Active and Passive Elec. Comp., 1988, Vol. 13, pp. 113–132
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Printed in Great Britain

DISTORTION ANALYSIS OF ACTIVE FILTERS

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(Received January 8, 1987, in final form November 13, 1987)

Second order active filters using single and two operational amplifiers at high frequencies and/or large-signal levels are investigated for their nonlinear performance. A general procedure using the Volterra series in presented regarding intermodulation distortion. Computer results by implementation in a program "Distortion Analysis" demonstrate the accuracy of the analysis technique.

INTRODUCTION

Analysis of the distortion of a filter is very important to improving its performance. In this paper a software package has been developed that measures the distortion at various desired frequencies of an active filter in tabular and are graphical form upon inputting parameters of this filter.

The analysis procedure used in this paper is based upon the Volterra series, described in section 2. If an operational amplifier is overdriven by a large input and/or high frequencies, the output slews at some limiting rate called slew rate. The nonlinear effect gives rise to distortion.-The study of this distortion and a model of an operational amplifiex using the distortion generator approach are discussed in section 3. Section 4 describes this procedure. Software implementation of this procedure has been dealt in section 5. Finally some conclusions have been drawn in section 6.

VOLTERRA SERIES REPRESENTATION

The Volterra series is a generalization of the convolution integral used to describe linear systems and is ideal for representing small non-linearities (3). If a non-linear system is time invariant, the relation between its input X(t) and output y(t) according to this representation is:

$$Y(t) = \int_{-\infty}^{+\infty} h_1(T_1)X(t - T_1)dT_1 + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_2(T_1, T_2)x(t - T_1)x(t - T_2)dT_1dT_2 + \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} h_n(T_1, T_2, \ldots, T_n)x(t - T_1)x(t - T_2) \ldots x(t - T_n)dT_1\, dT_2 \ldots d_T + \ldots \quad (1)$$

For n = 1, 2, ... $h_n(T_1, T_2, \ldots, T_n) = 0$ for any $T_j < 0$, j = 1, 2, ..., n $h_n(T_1, T_2, \ldots, T_n)$ is called nth order "Volterra Kernel".

The relation (1) can be written in compact from as:

$$y(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} h_n(T_1, T_2 \ldots, T_n) \prod_{k=1}^{n} x(t - T_k)dt_k \quad (2)$$

The nth fold transform of the nth order Volterra Kernel is defined by:

$$K(s_1, s_2, \ldots, s_n) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} h_n(T_1, T_2, \ldots, T_n) \cdot \exp[-(s_1, T_1 + s_2T_2 + \ldots + s_nT_n)]\, dT_1DT_2 \ldots dT_n \quad (3)$$

Where $K(s_1, s_2, \ldots, s_n)$ may be thought as the nth order transfer function. The physical interpretation can be given as follows. When n sinusodial signals of complex frequencies $s_1, s_2, \ldots, s_n$ of unit amplitude are applied to the filter, $K(s_1, s_2, \ldots, s_n)$ gives the magnitude of the nth order product signals at frequency $(s_1, s_2, \ldots, s_n)$.

Substituting expression (3) into eq. (2) and taking the Laplace transform we obtain:

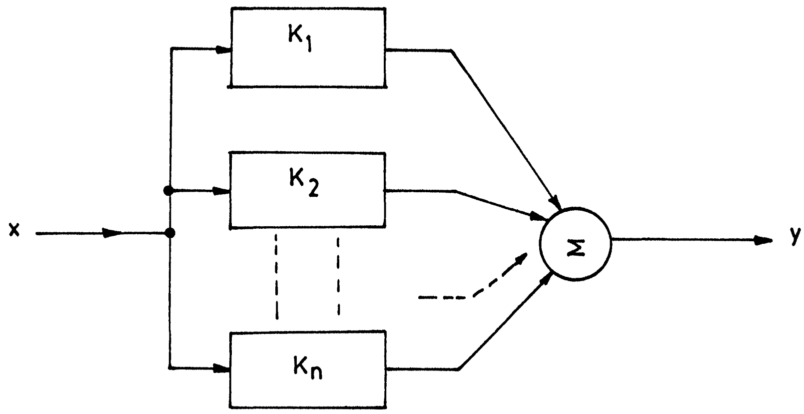

FIGURE 1 Volterra system representation

$$Y(s_1, s_2, \ldots, s_n) = \sum_{k=1}^{n} K(s_1, s_2, \ldots, s_k) \prod_{j=1}^{k} x(sj) \tag{4}$$

Here kernels up to nth order have been utilized to represent the system.

