in cartesian coordinates referred to ITRF, of the pair GRACE A/B used as initial conditions for the present analysis. They have been obtained from the files GNV1B_2003-09-14_A_00 and GNV1B_2003-09-14_B_00, retrieved from ftp://cddis.gsfc.nasa.gov/pub/slr/predicts/current/graceA_irvs_081202_0.gfz and ftp://cddis.gsfc.nasa.gov/pub/slr/predicts/current/graceB_irvs_081201_1.gfz. The epoch is 13 September 2003. See ftp://podaac.jpl.nasa.gov/pub/grace/doc/Handbook_1B_v1.3.pdf for the explanation of the GPS Navigation Data Format Record (GNV1B) format.

S/C	x_0 (m)	y_0 (m)	z_0 (m)	$\dot{x}_0$ (m s^{-1})	$\dot{y}_0$ (m s^{-1})	$\dot{z}_0$ (m s^{-1})
A	1075039.75	-3246695.67	-5943208.1	2088.38438	-6255.78024	3807.799
B	1007605.75	-3066941.52	-6049698.6	2138.63176	-6362.53248	3595.342

Table 2: Keplerian orbital elements of the pair GRACE A/B corresponding to the state vectors in cartesian coordinates of Table 1. They are the semi-major axis a, the eccentricity e, the inclination I to the Earth's equator, the longitude of the ascending node Ω, the argument of pericenter ω, and the mean anomaly $\mathcal{M}$. The Keplerian orbital periods of the GRACE pair is $P^{(\mathrm{Kep})} \doteq 2\pi\sqrt{a^3/GM_\oplus} = 1.56$ h$= 0.065$ d.

S/C	a_0 (km)	e_0	I_0 (deg)	Ω_0 (deg)	ω_0 (deg)	$\mathcal{M}_0$ (deg)
A	6841.11877	0.00272831	89.9395	-71.5742	119.916	-179.997
B	6839.80210	0.00298412	89.8374	-71.5081	118.082	-179.997

of GRACE-A due to the Lense-Thirring effect over $\Delta t = 30$ d, and by comparing its resulting shift with the well known analytical results [8] for the secular node precession, computed for GRACE-A: the agreement between the two outcomes was remarkable. The same has successfully been done for the Schwarzschild-type perigee precession of GRACE-A.

In Figure 1 we display the result of our numerical integrations for GTR whose[3] Schwarzschild and Lense-Thirring accelerations have been retrieved from Ref. [14]. Both the GTR pertur-

bations induce cumulative[4], long-term effects on the inter-satellite GRACE range; after $\Delta t = 30$ d the peak-to-peak amplitude of the Schwarzschild signal is $\Delta\rho_{\mathrm{Sch}} \sim 5 \times 10^5$ μm $= 50$ cm, while the Lense-Thirring range shift is $\Delta\rho_{\mathrm{LT}} = 2 \times 10^3$ μm $= 0.2$ cm. By assuming a present-day accuracy of $\sigma_\rho \sim 5$ μm in measuring the satellite-to-satellite range, it turns out that the Schwarzschild effect is, in principle, detectable with a relative accuracy of the order of 1×10^{-5}, while the Lense-Thirring signature would fall within the measurability domain at a $\sim 0.3\%$

---

[3]It is assumed that the de Sitter precession has been transformed away.

