# Higgs-induced spectroscopic shifts near strong gravity sources

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We explore the consequences of the mass generation due to the Higgs field in strong gravity astrophysical environments. The vacuum expectation value of the Higgs field is predicted to depend on the curvature of spacetime, potentially giving rise to peculiar spectroscopic shifts, named hereafter "Higgs shifts." Higgs shifts could be searched through dedicated multiwavelength and multispecies surveys with high spatial and spectral resolution near strong gravity sources such as Sagittarius A$^*$ or broad searches for signals due to primordial black holes. The possible absence of Higgs shifts in these surveys should provide limits to the coupling between the Higgs particle and the curvature of spacetime, a topic of interest for a recently proposed Higgs-driven inflationary model. We discuss some conceptual issues regarding the coexistence between the Higgs mechanism and gravity, especially for their different handling of fundamental and composite particles.

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## I. INTRODUCTION

One of the most important predictions of the standard model of particle physics is the existence of a scalar particle, called the Higgs particle, responsible for the spontaneous symmetry breaking of the electroweak sector, providing a dynamical mechanism to generate the mass of the intermediate vector bosons $W^{\pm}$ and $Z^0$ and of all fundamental fermionic matter fields [1]. The identification of the Higgs particle is considered an important milestone for the final validation of the standard model, and is the primary focus of research planned at the highest energy accelerators such as the Large Hadron Collider at CERN.

Although several implications of the Higgs coupling to fermions have been discussed in detail, little attention has been devoted so far, to our knowledge, to the fact that the Higgs particle should also play a crucial role in gravitational phenomena, provided that it satisfies the equivalence principle. If the Higgs field is coupled to the spacetime metric, its vacuum expectation value should differ from the one in a flat spacetime. As discussed in Sec. II (see also [2] for a preliminary account), different values for the mass of particles such as electrons and protons should then occur in the same region, with consequences for the energy levels of bound states of spectroscopic relevance. Peculiar "Higgs shifts" in the emission or absorption spectrum of atoms are expected, and it should be possible to distinguish them amidst the usual Doppler, gravitational, and cosmological shifts using multispecies spectroscopic analysis. In Sec. III we discuss in more detail what we consider the most promising cases for the observation of Higgs shifts from supermassive black holes near the Galactic center or primordial black holes − in particular, molecular tracers and neutral hydrogen in interstellar clouds and spectra of a star with high eccentricity, x-ray and $\gamma$-ray narrow lines. In the conclusions, we comment on general features of the Higgs-curvature connection, in particular potential instabilities due to the metric backreaction and the difference between Higgs physics and general relativity in dealing with the elementary or composite nature of particles.

## II. HIGGS FIELD IN CURVED SPACETIME

Quantum field theory in curved spacetime has been studied for several decades for both noninteracting and interacting fields (see [3] for an overview). The Lagrangian density for an interacting scalar field in a generic curved spacetime $g^{\mu\nu}$ is written as:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(\mu^2+\xi R)\phi^2 - \frac{\lambda}{4}\phi^4\right], \quad (1)$$

where $\mu$ and $\lambda$ are the mass parameter and the self-coupling quartic coefficient of the scalar field, respectively, $g$ is the determinant of the metric $g^{\mu\nu}$, and $\xi$ is a coefficient representing the coupling strength between the scalar field $\phi$ and the Ricci scalar $R$. This last coefficient is considered as a free parameter in all models analyzed so far, and only two prescriptions have been suggested on theoretical grounds. The so-called *minimal* coupling scenario simply assumes $\xi = 0$. This however is unnatural if the scalar field represents a Higgs field − leaving aside the doublet nature of the latter due to the $SU(2)_L \otimes U(1)_Y$ gauge symmetry which will be irrelevant in the following discussion. Indeed, if we believe that the standard model at some energy will merge with gravitation, we expect an interaction term between metric invariants and the scalar field. A minimal coupling instead minimizes the crosstalk between the standard model and the gravitational sectors, as in this case they will be only related via the metric tensor-scalar field kinetic term in Eq. (1). Moreover, this choice is not stable against quantum corrections [4], as confirmed by studying renormal-

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ization group features [3]. Proper renormalization behavior is instead fulfilled for a *conformal* coupling of $\xi = 1/6$, which has been shown to be a fixed point of the renormalization group equations [3, 5, 6]. It would be highly desirable to extract the Higgs-curvature coupling coefficient − or to obtain at least upper bounds − from the phenomenological analysis of particle observables in the presence of strong gravity, and from now on we discuss a possible scenario towards this direction.