Thus, the Volterra model for non-linear systems could be represented by the model shown in Figure 1, where the non-linear system is decomposed to a linear system and a finite number of homogeneous non-linear subsystems are completely characterized by their nonlinear transfer functions.

DISTORTION MODEL OF OPERATIONAL AMPLIFIER

Slew rate, which is the time rate of change of the closed loop amplifier output voltage under large signal conditions, can cause serious degradation of the filter frequency response sometimes leading to instability-(4). In this section slew-induced distortion is studied and a distortion model of an operational amplifiex presented.

The first order model of an operational amplifier is given in Figure 2a. The instantaneous differential input voltage $v_i(t)$ is used to control a current source, with the current of this voltage-controlled current source given by the function $f[v_i(t)]$. As shown in

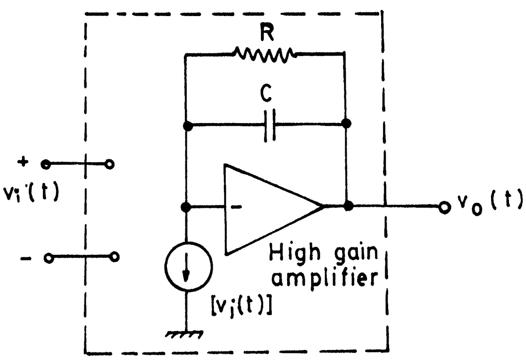

FIGURE 2(a) First-order model of an op amp.

Figure 2b, there is a region for small values of $v_i(t)$, in which $f[v_i(t)]$ is proportional to $v_i(t)$. However, for large values of $v_i(t)$ it saturates and becomes independent of $v_i(t)$. Then the output signal is distorted and in the limit approaches a triangle wave from where the slopes are equal to the slew rate of the op amp. Such distortions cannot be removed by the frequency response of filter and small-signal information is not passed by the filter.

There are two methods available to prevent slew-induced distortion (SID) from occuring:

(i) input to the filter can be clipped so as to keep the output slope of the op amp less than the slew rate. While this method allows larger signal levels at lower frequencies than amplitude limiting, it creates its own distortion in the form of higher harmonics.

(ii) An operational amplifier with a high slew rate, such as a biFET or CMOS type can be used if an operational amplifier is operating in the range of SID. Then its modelling using the Volterra series is difficult if exact methods are utilized, such as the "distortion generator" method (6). This method provides a sufficient accuracy in the estimation of small nonlinear distortions.

The relationship between input (v_i) and output (v_0) of an op amp possessing nonlinear and interial properties using eq. (4) is

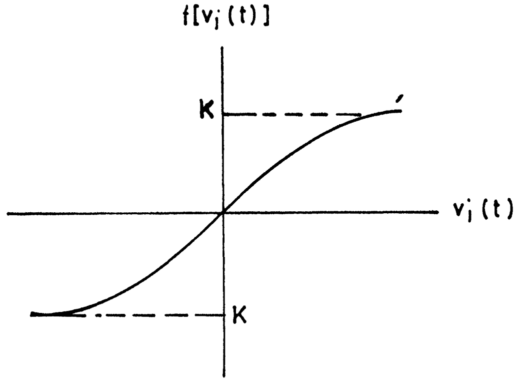

FIGURE 2(b) Dependence of the controlled current source of above circuit upon the differential input voltage v_i.

$$V_0(s_1, s_2, \ldots, s_n) = \sum_{k=1}^{n} K(s_1, s_2, \ldots, s_k) \prod_{j=1}^{k} v_i(s_j) \quad (5)$$

Transformation of eq. (5) by separating its linear part, gives

$$v_0(s_1, s_2, \ldots, s_n) = K(s_1)v_i(s_1) + \sum_{k=2}^{n} K(s_1, s_2, \ldots, s_n) \prod_{j=1}^{k} v_i(s_j) \quad (6)$$