[4]Note, from Table 2, that the orbital period of the GRACE spacecrafts is just 1.56 h.
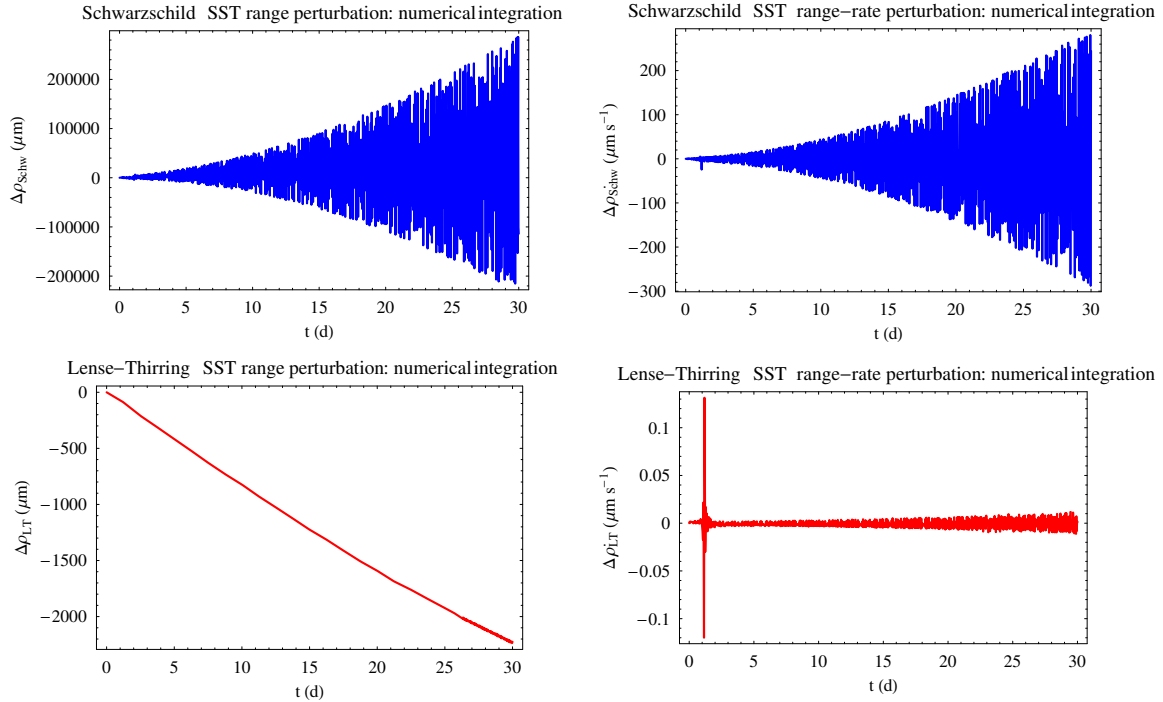
Figure 1: Differences of the numerically integrated satellite-to-satellite ranges (left column) and range-rates (right column) for GRACE A/B with and without the general relativistic Schwarzschild (blue line) and Lense-Thirring (red line) dynamical perturbations. The initial conditions, quoted in Table 1, are common to both the perturbed and un-perturbed integrations. The time span is $\Delta t = 30$ d. The units are μm (range) and μm s^{-1} (range-rate).

level. Concerning the range-rate, if we assume $\sigma_{\dot{\rho}} \sim 0.5$ μm s^{-1}, the Schwarzschild perturbation $\Delta \dot{\rho}_{\mathrm{Sch}}$, whose peak-to-peak nominal amplitude is 600 μm s^{-1} = 0.6 mm s^{-1}, would be detectable at a $\sim 0.08\%$ level. Instead, the Lense-Thirring effect on the range-rate $\Delta \dot{\rho}_{\mathrm{LT}}$ is too small, amounting to about 8×10^{-4} μm s^{-1} = 0.8 nm s^{-1}.

Thus, GTR affects the GRACE satellite-to-satellite range in a detectable way, in principle, given the present-day level of accuracy in measuring it. The same also holds for the range-rate, although only for the Schwarzschild signal, and

at a lower level of accuracy. Table 3 summarizes the results of this Section. However, we wish to point out that it should actually be checked if the relativistic signatures are not absorbed and removed from the range signal in estimating some of the various range parameters which are solved-for in the usual GRACE data processing. Indeed, it is exposed to some mismodeled device behavior, which requires estimating many empirical parameters in semi-dynamical orbit processing mode [15].

In Section 3 we will look at the competing effects induced on the range and range-rate of

Table 3: Peak-to-peak amplitudes of the inter-satellite GRACE range and range-rate perturbations $\Delta\rho$, $\Delta\dot{\rho}$ caused by the general relativistic Schwarzschild and Lense-Thirring effects, and by the first six zonal harmonics of the classical part of the geopotential over $\Delta t = 30$ d. Concerning the zonals, the values quoted for their perturbations have been obtained by setting $J_\ell = 1, \ell = 2, 3, 4, 5, 6, 7$; the nominal and mismodelled values of their range and range-rate perturbations can be obtained by multiplying the figures in this Table with those of Table 4 for J_ℓ and σ_{J_ℓ}. The present-day accuracy in measuring the satellite-to satellite GRACE range and range-rate is $\sigma_\rho \lesssim 10$ μm and $\sigma_{\dot{\rho}} \lesssim 1$ μm s^{-1}, respectively.

Dynamical effect	$\Delta\rho$ (μm)	$\Delta\dot{\rho}$ (μm s^{-1})
Schwarzschild	5×10^5	6×10^2
Lense-Thirring	2×10^3	8×10^{-4}
J_2	2×10^{15}	8×10^8
J_3	7×10^{15}	8×10^{12}
J_4	3×10^{15}	5×10^{10}
J_5	8×10^{15}	9×10^{12}
J_6	2×10^{15}	2×10^{11}
J_7	8×10^{15}	8×10^{12}

GRACE by some low-degree zonal harmonics $J_\ell, \ell \geq 2$ of the multipolar expansion of the Newtonian part of the Earth's gravitational potential accounting for its departures from spherical symmetry. Indeed, their unavoidably imperfect knowledge causes mismodeled range and range-rate signals which would corrupt the recovery of the relativistic ones at a level which has to be quantitatively assessed. On the other hand, such an investigation will also contribute to quantitatively evaluate the level of a possible a-priori "imprinting" of GTR itself, not solved-for so far in all the GRACE-based models, on the estimated values of such zonals. This issue has been treated, in the framework of the LAGEOS-based tests of the Lense-Thirring effect, in Ref. [16] as far as the node precessions of the orbital planes of the GRACE spacecrafts are concerned.