In the spontaneously broken phase the Higgs field develops a vacuum expectation value $v_0 = (-\mu^2/\lambda)^{1/2}$ in flat spacetime, with the masses of the elementary particles directly proportional to $v_0$ via the Yukawa coefficients of the fermion-Higgs Lagrangian density term, $m_i = y_i v_0/\sqrt{2}$. In a curved spacetime instead, the effective coefficient of the Higgs field $\mu^2 \mapsto \mu^2 + \xi R$, and the vacuum expectation value of the Higgs field will become spacetime dependent through the curvature scalar as:

$$v = \sqrt{-\frac{\mu^2 + \xi R}{\lambda}} \simeq v_0 \left(1 + \frac{\xi R}{2\mu^2}\right), \qquad (2)$$

where the last expression holds in a weak-curvature limit.

In the case of an elementary particle, such as the electron, provided that the Yukawa couplings $y_i$ are constants yet to be determined − presumably from algebraic or group-theoretic arguments of an underlying fundamental theory embedding the standard model − the mass $m_i$ will be simply changed proportionally to the Higgs vacuum expectation value, so that

$$\delta m_i = \frac{y_i}{\sqrt{2}}(v - v_0) \simeq \frac{y_i \xi R v_0}{2^{3/2}\mu^2} = \frac{\xi R}{2\mu^2} m_i. \qquad (3)$$

The situation for composite particles such as protons and neutrons is more involved. We assume that their masses are made of a flavor-dependent contribution proportional to the masses of the three valence quarks determined by the Higgs coupling, and a color-symmetrical term only dependent on the quark-quark and quark-gluon interaction, *i.e.* proportional to the QCD constant $\Lambda_{QCD} \simeq 300$ MeV. The latter term dominates for lighter, relativistic quarks constituting the *valence* component of protons and neutrons. Then, due to the universality of the QCD coupling constant for different flavors and for all gluons exchange, we can parametrize the proton and neutron masses in terms of flavor-dependent and flavor-independent parts as:

$$\begin{aligned} m_p &= (2y_u + y_d)v/\sqrt{2} + m_{QCD}, \\ m_n &= (y_u + 2y_d)v/\sqrt{2} + m_{QCD}, \end{aligned} \qquad (4)$$

where $y_u$ and $y_d$ are the Yukawa couplings of the up and down quarks, and $m_{QCD}$ is a flavor-independent contribution related to the gluon binding energy, depending on $\Lambda_{QCD}$. For a generic atom of atomic number $Z$ and atomic mass $A$ we then obtain:

$$\begin{aligned} &M(A,Z) = Zm_p + (A-Z)m_n = \\ &\frac{1}{\sqrt{2}}[y_u(Z+A) + y_d(2A-Z)]v + Am_{QCD}, \end{aligned} \qquad (5)$$

where we have neglected to first approximation the contributions of the electron mass, the electron-nucleus binding energy, and the nucleon-nucleon binding energy. The purely QCD-dependent mass term should be independent on the curvature of spacetime, since otherwise the gluon could acquire a mass giving rise to the explicit breaking of the color symmetry. This is analogous to the case of the other unbroken symmetry of the standard model, $U(1)_{\mathrm{em}}$, which leads to the electric charge conservation even in a generic curved spacetime. By considering the Yukawa couplings $y_u$ and $y_d$ as determining the current quark masses $m_u$ and $m_d$ (with central values quoted in the Particle Data Group of 2.25 and 5 MeV, respectively), it is evident that for composite states of quarks such as protons and neutrons and their combinations, the flavor/Yukawa coupling independent term dominates, and the effect of curved spacetime is therefore strongly suppressed. Therefore, the possibility of detecting Higgs shifts in atomic and molecular spectroscopy relies on the fact that electronic transitions depend primarily on the mass of the electron, while molecular transitions due to vibrational or rotational degrees of freedom depend upon the mass of the nuclei. While the electron mass is directly proportional to the Yukawa coupling to the Higgs particle, the mass of the nuclei is mainly due to the contribution of its proton and neutron constituents, which in turn depends mainly on the color binding energy. We therefore expect that molecular transitions will not be affected by the Higgs shifts to leading order, unlike electronic transitions.