Thus, the op amp can be represented as a linear voltage-controlled voltage source (VCVS) with transfer coefficient $K(s_1)$ and a distortion generator $\epsilon(s_1, s_2, \ldots, s_n)$ at the output as shown in Figure 3(9). the parameters of this generator are:

$$\epsilon(s_1, s_2, \ldots, s_n) = \sum_{k=2}^{n} K(s_1, s_2, \ldots s_k) \prod_{j=1}^{k} v_i(s_j) \quad (7)$$

For better accuracy a differential operational amplifier (DOA) model should be used DOA is regarded as a VCVS with added elements that indicate the finiteness of the input and output impedances. The model shown in Figure 4, reflect approximate properties of an actual DOA although and experimental method is preferable for determining the kernels of eq. (7). The DOA kernels obtained by this method are given below:

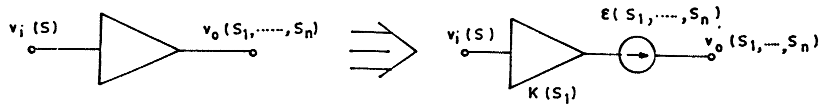

FIGURE 3(a) Operational amplifier with distortion.

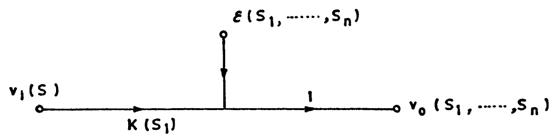

FIGURE 3(b) Signal flow-graph of above circuit.

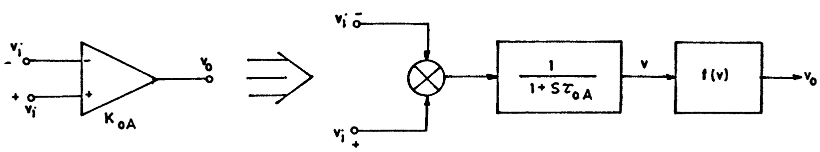

FIGURE 4 Equivalent circuit of operational amplifier based on DOA model.

$$K_{0_A}(s_1) = \frac{a_1}{1 + s_1 T_{0_A}}$$

$$K_{0_A}(s_1, s_2) = \frac{a_2}{(1 + s_1 T_{0_A})(1 + s_2 T_{0_A})} = 0 \quad (8)$$

$$K_{0_A}(s_1, s_2, s_3) = \frac{a_3}{\prod_{j=1}^{3} (1 + s_j T_{0_A})}$$

Here only three kernels have been used to characterize the op amp and the even order term is zero due to odd symetry characteristic in Figure 2b. This model gives accurate results for small nonlinearities.

ANALYSIS PROCEDURE

The analysis procedure adopted in this paper for any active filter is as follows:

(i) Identify various nodes of the circuit and draw a signal flow graph in such way that every node represents a voltage. If elements are given in the form of impedances, convert them into admittences.

(ii) Find the gain branch of the signal flow graph.

(iii) Find the voltage at VCVS input v_i^s.

(iv) Find the output voltage of the device excluding the distortion generators.

(v) Find the parameters of the distortion generator from equation 7 using equation 8.

(vi) Assuming the circuit to be linear, recompute the parameters of the distortion generators at the circuit output.

(vii) Find the total output signal by using equation 6.

(viii) From an analysis of the expression describing the filter output signal, kernels of the functions of Volterra series are determined. This series shows the relation between the input and output signals.

To illustrate the method, nonlinear properties of the bandpass filter shown in Figure 5a are investigated. The output impedance of the op amp (R_0) has also been considered.

(1) Various nodes are presented in Figure 5a, and a signal flow is drawn in Figure 5b.