3 A-priori "imprint" level of GTR in the even zonal KBR signature

The range and range-rate perturbations $\Delta\rho_{J_\ell}/J_\ell$, $\ell = 2, 4, 6$ and $\Delta\dot{\rho}_{J_\ell}/J_\ell$, $\ell = 2, 4, 6$ caused by the first three even zonals, divided by the nominal values of the even zonals themselves (see Table 4 for them and their 1-σ errors in one of the most recent global Earth's gravity field solution), are depicted in Figure 2 and summarized in Table 3. They have been computed as the relativistic effects in Section 2. Also in this case the code used has been tested by successfully reproducing the well-known analytical results for the node and perigee long-term variations caused by J_2, J_4, J_6 [18] in the case of GRACE-A. In order to obtain
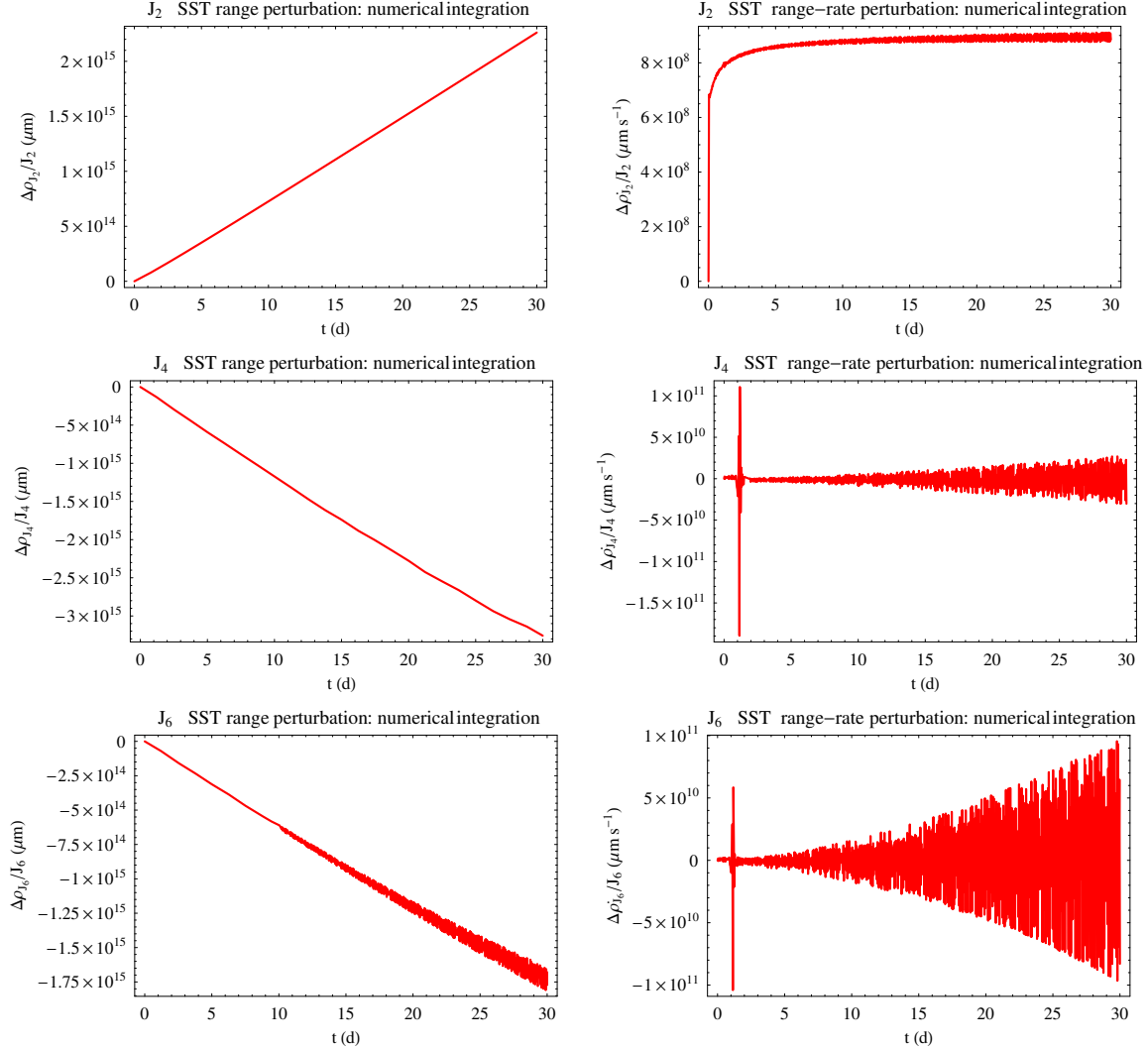
Figure 2: Differences of the numerically integrated ranges (left column) and range-rates (right column) for GRACE A/B with and without the classical dynamical perturbations due to the even zonal harmonics $J_\ell \doteq -\sqrt{2\ell+1}\ \overline{C}_{\ell,0}$, $\ell = 2, 4, 6$. For a better comparison with the relativistic signatures of Figure 1, the values of the even zonals have been set equal to unity, i.e. $\Delta\rho_{J_\ell}$ and $\Delta\dot{\rho}_{J_\ell}$ have been divided by J_ℓ, $\ell = 2, 4, 6$. According to the recent combined GOCE-GRACE solution GOCO01S [17], the formal, statistical uncertainties in the normalized Stokes coefficients are, $\sigma_{\overline{C}_{2,0}} = 0.44\times10^{-12}, \sigma_{\overline{C}_{4,0}} = 0.8\times10^{-13}, \sigma_{\overline{C}_{6,0}} = 0.4\times10^{-13}$ respectively. The initial conditions, quoted in Table 1, are common to both the perturbed and un-perturbed integrations. The time span is $\Delta t = 30$ d. The units are μm (range) and μm s^{-1} (range-rate).

