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Imprints of Anisotropic Inflation on the CMB

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ABSTRACT
We study the imprints of anisotropic inflation on the CMB temperature fluctuations and polarizations. The statistical anisotropy stems not only from the direction dependence of curvature and tensor perturbations, but also from the cross correlation between curvature and tensor perturbations, and the linear polarization of tensor perturbations. We show that off-diagonal TB and EB spectrum as well as on- and off-diagonal TT, EE, BB, TE spectrum are induced from anisotropic inflation. We emphasize that the off-diagonal spectrum induced by the cross correlation could be a characteristic signature of anisotropic inflation.

Key words: cosmology: inflation – cosmic background radiation

1 INTRODUCTION

Precise observations of the cosmic microwave background radiation (CMB) enable us to test the fundamental predictions of inflation on primordial fluctuations such as scale independence and Gaussianity. The statistical isotropy has been a robust prediction protected by the cosmic no-hair conjecture which claims that the inflation wash out classical anisotropy. Recently, however, its apparent violation has been reported and its origins including systematic effects have been widely discussed.(Armendariz-Picon 2006; Pullen & Kamionkowski 2007; Dvorkin et al. 2008; Hanson & Lewis 2009; Armendariz-Picon & Pekowsky 2009; Groeneboom et al. 2010; Hanson et al. 2010; Pontzen & Peiris 2010; Bennett et al. 2010)

The statistical anisotropy in primordial power spectrum of curvature fluctuations has been discussed so far. While, tensor perturbations (primordial gravitational waves) have not been taken into account. However, since the statistical anisotropy could be a relic of the breaking the rotational symmetry in the early universe, we can naturally expect that there exists cross correlation between curvature and tensor fluctuations since they used to interact with each other at that time. Its effects on the CMB would be relevant. In fact, an anisotropic inflation proposed by us exhibits such features (Watanabe et al. 2009).

We can test the statistical anisotropy in tensor perturbations with CMB B-mode polarization including cross correlation with temperature fluctuation and E-mode polarization. In the conventional cosmology, B-mode polarization is supposed to have no two-point cross correlation with temperature or E-mode. This results from the two assumptions: statistical isotropy and parity invariance. The former one obliges the correlations to be diagonal $\langle a^X_{lm} a^{X'}_{l'm'}\rangle = C^{XX'}_l \delta_{ll'}\delta_{mm'}$, while $a^{T,E}_{lm}$ and a^B_{lm} have parity $(-1)^l$ and $(-1)^{l+1}$ respectively, hence correlations $C^{TB,EB}_l$ have odd parity and vanish due to the latter assumption. In the presence of statistical anisotropy, off-diagonal $(l \neq l')$ correlations arise and therefore TB, EB correlations can appear even if the parity is conserved. These signals may be useful to distinguish the origins of apparent violation of statistical isotropy.

In this paper, we discuss statistical anisotropy from anisotropic inflation including tensor perturbations and show how they are imprinted in the two-point correlations of the CMB temperature fluctuation and polarizations. Then we compare signals of the primordial anisotropy induced by tensor perturbations with that induced purely by scalar perturbations, and see what we can expect as a signal peculiar to anisotropic inflation. In the next section, we define the statistical anisotropy which we deal with. In section 3, angular power spectrum are evaluated. The final section is devoted to the discussion.

2 PRIMORDIAL STATISTICAL ANISOTROPY

In the anisotropic inflation model, we have four kind of anisotropy: (i) direction dependence in primordial power spectrum of scalar (curvature) perturbations, (ii) that in tensor perturbations, (iii) cross correlation between curvature perturbations and a linear polarization mode of tensor perturbations, and (iv) linear polarization of tensor perturbations.

It is convenient to express the primordial power spectra in the following way:

$$\langle R^s(\mathbf{k}) R^{s'}(\mathbf{k}')^*\rangle = P^{ss'}(\mathbf{k})\delta^3(\mathbf{k}-\mathbf{k}'), \qquad (1)$$

where R^s denotes primordial scalar perturbations for R^0 and right- and left-handed circular polarizations of tensor perturbations for R^{+2} and R^{-2} respectively, and the delta function results from an assumption of statistical homogeneity (translational invariance). In the conventional isotropic inflation, where rotational invariance is conserved, $P^{ss'}(\mathbf{k})$ is proportional to $\delta_{ss'}$ and is a function dependent only on $|s|$ and $|\mathbf{k}|$. When we consider (iii) cross correlation and (iv) linear polarization, we need to take into account off-diagonal ($s \neq s'$) components.