In the relevant example of atomic hydrogen spectroscopy, the spectral lines depend on the reduced mass $\mu_H = m_e m_p/(m_e + m_p)$ and ultimately, due to the large mass ratio $m_p/m_e$, on the electron mass. At the molecular level, unless electronic transitions are excited, the Higgs shift is shown to be negligible even in the most favorable case of pyramidal molecules such as ammonia, for which tunneling provides exponentially higher sensitivity to the change in masses of the atoms. In particular, in the case of the nitrogen atom constituting the ammonia molecule, we have:

$$M_{\mathrm{N}} = \frac{21}{\sqrt{2}}(y_u + y_d)v + 14 m_{QCD}, \qquad (6)$$

with the effective mass for the inversion spectrum of ammonia equal to $\mu_{\mathrm{NH}_3} = 3M_{\mathrm{H}}M_{\mathrm{N}}/(3M_{\mathrm{H}} + M_{\mathrm{N}})$. For Yukawa couplings of $y_e = 2.89 \times 10^{-6}$, $y_u = 1.27 \times 10^{-5}$, $y_d = 2.83 \times 10^{-5}$, and a pure gluonic contribution of $m_{QCD} = 928$ MeV, mass shifts of $\delta\mu_{\mathrm{H}}/\mu_{\mathrm{H}} = 4 \times 10^{-3}$ for hydrogen and $\delta\mu_{\mathrm{NH}_3}/\mu_{\mathrm{NH}_3} = 3.4 \times 10^{-5}$ for ammonia are obtained for a variation of $\delta v = 1$ GeV around $v_0=$ 250 GeV ($\delta v/v_0 = 4 \times 10^{-3}$). Therefore, it is clear that, even if the ammonia inversion spectrum is in principle more sensitive (by a factor $\simeq 4 \div 5$ as discussed in [7]) to the masses of its constituents than spectra from other molecular and nonpyramidal species, under the hypothesis that $m_{QCD}$ does not couple to the Higgs vacuum its sensitivity does not match the one of atomic hydrogen.

## III. ASTROPHYSICAL CONSIDERATIONS

We now discuss qualitatively the possibility of observing Higgs shifts from astrophysical objects. This implies a number of restrictive hypotheses both on the gravitational sources and their coupling to the Higgs particle, and on the detectability of the Higgs shift amidst other sources of wavelength shift. As remarked above, it is important to detect both spectroscopic lines due to electronic transitions and nuclear (vibrational or rotational) transitions. This is difficult to achieve in the same region of space from the same species for a gas at thermal equilibrium, due to the large energy scale difference required for effectively producing these excitations. A comparative analysis of wavelength shifts from different species seems then necessary. This should allow for discrimination from the Doppler shift and the purely gravitational shift. The Doppler shift should be the same for molecules belonging to the same comoving cloud, while the wavelength shift expected from general relativity will act universally on all particles, so unlike the Higgs shift it will not distinguish between fundamental particles and interactions binding energies.

A further difficulty is that the Ricci scalar $R$ is zero in the case of symmetrical gravitational sources, which are described by the Schwarzschild or the Kerr metric. We will then make the hypothesis that the Higgs field couples to another scalar invariant, for instance the Kretschmann invariant defined as $K_1 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, where $R^{\mu\nu\rho\sigma}$ is the Riemann curvature tensor. This invariant plays an important role in quadratic theories of gravity [8–10] and more in general in modified $f(R)$ theories [11]. In the case of the Schwarzschild metric the Kretschmann invariant is $K_1 = 12R_s^2/r^6$, with $R_s$ the Schwarzschild radius $R_s = 2GM/c^2$, and $r$ the distance from the center of the mass $M$. If we replace the Higgs-Ricci curvature coupling term $\xi\phi^2 R/2$ in Eq. (1) with a Higgs-Kretschmann coupling term we obtain the modified Lagrangian density

$$\mathcal{L}_K = \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(\mu^2+\xi' K_1^{1/2})\phi^2 - \frac{\lambda}{4}\phi^4\right], \tag{7}$$