Table 4: Estimated values and formal, statistical errors of the normalized Stokes coefficients $\overline{C}_{\ell,0}$ of the geopotential for $\ell = 2, 3, 4, 5, 6, 7$ from the GOCE/GRACE-based solution GOCO01S [17]. Recall that $J_\ell \doteq -\sqrt{2\ell+1}\ \overline{C}_{\ell,0}$.

Degree ℓ	$\overline{C}_{\ell,0}$	$\sigma_{\overline{C}_{\ell,0}}$
2	$-0.484164968978546 \times 10^{-3}$	$0.445570221091724 \times 10^{-12}$
3	$0.957198053205652 \times 10^{-6}$	$0.278872574524992 \times 10^{-12}$
4	$0.540003306665179 \times 10^{-6}$	$0.861279816394233 \times 10^{-13}$
5	$0.686701781984240 \times 10^{-7}$	$0.602680991556504 \times 10^{-13}$
6	$-0.149958175526140 \times 10^{-6}$	$0.451610382407952 \times 10^{-13}$
7	$0.905106214220478 \times 10^{-7}$	$0.362372315970691 \times 10^{-13}$

the magnitudes of the nominal and of the mismodelled range and range-rate signatures it is sufficient to multiply the figures in Figure 2 by J_ℓ and σ_{J_ℓ} in Table 4, respectively.

According to the latest GOCE-GRACE solution GOCO01S [17], the formal, statistical uncertainties are, $\sigma_{J_2} = 9.8 \times 10^{-13}, \sigma_{J_4} = 2.4 \times 10^{-13}, \sigma_{J_6} = 1.4 \times 10^{-13}$ respectively. Thus, the resulting range and range-rate mismodeled signals are as large as 1960 μm and 7.8×10^{-4} μm s^{-1} (σ_{J_2}), 720 μm and 1.2×10^{-2} μm s^{-1} (σ_{J_4}), 245 $= \mu$m and 3×10^{-2} μm s^{-1} (σ_{J_6}). They are smaller than the corresponding general relativistic Schwarzschild effects, especially as far as the range-rate is concerned ($\Delta\rho_{\sigma_{J_\ell}}/\Delta\rho_{\text{Sch}} \sim 10^{-4} - 10^{-3}$, $\Delta\dot{\rho}_{\sigma_{J_\ell}}/\Delta\dot{\rho}_{\text{Sch}} \sim 10^{-6} - 10^{-5}, \ell = 2, 4, 6$). Concerning the Lense-Thirring effect, the mismodelled even zonals ranges are of the same order of magnitude or smaller than the gravitomagnetic range shifts by one order of magnitude ($\Delta\rho_{\sigma_{J_\ell}}/\Delta\rho_{\text{LT}} \sim 10^{-1} - 1$), while the range-rate ones are larger by about two orders of magnitude. However, it must be considered that, given a specific Earth's gravity field model, the realistic errors in its estimated even zonals may be up

to one order of magnitude larger. Moreover, an even more conservative approach to realistically evaluate the true uncertainties in the even zonals consists of comparing their estimated values from different global gravity field solutions.