Note that, unlike diagonal ones, these components change their values with the rotation of the polarization bases, hence the bases have to be specified. The simplest choice is to make use of spherical coordinates with a certain fixed preferred direction, i.e. $\sqrt{2}e^{\pm 2}_{ij} = e^{\rm d}_{ij} \pm i e^{\times}_{ij}$, $\sqrt{2}e^{\rm d}_{ij} = \mathbf{e}^{\theta}_i\mathbf{e}^{\theta}_j - \mathbf{e}^{\phi}_i\mathbf{e}^{\phi}_j$, $\sqrt{2}e^{\times}_{ij} = \mathbf{e}^{\theta}_i\mathbf{e}^{\phi}_j + \mathbf{e}^{\phi}_i\mathbf{e}^{\theta}_j$, here θ and ϕ are polar and azimuthal angles. This convention is adopted throughout this paper. The power spectra in linear polarization modes $\sqrt{2}R^{\rm d} = R^{+2} + R^{-2}$, $\sqrt{2}R^{\times} = i(R^{+2} - R^{-2})$ are similarly defined by $\langle R^{\alpha}(\mathbf{k})R^{\beta *}(\mathbf{k}')\rangle = P^{\alpha\beta}(\mathbf{k})\delta^3(\mathbf{k}-\mathbf{k}')$, where α, β denote $0, {\rm d}$ or $\times$. Then, the power spectra of helicity bases are expressed by those of linear bases as

$$\begin{aligned} P^{0\pm 2} &= (P^{0\rm d} \pm iP^{0\times})/\sqrt{2},\ P^{\pm 20} = (P^{\rm d0} \mp iP^{\times 0})/\sqrt{2}, \\ P^{+2+2} &= P^{-2-2} = (P^{\rm dd} + P^{\times\times})/2 \equiv P_t^{\rm unp} \\ P^{+2-2} &= P^{-2+2} = (P^{\rm dd} - P^{\times\times})/2 \equiv P_t^{\rm pol}, \end{aligned} \quad (2)$$

We hereafter neglect circular polarization of the tensor modes in this study.

We consider anisotropic inflation models we proposed in Watanabe et al. (2009). This model assumes a vector field coupled to the inflaton field ϕ through a kinetic term of the form $\mathcal{L}_{\rm vec} = -1/4 f(\phi)^2 F_{\mu\nu}F^{\mu\nu}$. The anisotropic inflation predicts the following modification to primordial power spectra (Dulaney & Gresham 2010; Gumrukcuoglu et al. 2010; Watanabe et al. 2010):

i) $P^{00}(\mathbf{k}) = P_{\rm s}(k)[1 + g\sin^2\theta]$,

ii) $P_t^{\rm unp}(\mathbf{k}) = P_{\rm t}(k)[1 + g_h \sin^2\theta]$,

iii) $P_t^{\rm pol}(\mathbf{k}) = P_{\rm t}(k) g_l \sin^4\theta$,

iv) $P^{0\rm d}(\mathbf{k}) = P^{\rm d0}(\mathbf{k}) = \sqrt{P_{\rm s}(k)P_{\rm t}(k)}\, g_c \sin^2\theta$, (3)

where $P_{\rm s}(k), P_{\rm t}(k)$ are isotropic parts of scalar and tensor power spectra, the θ is the angle between $\hat{\mathbf{k}}$ and a certain privileged direction, and $k \equiv |\mathbf{k}|$. For simplicity, here we neglected the scale dependence of g, g_h, g_l, g_c which is not significant for the scales relevant to the CMB. The model predicts the consistency relation $g_h = \frac{1}{4}\epsilon g$, $g_c = \sqrt{\epsilon} g$, where ϵ is a slow-roll parameter, in addition to the usual relation for tensor-to-scalar ratio $r = 2P_{\rm t}(k)/P_{\rm s}(k) = 16\epsilon$, and the linear polarization is relatively small $g_l \sim \mathcal{O}(g_h^2)$ and there is no cross correlation between scalar perturbations and cross mode tensor perturbations $P^{\times 0} = P^{0\times} = 0$.