in which the curvature-scalar interaction term appears proportional to $K_1^{1/2}$ for dimensional reasons. This could appear problematic in regions of weak spacetime curvature where $K_1 \to 0$ since divergencies may occur, but in the context of spacetime regions analyzed here this coupling may be considered as arising from an effective interaction Lagrangian valid for strong and static gravitational fields. In this case the mass term $\mu^2$ maps onto

$$\mu^2 \mapsto \mu^2(1 + 2\sqrt{3}\xi' R_s \lambda_\mu^2/r^3), \tag{8}$$

where, in preparation for concrete estimates, we have introduced the Compton wavelength associated to the Higgs mass parameter $\mu$ as $\lambda_\mu = \hbar/(\mu c)$. Assuming a Higgs mass of 160 GeV and a vacuum expectation value of $v_0$=250 GeV, we obtain a Compton wavelength for the Higgs mass parameter $\lambda_\mu \simeq 2\times 10^{-18}$ m: this is the length scale with which the Kretschmann invariant has to be confronted in any astrophysical setting.

If we imagine collecting electromagnetic signals emitted from the innermost stable orbit of a Schwarzschild black hole, assuming that the Kretschmann invariant does not perturb significantly the stability analysis of black holes, we obtain for $r = 3R_s$, $K_1^{1/2} = (4/243)^{1/2}R_s^{-2}$. The frequency shift is therefore inversely proportional to the square of the Schwarzschild radius, and it gets larger by considering rotating black holes due to the smaller innermost stable orbits allowed in the Kerr metric [12]. For supermassive black holes such as the one located in our Galaxy, Sagittarius A$^*$ with an estimated mass of $M \simeq 2.6\times 10^6$ solar masses [13–17], the Schwarzschild radius is equal to $R_s =\simeq 8\times 10^9$ m. For a solar mass black hole we obtain $K^{1/2} = 1.5\times 10^{-8}$ m$^{-2}$. In the two cases the product $\lambda_\mu^2 K_1^{1/2}$ is $\simeq 8\times 10^{-57}$ and $\simeq 6\times 10^{-44}$ respectively, leading to tiny Higgs shifts quite far from what can be achieved with any foreseeable survey unless quite large values of the Higgs-curvature coupling parameter $\xi'$ are allowed. If we consider mini black holes with a mass of the order of $10^{11}$ kg, which should survive evaporation via quantum tunneling [18–20], we obtain $R_s \simeq 10^{-16}$ m, and $\delta m_i/m_i \simeq 2\times 10^{-5}\xi' \simeq \delta\nu/\nu$. Recent surveys of molecular clouds, for instance containing ammonia [21, 22], have a spectral sensitivity corresponding to a Doppler shift of about 2-3 km/s, *i.e.* $\delta\nu/\nu \simeq 10^{-5}$, comparable to the expected estimates based on mini black holes. If the same spectral sensitivity could be maintained in a broad survey of other spectroscopic transitions, upper limits of the order of $\xi' \simeq 1$ could be achieved.

In spite of the pessimistic estimates reported above, it may be worthwhile to perform surveys near the Galactic center, especially keeping in mind the absolute lack of information on the Higgs-Kretschmann coupling $\xi'$. With a 1 pc resolution survey one should be able to obtain spectra of atoms or molecules at a distance of $r \simeq 2\times 10^{16}$ m from the Galactic center. While detailed surveys of the Galactic center have been performed for various molecular species such as for instance $NH_3$ [23–27], CO [28], $H_2CO$ [29, 30], and multispecies [31–33], observation of atomic lines from the same region is difficult due to the strong absorption at optical wavelengths. This issue may be circumvented by focusing on high-precision observations of the 21 cm line of neutral hydrogen which still depends on the electron-to-proton mass ratio. A further refinement on this proposal is obtained by monitoring neutral hydrogen surrounding stars with highly eccentric orbits around Sagittarius A$^*$. This should provide clearer signatures, especially in regard to a possible temporal variability of the 21 cm line related to the proximity of the star to the source of strong gravity.