Conversely, we can use Figure 1 and Figure 2 to obtain preliminary, quantitative evaluations of a possible a-priori "imprinting" of GTR itself in the even zonals considered. It can be done by posing

$$\Delta x_{\text{GTR}} = \Delta x_{J_\ell}, \ x = \rho, \dot{\rho}. \quad (1)$$

Thus, by dividing the relativistic range and range-rate perturbations by the corresponding classical ones for each degree ℓ considered gives us a sort of "effective" relativistic even zonal $\Delta J_\ell^{(\text{eff})}$, i.e. the part of the even zonal of degree ℓ which would give a signal as large as those due to GTR. The Schwarzschild range perturbation yields $\Delta J_2^{(\text{Sch})} = 2.5 \times 10^{-10}, \Delta J_4^{(\text{Sch})} = 1.7 \times 10^{-10}, \Delta J_6^{(\text{Sch})} = 2.8 \times 10^{-10}$; given the present-day level of accuracy in determining the even zonals in the GRACE-based models, it appears that the "imprint" of the Schwarzschild perturbation cannot be neglected. Concerning the Lense-Thirring range effect, its a-priori im-

pact on the even zonals is rather small, amounting to $\Delta J_2^{(LT)} = 1 \times 10^{-12}, \Delta J_4^{(LT)} = 7 \times 10^{-13}, \Delta J_6^{(LT)} = 1 \times 10^{-12}$. It barely falls within the (formal) measurability domain of the most recent GRACE-based models. Table 5 summarizes the results for the range. The situation for the range-rate, depicted in Table 6, is $\Delta J_2^{(Sch)} = 7.5 \times 10^{-7}, \Delta J_4^{(Sch)} = 1.2 \times 10^{-8}, \Delta J_6^{(Sch)} = 3 \times 10^{-9}$, and $\Delta J_2^{(LT)} = 1 \times 10^{-12}, \Delta J_4^{(LT)} = 1.6 \times 10^{-14}, \Delta J_6^{(LT)} = 4 \times 10^{-15}$. In this case, the a-priori "imprinting" of the Schwarzschild perturbation on the even zonals is larger than for the range, while the bias due to the Lense-Thirring effect is practically not detectable, even for the most recent ones. Following the approach of Ref. [16], it can be shown that a Lense-Thirring "imprint" in J_4 and J_6 as large as that of Table 5 would correspond to a $0.2 - 0.3\%$ systematic bias in the LAGEOS-based tests[5] of such a relativistic effect.

In Figure 3 we repeat the same analysis for the first three odd zonals; see also Table 3. Since the formal, statistical errors are $\sigma_{J_3} = 7 \times 10^{-13}, \sigma_{J_5} = 2 \times 10^{-13}, \sigma_{J_7} = 1 \times 10^{-13}$, respectively [17], the mismodeled signals amount to 4900 μm, 5.6 μm s^{-1} (σ_{J_3}), 1600 μm, 1.8 μm s^{-1} (σ_{J_5}), and 800 μm, 0.8 μm s^{-1} (σ_{J_7}). Although their time signature is different, the corrupting impact of the odd zonals on the GTR is larger than that of the even zonals. Indeed, for the Schwarzschild perturbation we have $\Delta\rho_{\sigma_{J_\ell}}/\Delta\rho_{Sch} \sim 0.2 - 1 \times 10^{-2}$, $\Delta\dot\rho_{\sigma_{J_\ell}}/\Delta\dot\rho_{Sch} \sim 1 - 9 \times 10^{-3}, \ell = 3, 5, 7$, while for the Lense-Thirring effect the situation is worse since $\Delta\rho_{\sigma_{J_\ell}}/\Delta\rho_{LT} \sim 0.4 - 2.45$, $\Delta\dot\rho_{\sigma_{J_\ell}}/\Delta\dot\rho_{LT} \sim 1 -$

---

[5]They are mainly affected by such even zonals, being the other ones of higher degree negligible. This is a reason for which we do not consider other even zonals in the present study.

$7 \times 10^3, \ell = 3, 5, 7$. The possible "imprinting" of GTR on J_3, J_5 and J_7 amounts to $\Delta J_3^{(Sch)} = 7 \times 10^{-11}, \Delta J_5^{(Sch)} = 6 \times 10^{-11}, \Delta J_7^{(Sch)} = 6 \times 10^{-11}$, and $\Delta J_3^{(LT)} = 3 \times 10^{-13}, \Delta J_5^{(LT)} = 2.5 \times 10^{-13}, \Delta J_7^{(LT)} = 2.5 \times 10^{-13}$ for the range, as resumed in Table 5. The potential "imprinting" of GTR from the range-rate summarized in Table 6, is $\Delta J_3^{(Sch)} = 7.5 \times 10^{-11}, \Delta J_5^{(Sch)} = 7 \times 10^{-11}, \Delta J_7^{(Sch)} = 7.5 \times 10^{-11}$, and $\Delta J_3^{(LT)} = 1 \times 10^{-16}, \Delta J_5^{(LT)} = 9 \times 10^{-17}, \Delta J_7^{(LT)} = 1 \times 10^{-16}$. Thus, also for the odd zonals, the a-priori potential impact of the Schwarzschild perturbations to the range and the range-rate is not negligible.