3 ANGULAR POWER SPECTRUM

In this section, we evaluate the following angular power spectrum:

$$C^{XX'}_{ll'mm'} \equiv \langle a^X_{lm} a^{X'*}_{l'm'}\rangle,$$

$$a^X_{lm}(\eta_o, \mathbf{x}_o) = \int d\Omega_{\hat{\mathbf{p}}} X(\hat{\mathbf{p}}, \eta_o, \mathbf{x}_o) Y^*_{lm}(\hat{\mathbf{p}}). \quad (4)$$

where $*$ denotes complex conjugate, Y_{lm} is a spherical harmonics, and $X = T, E, B$ designates fluctuations in temperature and polarization modes of the CMB.

This fluctuation X can be associated to the primordial fluctuations in the following way:

$$\begin{aligned} X(\hat{\mathbf{p}}, \eta_o, \mathbf{x}_o) &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_L \sum_{s=-2}^{2} R^s(\mathbf{k}) Y_{Ls}(\hat{\mathbf{p}}; \hat{\mathbf{k}}) \\ &\quad \times \Delta^X_{Ls}(k, \eta_o) e^{i\mathbf{k}\cdot\mathbf{x}_o}. \end{aligned} \quad (5)$$

where $s = 0, \pm 2$ denotes contributions from 3d-scalar, tensor mode respectively. The vector modes are hereafter neglected for simplicity. Note that $Y_{Ls}(\hat{\mathbf{p}}; \hat{\mathbf{k}})$ explicitly indicates that the polar angle of the spherical harmonics is measured from the direction $\hat{\mathbf{k}}$ while the azimuthal angle is assumed to be defined by the polarization bases of R^s. And the transfer functions Δ satisfy the relations

$$\Delta^M_{L,-s} = \Delta^M_{L,s},\ \Delta^E_{L,-s} = \Delta^E_{L,s},\ \Delta^B_{L,-s} = -\Delta^B_{L,s}. \quad (6)$$

Substituting Eq.(5) into Eq.(4) and using the following formula

$$Y_{lm}(\hat{\mathbf{p}}; \hat{\mathbf{k}}) = \sqrt{\frac{4\pi}{2l+1}} \sum_{m'} Y_{lm'}(\hat{\mathbf{p}}; \mathbf{e})\ {}_{-m}Y^*_{lm'}(\hat{\mathbf{k}}; \mathbf{e}), \quad (7)$$

we have

$$\begin{aligned} C^{XX'}_{ll'mm'} &= \int \frac{k^2 dk}{(2\pi)^6} \sum_{s,s'} \tilde{\Delta}^X_{l,s}(k,\eta_o) \tilde{\Delta}^{X'}_{l',s'}(k,\eta_o) \\ &\quad \times \int d\Omega_{\hat{\mathbf{k}}} P^{ss'}(\mathbf{k})\ {}_{-s}Y^*_{lm}(\hat{\mathbf{k}})\ {}_{-s'}Y_{l'm'}(\hat{\mathbf{k}}), \end{aligned} \quad (8)$$

where we defined $\tilde{\Delta}^X_{l,s}(k,\eta_o) \equiv \sqrt{\frac{4\pi}{2l+1}} \Delta^X_{l,s}(k,\eta_o)$. Then $P^{ss'}$ is associated to the linear polarizations and scalar-tensor correlation via Eq.(2) and the property of transfer function Eq.(6) helps to simplify the expression. We see that in the conventional cosmology the assumption $P^{ss'}(\mathbf{k}) = \delta_{ss'} P^{|s|}(k)$ (i.e. statistically isotropic, no cross correlation between scalar and tensor, no circular polarization), together with the relation $\Delta^B_{l,-s} = -\Delta^B_{l,s}$ results in $C^{TB}_{ll'mm'} = C^{EB}_{ll'mm'} = 0$

Now, we can see the imprints of statistical anisotropy on the CMB.