Admittances of given impedances are:

$Y_1 = 1/R_1$
$Y_2 = 1/sC_2$
$Y_3 = 1/R_3$
$Y_4 = 1/sC_4$
$Y_5 = 1/R_5$
and $Y_6 = 1/R_0$

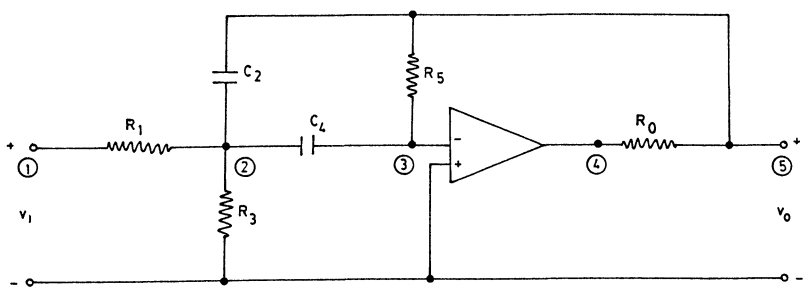

FIGURE 5(a) Band-pass filter.

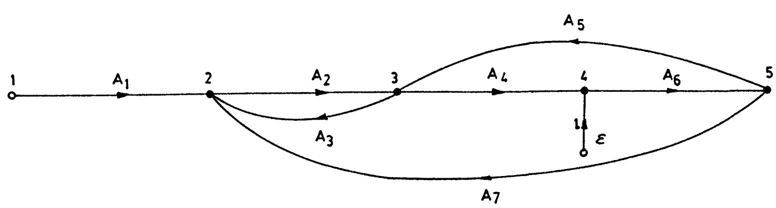

FIGURE 5(b) Signal flow graph of above filter.

(2) For finding the gain of a branch the following formula is used with fairly good (if not exact) results:

$$\text{Gain of a Branch} = \frac{\text{Value of admittance between two nodes of branch}}{\text{sum of the admittances at second node}} \tag{9}$$

Using this formula in the present analysis:

$$A_1 = \frac{Y_1}{Y_1 + Y_2 + Y_1 + Y_4}$$

$$A_2 = \frac{Y_4}{Y_4 + Y_5}$$

$$A_3 = \frac{Y_4}{Y_1 + Y_2 + Y_3 + Y_4}$$

$$A_4 = \frac{a_1}{1 + sT_{0_A}}$$

$$A_5 = \frac{Y_5}{Y_4 + Y_5}$$

$$A_6 = \frac{Y_0}{Y_1 + Y_2 + Y_3 + Y_4}$$

(3) The following general gain formula (2) is used to find the voltage as a node for a gain voltage at any other node of signal flow graph

$$M_{mn} = \sum_{k=1}^{N} \frac{M_k \Delta k}{\Delta} \tag{10}$$

where M_{mn} = gain between nodes m and n
N = total number of forward paths
M_k = gain of kth forward path
Δ_k = for that part of the signal flow graph which is nontouching with the kth forward path

$$\Delta = 1 - \sum_m P_{m1} + \sum_m P_{m2} + \ldots + (-1)^r \sum_m P_{mr} + \ldots \tag{11}$$

P_{mr} = gain product of the mth possible combination of all nontouching loops.

Two parts of a signal flow graph are said to be nontouching if they do not share a common node.

Using this formula the signal at the input of the op amp is:

$$V_3(s) = \frac{A_1A_2}{1 - A_2A_3 - A_4A_5A_6 - A_2A_4A_6A_7} v_1(s)$$
$$= T_{1_3}(s)v_1(s)$$

(4) Use of eq. (10) gives a linear part of the output signal as:

$$V_5(s) = \frac{A_1A_2A_4A_6}{1 - A_2A_3 - A_4A_5A_6 - A_2A_4A_6A_7} v_1(s)$$

$$= T_f(s)v_1(s)$$

(5) Parameters of the distortion generator for the present case using eq. (7)

$$\epsilon(s_1, s_2, \ldots, s_n) = \sum_{k=2}^{\infty} K_{0_A}(s_1, s_2, \ldots, s_n) \prod_{j=1}^{k} [-v_3(s_j)]$$

(6) The transfer function from the distortion generator to the filter output is (using eq. 10)

$$T_{\epsilon 5}(s) = \frac{A_6(1 - A_2A_3)}{1 - A_2A_3 - A_4A_5A_6 - A_2A_4A_6A_7}$$

(7) Recompute the signal from the distortion generator to the filter output and find the total output signal (using eq. 6.)

$$V_0(s_1, s_2, \ldots, s_n) = T_f(s)V_i(s) + \sum_{k=1}^{n} K_{0A}(s_1, s_2, \ldots, s_n) \times (-1)^k \prod_{j=1}^{n} [v_3(s_j) \times T_{\epsilon 5}(\epsilon s_j)] \qquad (12)$$

To this point all the values used on RHS of above equation have been determined so that output can be calculated.