4   Summary and conclusions

Given the present-day high level of accuracy in measuring the GRACE satellite-to-satellite range ($\sigma_\rho \lesssim 10$ μm) and range-rate ($\sigma_{\dot\rho} \lesssim 1$ μm s^{-1}), we preliminarily investigated the possibility of measuring the dynamical orbital effects of GTR, in both its Schwarzschild and Lense-Thirring components, on such directly observable quantities. A further motivation is given by the currently ongoing efforts to design a follow-on of GRACE accurate to nm s^{-1} or better. Moreover, it must be recalled that GTR has never been solved-for in all the GRACE-based Earth's global gravity field solutions produced so far, so that the multipoles of the terrestrial gravitational field estimated in them may, in principle, retain an a-priori "imprinting" of GTR itself. This is important in view of the fact that such Earth's gravity field solutions have been used as background reference models in some tests of GTR itself performed with other satellites.

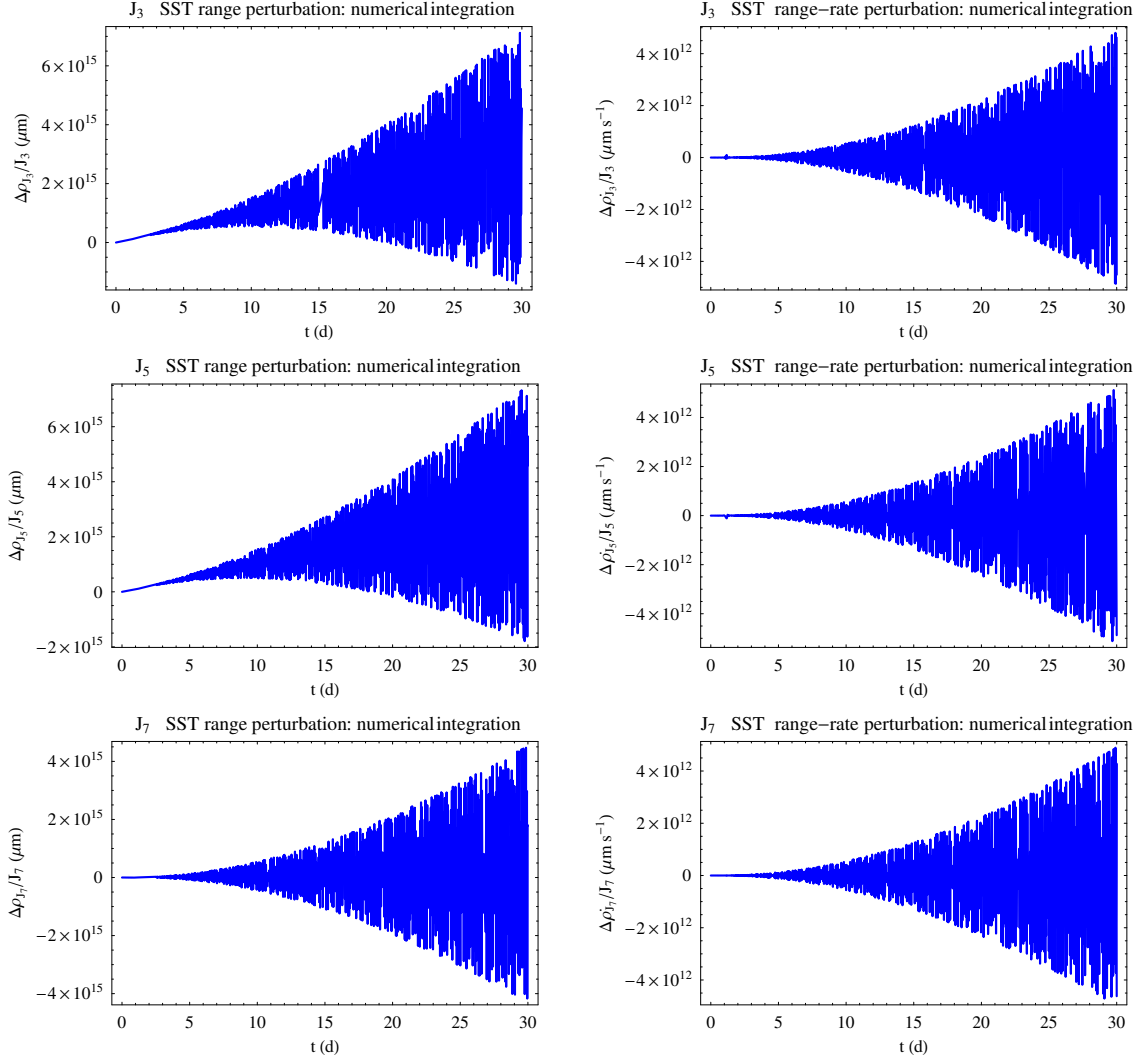By numerically integrating the GRACE A/B

Figure 3: Differences of the numerically integrated ranges (left column) and range-rates (right column) for GRACE A/B with and without the classical dynamical perturbations due to the odd zonal harmonics $J_\ell \doteq -\sqrt{2\ell+1}\ \overline{C}_{\ell,0}$, $\ell = 3, 5, 7$. For a better comparison with the relativistic signatures of Figure 1, the values of the odd zonals have been set equal to unity, i.e. $\Delta\rho_{J_\ell}$ and $\Delta\dot{\rho}_{J_\ell}$ have been divided by J_ℓ, $\ell = 3, 5, 7$. According to the recent combined GOCE-GRACE solution GOCO01S [17], the formal, statistical uncertainties in the normalized Stokes coefficients are, $\sigma_{\overline{C}_{3,0}} = 0.27 \times 10^{-12}$, $\sigma_{\overline{C}_{5,0}} = 0.6 \times 10^{-13}$, $\sigma_{\overline{C}_{7,0}} = 0.4 \times 10^{-13}$ respectively. The initial conditions, quoted in Table 1, are common to both the perturbed and un-perturbed integrations. The time span is $\Delta t = 30$ d. The units are μm (range) and μm s^{-1} (range-rate).

Table 5: "Effective" general relativistic parts $\Delta J_\ell^{(\text{eff})} = 1, \ell = 2, 3, 4, 5, 6, 7$ of the zonals considered due to the range perturbations $\Delta\rho$. They are a measure of the possible a-priori "imprinting" of GTR itself, not explicitly solved-for in all the GRACE-based solutions produced so far, on the zonals.