The presence of spectroscopic shifts related to the Higgs field could also be investigated in high-energy astrophysics phenomena. For instance, there should be a further contribution in the redshift of the $K_\alpha$ emission line from ionized iron of stars orbiting in proximity of the source of spacetime curvature [34]. Another possibility is the detection of shifts in the annihilation spectrum near the Galactic center. Recent surveys have been performed with an energy resolution $\Delta E/E = 1.47 \times 10^{-4}$ at the positron annihilation peak [35]. In this case it is crucial to achieve a high angular resolution of the detector, since the putative shifted signal from the Galactic center will be otherwise smeared out by the nearby unshifted contributions. The intrinsic resolution of the 511 keV peak is limited by the environmental temperature around the Galactic center, estimated to be $T \leq 5 \times 10^4$ K [36], which leads to a relative energy spread at the annihilation peak of $K_B T/E_\gamma \simeq 10^{-5}$. With a measured positronium fraction close to unity ($0.93 \pm 0.04$ from [37]) and in the presence of neutral H or $H_2$ gases, the influence of external magnetic or electric fields on the annihilation spectrum should be minimized. A comparative analysis between signals for electron-positron and proton-antiproton annihilation from strong gravity sources using telescopes with both large energy and angular resolution (with Fermi/GLAST being the best candidate available now for the hadronic annihilation channel), might allow a detailed test of the presence of Higgs shifts.

## IV. CONCLUSIONS

We have discussed the interplay between the Higgs particle and the curvature of spacetime and the possibility of observing peculiar spectroscopic shifts from strong gravity astrophysical sources. Some final comments are in order. Although the discussion relies on strong gravity being associated to a nonzero Ricci scalar, or coupling through the Kretschmann invariant, the main message discussed in this note is to search for frequency shifts which discriminate between transitions associated to electronic or baryonic states. While we have focused on sources of astrophysical interest, similar considerations could be extended in a cosmological framework, for instance by looking at the presence of specific frequency shifts in high redshifts systems such as the quasar emission or absorption spectra. This could proceed along parallel lines, providing alternative interpretations to the already developed analysis of the possible time dependence of the proton-electron mass ratio [7, 38], acquiring information about the time evolution of the Higgs field. Based on the alternative assumption that quasars redshifts are not necessarily of cosmological origin, the possibility that quasars are naked singularities [39–41] with strong gravitational redshifts possibly containing also a Higgs component should be left open as a possibility. Astrophysical limits arising from the analysis as suggested here are critical to test proposals that rely on having the Higgs boson being responsible for the inflationary model, as discussed in [42], especially considering the large curvature-Higgs coupling (order of $\xi \simeq 10^4$) required in this scenario.

Furthermore, it is worth noticing that for the conformal coupling the vacuum expectation value is increased in a curved spacetime corresponding to positive ($R > 0$ or $K_1 > 0$) scalar invariants. Conceptually, the presence of a positive feedback on the Higgs expectation value due to a finite curvature may lead to gravitational instabilities. If we consider a test mass located near a source of curved spacetime, due to the Higgs field its mass will increase with respect to the flat spacetime, consequently increasing the local curvature, which in turn will increase the value of the test mass. In principle, this positive feedback mechanism could generate a conceptual issue for the coexistence of general relativity and Higgs couplings, at least in its nonminimal version. Alternatively, if the feedback turns out to be negative, an oscillatory behavior for the spacetime metric is expected, leading to a Higgs-driven mechanism for the emission of gravitational radiation, with potential implications on the spectrum of primordial density fluctuations imprinted in the temperature anisotropies of the cosmic microwave background. A general analysis on a scalar field with polynomial potential terms up to the fourth order has been carried out in [43], implying $\xi \leq 0$ or $\xi \geq 1/6$ for a stable Higgs field.

Lastly, we want to point out that in the standard model the mass of fundamental particles have a different treatment as compared to the mass of composite particles. Assuming validity of the equivalence principle − an assumption which will be analyzed in detail in a future contribution − the gravitational mass of the electrons constituting a test body will change if the Higgs field is coupled to curvature, while the nucleons will continue to keep, at leading order, the usual gravitational charge. This is in striking contrast with the standard general relativity scenario, whereby all sources of energy contribute without any distinctive feature. In turn, this originates an unappealing contrast in dealing with the masses, based on their classification as fundamental or composite − a classification which has been proven to change in time as further layers of elementary particles have emerged.

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