3.1 (i) anisotropy in scalar perturbations

We can calculate TT,TE,EE spectrum in the following way:

$$\begin{aligned} C^{XX'({\rm i})}_{ll'mm'} &= \int \frac{k^2 dk}{(2\pi)^6} \tilde{\Delta}^X_{l,0} \tilde{\Delta}^{X'}_{l',0} I^{({\rm i})}_{ll'mm'}, \\ I^{({\rm i})}_{ll'mm'} &\equiv \int d\Omega_{\hat{\mathbf{k}}} P^{00}(\mathbf{k})\ Y^*_{lm}(\hat{\mathbf{k}})\ Y_{l'm'}(\hat{\mathbf{k}}), \end{aligned} \quad (9)$$

This can be evaluated by expanding the spectrum into spherical harmonics $P^{00}(\mathbf{k}) = \sum_{LM} a^{00}_{LM}(k) Y_{LM}(\hat{\mathbf{k}})$ where $a^{00}_{LM}(k) = 0$ for odd L. Then using the relation

$$\int d\Omega\ Y_{LM}\ {}_{-s}Y^*_{lm}\ {}_{-s}Y_{l'm'}$$

$$= \sqrt{\frac{(2L+1)(2l'+1)}{4\pi(2l+1)}}\mathcal{C}^{lm}_{LMl'm'}\mathcal{C}^{ls}_{L0l's}, \quad (10)$$

we have

$$I^{(\mathrm{i})}_{ll'mm'} = \sum_{LM} a^{00}_{LM}\sqrt{\frac{(2L+1)(2l'+1)}{4\pi(2l+1)}}\mathcal{C}^{lm}_{LMl'm'}\mathcal{C}^{l0}_{L0l'0} . \quad (11)$$

As L is even, this is only non-zero for even $l-l'$. The statistical anisotropy is characterized by the component $a^{00}_{20} = -\frac{4}{3}\sqrt{\frac{\pi}{5}}gP_s(k)$ and hence we have

$$I^{(\mathrm{i})}_{ll'mm'} = -\frac{2}{3}gP_s(k)\sqrt{\frac{2l'+1}{2l+1}}\mathcal{C}^{lm}_{20l'm'}\mathcal{C}^{l0}_{20l'0} , \quad (12)$$

where $\mathcal{C}$ denotes Clebsch-Gordan coefficient. This has m dependent contribution for $l'=l, l\pm 2$. The correlation is proportional to $\delta_{mm'}$ due to the axisymmetry of the system and the fact that we have chosen a specific coordinate in decomposing into spherical harmonic coefficients. For the general coordinate system, modes with different m are cross correlated. Similar analyses are made in Ackerman et al. (2007); Gumrukcuoglu et al. (2007)

3.2 (ii) anisotropy in tensor perturbations

The induced spectrum is

$$\begin{aligned} C^{XX'(\mathrm{ii})}_{ll'mm'} &= \int \frac{k^2dk}{(2\pi)^6}\tilde{\Delta}^X_{l,2}\tilde{\Delta}^{X'}_{l',2}I^{(\mathrm{ii})\pm}_{ll'mm'}, \\ I^{(\mathrm{ii})\pm}_{ll'mm'} &= \int d\Omega_{\hat{\mathbf{k}}}P^{\mathrm{unp}}_t(\mathbf{k})\Big({}_{-2}Y^*_{lm}(\hat{\mathbf{k}})\ {}_{-2}Y_{l'm'}(\hat{\mathbf{k}}) \\ &\qquad \pm\ {}_{+2}Y^*_{lm}(\hat{\mathbf{k}})\ {}_{+2}Y_{l'm'}(\hat{\mathbf{k}})\Big) , \end{aligned} \quad (13)$$

where the upper / lower sign appears in TT, EE, TE, BB / TB, EB correlations. We decompose it as: $P^{\mathrm{unp}}(\mathbf{k}) = \sum_{LM} a^{\mathrm{unp}}_{LM}(k)Y_{LM}(\hat{\mathbf{k}})$ where $a^{\mathrm{unp}}_{LM}(k) = 0$ for odd L. Then, we have

$$\begin{aligned} I^{(\mathrm{ii})\pm}_{ll'mm'} &= \sum_{LM} a^{\mathrm{unp}}_{LM}\sqrt{\frac{(2L+1)(2l'+1)}{4\pi(2l+1)}} \\ &\qquad \times\mathcal{C}^{lm}_{LMl'm'}\left(\mathcal{C}^{l2}_{L0l'2} \pm \mathcal{C}^{l(-2)}_{L0l'(-2)}\right) . \end{aligned} \quad (14)$$

From the symmetry of Clebsch-Gordan coefficient $\mathcal{C}^{c\gamma}_{a\alpha b\beta} = (-1)^{a+b-c}\mathcal{C}^{c-\gamma}_{a-\alpha b-\beta}$, we have following relations: $I^{(\mathrm{ii})+}_{ll'mm'} = 0$ for odd $l-l'$, and $I^{(\mathrm{ii})-}_{ll'mm'} = 0$ for even $l-l'$. This is a manifestation of parity symmetry of the system. In the case of anisotropic inflation, we have a component $a^{\mathrm{unp}}_{20} = -\frac{4}{3}\sqrt{\frac{\pi}{5}}g_hP_t(k)$, which give rise to non-zero TT, EE, BB, TE correlations for $l'=l, l\pm 2$ and TB, EB correlations for $l' = l\pm 1$.