Dynamical effect	$\Delta J_2^{(\text{eff})}$	$\Delta J_3^{(\text{eff})}$	$\Delta J_4^{(\text{eff})}$	$\Delta J_5^{(\text{eff})}$	$\Delta J_6^{(\text{eff})}$	$\Delta J_7^{(\text{eff})}$
Schwarzschild	2.5×10^{-10}	7×10^{-11}	1.7×10^{-10}	6×10^{-11}	2.8×10^{-10}	6×10^{-11}
Lense-Thirring	1×10^{-12}	3×10^{-13}	7×10^{-13}	2.5×10^{-13}	1×10^{-12}	2.5×10^{-13}

Table 6: "Effective" general relativistic parts $\Delta J_\ell^{(\text{eff})} = 1, \ell = 2, 3, 4, 5, 6, 7$ of the zonals considered due to the range-rate perturbations $\Delta\dot{\rho}$. They are a measure of the possible a-priori "imprinting" of GTR itself, not explicitly solved-for in all the GRACE-based solutions produced so far, on the zonals.

Dynamical effect	$\Delta J_2^{(\text{eff})}$	$\Delta J_3^{(\text{eff})}$	$\Delta J_4^{(\text{eff})}$	$\Delta J_5^{(\text{eff})}$	$\Delta J_6^{(\text{eff})}$	$\Delta J_7^{(\text{eff})}$
Schwarzschild	7.5×10^{-7}	7.5×10^{-11}	1.2×10^{-8}	7×10^{-11}	3×10^{-9}	7.5×10^{-11}
Lense-Thirring	1×10^{-12}	1×10^{-16}	1.6×10^{-14}	9×10^{-17}	4×10^{-15}	1×10^{-16}

equations of motion over a time span 30 d long we found that the GTR range signals are as large as 50 cm (Schwarzschild) and 0.2 cm (Lense-Thirring), while the magnitude of the range-rate effects are 0.6 mm s^{-1} (Schwarzschild) and 0.8 nm s^{-1} (Lense-Thirring). If, on the one hand, they would be, in principle, measurable (apart from the Lense-Thirring range-rate effect) with a good level of accuracy, on the other hand the imperfect knowledge of some low-degree zonal harmonics of the geopotential causes competing range and range-rate perturbations which would corrupt the recovery of the relativistic signals of interest. According the latest GOCE/GRACE-based model, the mismodelled signatures of the even zonals are smaller than the Schwarzschild range and range-rate ones by several orders of magnitude, but are about of the same order of magnitude, or smaller by one order of magnitude, than the Lense-Thirring range perturbation. The impact of the odd zonals is larger, causing mismodelled range signals overwhelming the Lense-Thirring one. Conversely, by comparing the relativistic and the zonals range and range-rate orbital effects it has been possible to quantitatively assess the level of a possible a-priori "imprinting" of GTR itself on the zonals considered. It turned out that it is not negligible as far as the Schwarzschild component is considered, while the Lense-Thirring "imprint" on some zonals is just at the edge of the present-day level of accuracy in determining them.