(8) From the expression obtain in eq. 12, kernels of the functions are determined using eq. 6:

$$K_f(s_1) = T_f(s_1)$$

$$\vdots$$

$$K_f(s_1, s_2, \ldots, s_p) = K_{0_A}(s_1, s_2, \ldots, s_p)(-1)^p \prod_{j=1}^{p} [T_{13}(s_j)\, T_{\epsilon 5}(s_j)] \qquad (13)$$

Utilizing an expression for the complex amplitude of the first harminic, we can find the amplitude frequency response from the first harmonic:

$$|K_{eq}(iw)| = \left| K_f(i\omega) + \frac{3}{4} U_m^2 K_j(iw, iw, -iw) \right| \quad (14)$$

If measure of distortion of a particular order (e.g., 3) is of interest, it can be determined by calculating the third order intermodulation index, which is defined as (4):

$$IM_3 = 20 \log \left| \frac{K_3(s_1, s_1, -s_2)}{\prod_{j=1}^{3} K_1(s_j)} \right|$$

Similarly, other intermodulation indices can also be calculated if required.

SOFTWARE IMPLEMENTATION

The software package made using the algorithm developed in last section is given in Appendix 1. The program developed is quite general and can be used for any type of filter with the following restrictions:

(1) The amplitude of input harmonic signal should not be too large and it must satisfy the inequality:

$$u_m \leqslant 1.2\xi \left[\frac{|K_j(iw, iw, -iw)|}{|K_j(iw, iw, -iw, iw, -iw)|} \right]^{1/2}$$

where ξ is given order of smallness.

(2) The signal flow graph of filter should not have more than two nontouching loops.

Thus, for small nonlinearities and for not every complex circuits, this program can be used, to determine an amplitude frequency response.

The band pass filter, that was used to illustrate the procedure in last section is given here to also show performance of program. Input data i.e., parameters of the filter and op amp used are given in the data file in Appendix. The variation of amplitude with frequency is also given in Appendix 2.

From these results of this filter, the following conclusion can be drawn. The major contribution to the nonlinear distortion of the filter is made by the signals in the filter pass band.

CONCLUSIONS

Although alternative approaches were available for the problem, a distortion generator method is described in this paper because it simplifies the computations so that more complex circuits (difficult with other methods such as in Nasseret al's method (8)) can be analysed.

The importance of this work lies in the fact that nonidealities of active elements have a stronger effect than that of passive ones on the nonlinear properties of the filter (5). The program developed can be used to calculate the distortion in filter output due to nonidealities of active elements so that filter performance can be improved.

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APPENDIX-I LISTINGS OF PROGRAM "DISTORTION ANALYSIS"

List of Symbols used in Program "Distortion Analysis"

N	: Number of elements in network
NN	: Number of nodes in network
N1 & N2	: Node numbers
A	: Element in branch connecting nodes N1 and N2
N3	: Signifies type of element = 1 to indicate presence of resistance = 2 to indicate presence of capacitor = 3 to indicate presence of inductor = 4 to indicate presence of positive input to op amp = 5 to indicate presence of negative input to op amp
XK	: a_1 of equation (8)
T	: T_{0A} of equation (8)
A13	: a_3 of equation (8)
L1	: Lower frequency limit from which frequency response is required
L2	: Upper frequency limit upto which frequency response is required
L3	: Frequency interval at which $\|K_{eq}(iw)\|$ is to be calculated
U	: Input voltage level
I11	: Number of times $\|K_{eq}(iw)\|$ is calculated
J11	: Frequencies for which $\|K_{eq}(iw)\|$ is calculated
Y	: $\|K_{eq}(iw)\|$
P	: Character array in which graph is plotted
B	: Admittance corresponding to given impedance A
C	: Gain of a branch
E	: Resultant gain between two unconnected nodes
E1, E2, E3, E4, E5	: Values of E for different nodes
A1	: $K_{0A}(s_1)$ of equation (8)
A2	: $1/(1 + s_1T_{0A})$ of equation (8)
A3	: complex form of A_{13}
E11, E12	: Values of A2 for different frequencies
E21, E22	: Values of A1 for different frequencies
KP	: $\prod_{j=1}^{n} [T_{13}(s_j)\ T_{\pm}67_5(s_j)]$ of equation (13) for all distortion generators.
KEQ3	: $\|K_{eq}(iw)\|$ of equation (14)
Z	: Complex form of U
D	: Gain of a branch
I1F	: Parameter used to get negative and double of frequency at which $\|K_{eq}(iw)\|$ is being calculated.
N1N	: Parameter used to limit number of values to 80 if I11 exceeds 80.
JMAX	: Maximum value of all the J11s
JMIN	: Minimum value of all the J11s
YMAX	: Maximum value of all the Ys
YMIN	: Minimum value of all the Ys