3.3 (iii) cross correlation

We have TT,EE,TE spectrum induced by the cross correlation of scalar perturbations and plus mode tensor perturbations:

$$\begin{aligned} C^{XX'(\mathrm{iii})}_{ll'mm'} &= \frac{1}{\sqrt{2}}\int\frac{k^2dk}{(2\pi)^6}\Big[\tilde{\Delta}^X_{l0}\tilde{\Delta}^{X'}_{l'2}I^{(\mathrm{iii})+}_{ll'mm'} \\ &\qquad +(-1)^{l+l'+m+m'}\tilde{\Delta}^X_{l2}\tilde{\Delta}^{X'}_{l'0}I^{(\mathrm{iii})+}_{l',l,-m',-m}\Big], \end{aligned} \quad (15)$$

and TB,EB spectrum:

$$C^{XB(\mathrm{iii})}_{ll'mm'} = \frac{1}{\sqrt{2}}\int\frac{k^2dk}{(2\pi)^6}\tilde{\Delta}^X_{l0}\tilde{\Delta}^B_{l'2}I^{(\mathrm{iii})-}_{ll'mm'} . \quad (16)$$

Here, we have defined

$$\begin{aligned} I^{(\mathrm{iii})\pm}_{ll'mm'} &= \int d\Omega_{\hat{\mathbf{k}}}P^{0\mathrm{d}}(\mathbf{k})\ Y^*_{lm}(\hat{\mathbf{k}}) \\ &\qquad \times\left({}_{-2}Y_{l'm'}(\hat{\mathbf{k}}) \pm {}_{+2}\ Y_{l'm'}(\hat{\mathbf{k}})\right) , \end{aligned} \quad (17)$$

To derive these relations we used the property $P^{d0}(\mathbf{k}) = P^{0d}(-\mathbf{k})$ and ${}_sY^*_{lm}(\mathbf{k}) = (-1)^{l+m}\ {}_{-s}Y_{l,-m}(-\mathbf{k})$.

The direction dependence of cross correlation is $\sin^2\theta$. Hence, TT,EE,TE spectrum can be evaluated as:

$$\begin{aligned} I^{(\mathrm{iii})+}_{ll'mm'} &= 2\Big(\alpha^{-2}_{l+2,m}\delta_{l',l+2} + \alpha^0_{l,m}\delta_{l',l} + \alpha^{+2}_{l-2,m}\delta_{l',l-2}\Big) \\ &\qquad \times\delta_{mm'}g_c\sqrt{P_s(k)P_t(k)}, \\ I^{(\mathrm{iii})-}_{ll'mm'} &= 2\Big(\beta^{-1}_{l+1,m}\delta_{l',l+1} + \beta^{+1}_{l-1,m}\delta_{l',l-1}\Big) \\ &\qquad \times\delta_{mm'}g_c\sqrt{P_s(k)P_t(k)} , \end{aligned} \quad (18)$$

where the coefficients are given by:

$$\begin{aligned} \alpha^{+2}_{l,m} &\equiv \sqrt{\frac{l(l-1)(l+m+1)(l-m+1)(l+m+2)(l-m+2)}{(l+1)(l+2)(2l+1)(2l+3)^2(2l+5)}} , \\ \alpha^{-2}_{l,m} &\equiv \sqrt{\frac{(l+1)(l+2)(l+m)(l-m)(l+m-1)(l-m-1)}{(l-1)l(2l-3)(2l-1)^2(2l+1)}} , \\ \alpha^0_{l,m} &\equiv \frac{2\left\{3m^2 - l(l+1)\right\}}{(2l-1)(2l+3)}\sqrt{\frac{(l-1)(l+2)}{l(l+1)}} , \\ \beta^{+1}_{l,m} &\equiv 2m\sqrt{\frac{(l-1)(l+m+1)(l-m+1)}{l(l+1)(l+2)(2l+1)(2l+3)}} , \\ \beta^{-1}_{l,m} &\equiv -2m\sqrt{\frac{(l+2)(l+m)(l-m)}{(l-1)l(l+1)(2l+1)(2l-1)}} . \end{aligned} \quad (19)$$

The derivation is given in Appendix A.