As a caveat concerning the measurability of the GTR effects considered here, we stress the

need of checking with extensive numerical simulations if the relativistic signatures are not actually absorbed and removed from the range signal in estimating some of the various range parameters which are solved-for in the usual GRACE data processing.

Finally, let us note that the approach presented here can, in principle, also be extended to other satellite-to-satellite orbital configurations suitably designed to enhance the relativistic signatures, and to the dynamical effects caused by various modified models of gravity.

References

[1] B.D. Tapley, and Ch. Reigber, "The GRACE mission: status and future plans." EOS Trans. AGU 82 (47), Fall Meet. Suppl. G41 C-02, 2001.

[2] B.D. Tapley, S. Bettadpur, M. Watkins, and Ch. Reigber, "The gravity recovery and climate experiment: mission overview and early results", *Geophys. Res. Lett.* **31**(9), L09607 2004.

[3] R. Schmidt, F. Flechtner, U. Meyer, K.-H. Neumayer, C. Dahle, R. König, and J. Kusche, "Hydrological signals observed by the GRACE satellites", *Surv. Geophys.* **29**(4-5), 319334, 2008.

[4] Ch. Reigber, R. Schmidt, F. Flechtner, R. König, U. Meyer, K.-H. Neumayer, P. Schwintzer, and S.Y. Zhu, "An Earth gravity field model complete to degree and order 150 from GRACE: EIGEN-GRACE02S", *J. Geodyn.* **39**(1), 1-10, 2005.

[5] J. Kim, P. Roesset, S. Bettadpur, B. Tapley, and M. Watkins, "Error analysis of the Gravity Recovery and Climate Experiment (GRACE)", in: (Ed.) M. Sideris, IAG Symposium Series 123, p. 103108, Springer-Verlag Berlin Heidelberg, 2001.

[6] Z. Wei, H. Hou-Tse, Z. Min, and Y. Mei-Juan, "Accurate and rapid error estimation on global gravitational field from current GRACE and future GRACE Follow-On missions", *Chinese Physics B* **18**(8), 3597-3604, 2009.

[7] B. Loomis, R.S. Nerem, D. Rowlands, and S. Luthcke, "Performance Simulations for a GRACE Follow-On Mission Using a Masscon Approach", American Geophysical Union, Fall Meeting 2006, abstract #G13A-0024, 2006.

[8] J. Lense, and H. Thirring, "Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie," *Physikalische Zeitschrift,* **19**, 156-163, 1918.

[9] C. Chicone, and B. Mashhoon, "Tidal Dynamics in Kerr Spacetime", *Class. Quantum Gravit.* **23**(12), 4021-4033, 2006.

[10] K.M. Larson, N. Ashby, C. Hackman, and W. Bertiger, "An assessment of relativistic effects for low Earth orbiters: the GRACE satellites", *Metrologia* **44**(6), 484-490, 2007.

[11] I. Ciufolini, A. Paolozzi, E. C. Pavlis, J.C. Ries, R. König, R.A. Matzner, G. Sindoni and H. Neumayer, "Towards a One Percent Measurement of Frame Dragging by Spin with Satellite Laser Ranging to LAGEOS, LAGEOS 2 and LARES and GRACE Gravity Models," *Space Sci. Rev.*, **148**(1-4), 71-104, 2009.

[12] L. Iorio, H.I.M. Lichtenegger, M.L. Ruggiero, and C. Corda, "Phenomenology of the Lense-Thirring effect in the solar system", *Astrophys. Space Sci.* doi:10.1007/s10509-010-0489-5, 2010.

[13] I. Haranas, O. Ragos, and V. Mioc, "Yukawa-type potential effects in the anomalistic period of celestial bodies", *Astrophys. Space Sci.*, doi: 10.1007/s10509-010-0497-5, 2010.

[14] D.D. McCarthy, and G. Petit: IERS Conventions (2003), Frankfurt am Main: Verlag des Bundesamtes für Kartographie und Geodäsie, pp. 106, 2004.

[15] G. Balmino, R. Biancale, S. Bruinsma, J. Lemoine, F. Flechtner, R. Schmidt, and U. Meyer, "On Pertinent Use of GRACE K-Band Data", American Geophysical Union, Spring Meeting 2005, abstract #G13A-02

[16] L. Iorio, "On Possible A-Priori "Imprinting" of General Relativity Itself on the Performed Lense-Thirring Tests with LAGEOS Satellites", *Communications and Network* **2**(1), 26-30, 2010.

[17] R. Pail, H. Goiginger, W.-D. Schuh, E. Höck, J. M. Brockmann, T. Fecher, T. Gruber, T. Mayer-Gürr, J. Kusche, A. Jäggi, D. Rieser, "Combined satellite gravity field model *GOCO01S* derived from GOCE and GRACE", *Geophys. Res. Lett.* **37**, L20314, 2010.

[18] M. Capderou, Satellites. Orbits and missions. Springer-Verlag France, 2005.