3.4 (iv)linear polarization

For this case, we have

$$\begin{aligned} C^{XX'(\mathrm{iv})}_{ll'mm'} &= \int\frac{k^2dk}{(2\pi)^6}\tilde{\Delta}^X_{l,2}\tilde{\Delta}^{X'}_{l',2}I^{(\mathrm{iv})\pm}_{ll'mm'}, \\ I^{(\mathrm{iv})\pm}_{ll'mm'} &= \int d\Omega_{\hat{\mathbf{k}}}P^{\mathrm{pol}}_t(\mathbf{k})\ \Big({}_{-2}Y^*_{lm}(\hat{\mathbf{k}})_{+2}Y_{l'm'}(\hat{\mathbf{k}}) \\ &\qquad \pm_{+2}Y^*_{lm}(\hat{\mathbf{k}})_{-2}Y_{l'm'}(\hat{\mathbf{k}})\Big) , \end{aligned} \quad (20)$$

where upper and lower sign correspond to TT, EE, BB, TE and TB, EB spectrum, respectively. In the case of anisotropic inflation, where direction dependence is proportional to $\sin^4\theta$ we can evaluate them as:

$$\begin{aligned} &\frac{1}{2}I^{(\mathrm{iv})+}_{ll'mm'} \\ &= \Big[\alpha^{+2}_{lm}\alpha^{-2}_{l+4,m}\delta_{l',l+4} \\ &+ \left(\alpha^{+2}_{lm}\alpha^0_{l+2,m} + \alpha^0_{l,m}\alpha^{-2}_{l+2,m} - \beta^{+1}_{lm}\beta^{-1}_{l+2,m}\right)\delta_{l',l+2} \\ &+ \left((\alpha^{+2}_{lm})^2 + (\alpha^0_{lm})^2 + (\alpha^{-2}_{lm})^2 - (\beta^{-1}_{lm})^2 - (\beta^{+1}_{lm})^2\right)\delta_{l'l} \\ &+ \left(\alpha^0_{lm}\alpha^{+2}_{l-2,m} + \alpha^{-2}_{lm}\alpha^0_{l-2,m} - \beta^{-1}_{lm}\beta^{+1}_{l-2,m}\right)\delta_{l',l-2} \end{aligned}$$

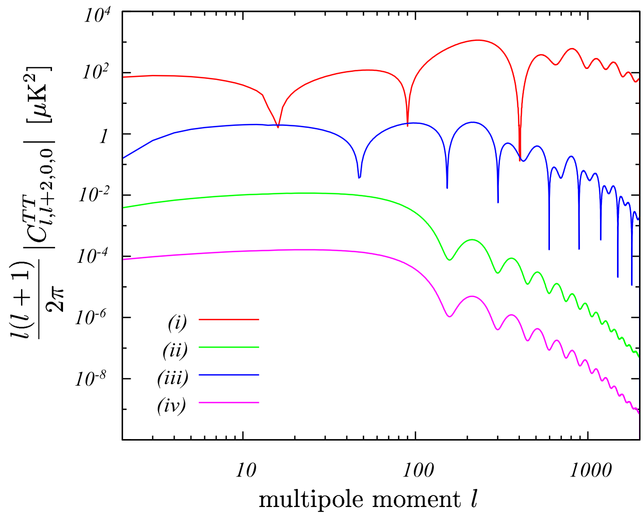

Figure 1. The TT spectra induced by (i) anisotropy in scalar perturbations, (ii) that in tensor perturbations, (iii) cross correlation, and (iv) linear polarization of tensor perturbations. The parameters are chosen as $g = 0.3,\ r = 0.3$.

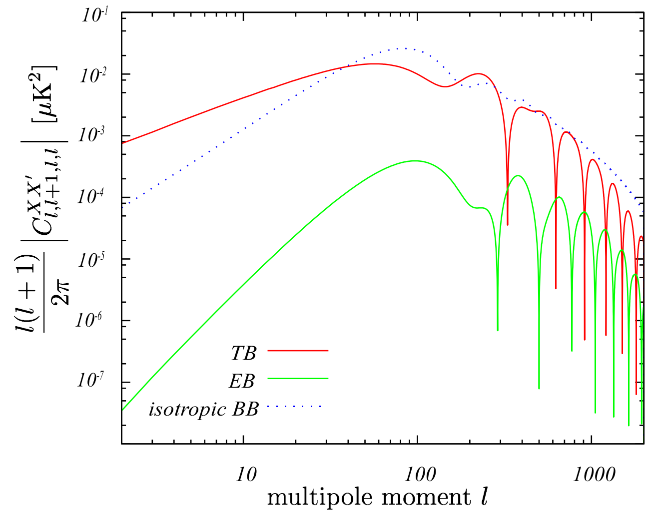

Figure 2. The TB and EB spectrum induced by (iii) cross correlation. As a reference of magnitude, the conventional BB spectrum $l(l+1)C^{BB}_{llmm}/2\pi$ induced by isotropic part of the tensor perturbations is plotted with a dotted line. The parameters are chosen as $g = 0.3,\ r = 0.3$.

$$+\alpha^{-2}_{lm}\alpha^{+2}_{l-4,m}\delta_{l',l-4}\Big]\delta_{mm'}g_lP_t(k) \qquad (21)$$

and

$$\frac{1}{2}I^{(\mathrm{iv})-}_{ll'mm'}$$
$$=\Big[(\beta^{+1}_{lm}\alpha^{-2}_{l+3,m}-\alpha^{+2}_{lm}\beta^{-1}_{l+3,m})\delta_{l',l+3}$$
$$+(\beta^{+1}_{lm}\alpha^{0}_{l+1,m}+\beta^{-1}_{lm}\alpha^{-2}_{l+1,m}-\alpha^{+2}_{lm}\beta^{+1}_{l+1,m}$$
$$-\alpha^{0}_{lm}\beta^{-1}_{l+1,m})\delta_{l',l+1}$$
$$+(\beta^{+1}_{lm}\alpha^{+2}_{l-1,m}+\beta^{-1}_{lm}\alpha^{0}_{l-1,m}-\alpha^{0}_{lm}\beta^{+1}_{l-1,m}$$
$$-\alpha^{-2}_{lm}\beta^{-1}_{l-1,m})\delta_{l',l-1}$$
$$+(\beta^{-1}_{lm}\alpha^{+2}_{l-3,m}-\alpha^{-2}_{lm}\beta^{+1}_{l-3,m})\delta_{l',l-3}\Big]\delta_{mm'}g_lP_t(k). \qquad (22)$$

4 DISCUSSION

In the previous section, we have shown that anisotropy related to tensor perturbations generally induces off-diagonal TB, EB spectra as well as on- and off-diagonal TT, EE, BB, TE spectrum. Here we discuss its significance.

First, we compare the amplitude of signals induced by the four components of anisotropy. In Fig 1, we have depicted contributions of each component to an off-diagonal TT correlation $\frac{l(l+1)}{2\pi}|C^{TT}_{l,l+2,0,0}|$. Here we chose the anisotropy in scalar perturbations $g = 0.3$ which is the magnitude reported as a tentative detection in Groeneboom et al. (2010) in addition to the tensor-to-scalar ratio of $r = 0.3$. The other quantities are determined by the consistency relations of this model: $r = 16\epsilon,\ g_h = \frac{1}{4}\epsilon g,\ g_c = \sqrt{\epsilon}g,\ g_l \sim g_h^2$.

We see that the contributions of tensor perturbations (ii),(iii) and (iv) are suppressed in comparison to that of (i) anisotropy in scalar perturbations and that (iii) cross correlation has the largest contribution next to (i). This reflects the hierarchy of $rg_h = \mathcal{O}(g\ \epsilon^2), \sqrt{r}g_c = \mathcal{O}(g\ \epsilon), rg_l = \mathcal{O}(g^2\ \epsilon^3)$ and is also true of EE and TE spectra. The ratio of (i), (ii) and (iii) are given by the slow-roll parameter ϵ (or tensor-to-scalar ratio r) and are not dependent on the value of g.

Next we consider peculiar signals of anisotropic inflation. The components (ii), (iii), (iv) induce B mode polarization, and its largest correlation is induced by (iii) cross correlation. In Fig 2, we have depicted examples of TB and EB correlations $\frac{l(l+1)}{2\pi}|C^{TB}_{l,l+1,l,l}|, \frac{l(l+1)}{2\pi}|C^{EB}_{l,l+1,l,l}|$. The parameters are again $r = 0.3, g = 0.3$. As a reference of magnitude, the conventional BB spectrum induced by the isotropic part of tensor perturbations $\frac{l(l+1)}{2\pi}C^{BB}_{llmm}$ (independent of m) is also plotted with a dotted line.

The ratio of the isotropic BB correlation and TB correlation induced by cross correlation is not dependent on ϵ (or r) for a fixed value of g in this model of anisotropic inflation, and for $g \sim 0.3$ their amplitudes become comparable. Although the detailed detectability needs to be discussed, off-diagonal TB correlation can be a potential target of future CMB observations. This will be a smoking-gun signal of existence of cross correlation and anisotropic inflation.

It should be emphasized that both figures vanish for statistically isotropic fluctuations.

The correlation induced by (iv) linear polarization of tensor perturbations is highly suppressed and lacks any distinctive signature in contrast to circular polarization of tensor perturbations, which predicts odd-parity correlations (Lue et al. 1999; Saito et al. 2007) in the CMB.

The predicted correlations are m dependent (coordinate dependent) and hence simple summation over m doesn't make sense in constructing the observables which presents the characteristics of statistical anisotropy. Instead bipolar power spectrum (Hajian & Souradeep 2003) can be extended to include these signature and used as a spectroscopic tool to distinguish it from other origins of (apparent) statistical anisotropy.

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APPENDIX A: DEFINITION OF $\alpha_{LM}^{\pm 2,0}, \beta_{LM}^{\pm 1}$

In this appendix we present a tool for evaluating $I_{ll'mm'}^{(\mathrm{iii})\pm}$ and $I_{ll'mm'}^{(\mathrm{iv})\pm}$ in a case of anisotropic inflation, where the direction dependence in cross correlation and linear polarization are given by $P^{0\mathrm{d}} \propto \sin^2\theta$, $P_t^{\mathrm{pol}} \propto \sin^4\theta$. First, spin weighted spherical harmonics is associated to spherical harmonics as:

$$\begin{aligned} {}_{\pm 2}Y_{lm} &= (\hat{A} \pm i\hat{B})Y_{lm} \\ \hat{A} &= \frac{-\cos\theta\sin\theta\partial_\theta + \sin^2\theta\partial_\theta^2 - \partial_\phi^2}{\sin^2\theta\sqrt{(l-1)l(l+1)(l+2)}}, \\ \hat{B} &= \frac{2(\sin\theta\partial_\theta - \cos\theta)\partial_\phi}{\sin^2\theta\sqrt{(l-1)l(l+1)(l+2)}}. \end{aligned} \tag{A1}$$

Using $\partial_\phi Y_{lm} = imY_{lm}$ and the recursion relations:

$$\begin{aligned} \sin\theta\partial_\theta Y_{lm} &= l\sqrt{\frac{(l+1)^2 - m^2}{(2l+1)(2l+3)}}Y_{l+1,m} \\ &\quad -(l+1)\sqrt{\frac{l^2-m^2}{(2l+1)(2l-1)}}Y_{l-1,m}, \\ \cos\theta Y_{lm} &= \sqrt{\frac{(l+1)^2 - m^2}{(2l+1)(2l+3)}}Y_{l+1,m} \\ &\quad +\sqrt{\frac{l^2-m^2}{(2l+1)(2l-1)}}Y_{l-1,m}, \end{aligned}$$

we obtain the relations:

$$\begin{aligned} \sin^2\theta\hat{A}Y_{lm} &= \alpha_{lm}^{+2}Y_{l+2,m} + \alpha_{lm}^{0}Y_{lm} + \alpha_{lm}^{-2}Y_{l-2,m}, \\ \sin^2\theta\hat{B}Y_{lm} &= i\beta_{lm}^{+1}Y_{l+1,m} + i\beta_{lm}^{-1}Y_{l-1,m}. \end{aligned} \tag{A2}$$

The coefficients α, β are given in Eq.(19). Then the orthonormality of spherical harmonics leads to Eq.(18) and Eq.(22).

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