D1	: Denominator of equation (9)
X	: Numerator of equation (9)
I1, I2	: Node numbers used for transfer from main segment to fuı.ction sub-program E
N5	: Array used to compare nodes with previously used nodes while calculating D1 to ensure that no branch is used more than once.
NIJ	: Two dimensional array to give information whether a branch exists between two nodes or not.
Q	: M_K of equation (10)
Q1	: Gain product of a loop
Q2	: P_{m1} of equation (10)
Q3	: P_{m2} of equation (10)

Besides these symbols I, J, II, JJ and L1 have been used as indices of DO loops.

```
PROGRAM DISTORTIONANALYSIS
MAIN SEGMENT
DIMENSION N1(25), N2(25), N3(25), A(25), J11(200), Y(200)
CHARACTER P(80, 80)
COMPLEX E(25), C(25), E1(15), E2(15), E3(15), E4(15),
E21, E22, KP, KEQ3, Z, D, E, 45, A1, A2, A3, E11, E12
COMMON N1, N2, N/BL1/B/BL2/C, NN
DATA P/6400*"/
READING NUMBER OF ELEMENTS, NUMBER OF NODES, 1, T
AND 3 OF OP AMP, LOWER, UPPER LIMITS AND FREQUENCY
INTERVAL OF FREQUENCY RESPONSE, INPUT VOLTAGE LEVEL
READ (5, *)N, NN, XK, T, A13, L1, L2, L3 U
READING NUMBER OF NODES AND ELEMENTS IN BRANCH
CONNECTING THESE NODES
READ (5, *) (N1(I), N2(I), N3(I), A(I), I = 1, N)
Z = CMPLX(0, 0.0)
I11 = 0
D0 6 I1F = L1, L2, L3
KP = CMPLX(0.0, 0.0)
JF = I1F
A3 = CMPLX(A13, 0.0)
A2 = (CMPLX (1.0, 0.0))/(CMPLX(1.0, I1F*6.28319*T))
A1 = (CMPLX(XK, 0.0))*A2
CONVERSION OF IMPEDANCES INTO ADMITTANCES
DO 1 I = 1, N
IF(N3(I).EQ.1)B(I) = CMPLX(I.0/A(I), 0.0)
IF(N3(I).EQ.2)B(I) = CMPLX(0.0, I1F*6.28319*A(I))
IF(N3(I).EQ.3)B(I) = CMPLX(0.0, 1.0/(I1F*6.28319*A(I)))
IF((N3(I).EQ.4).OR.(N3(I).EQ.5))B(I) = KP
CONTINUE
DO 2 I = 1, N
```

```
IF((N3(I).EQ.1).OR.(N3(I).EQ.2).OR.(N3(I).EQ.3))
C(I) = D(N2(I), B(I))
IF((N3(I).EQ.4).AND.(N1(I).NE.0))C(I) = A1
IF((N3(I).EQ.5).AND.(N1(I).NE.0))C(I) = -A1
CONTINUE
IF(I1F.LT.C)GO TO 4
IF(I1F.EQ.(2*JF))GO TO 8
E11 = A2
E21 = A1
CALCULATION OF GAIN AT INPUT OF OP AMP ... INPUT
NODE OF FILTER AND GAIN AT OUTPUT NODE OF FILTER
...DISTORTION GENERATOR
DO 3 I = 1, N
IF((N3(I).NE.4).OR.(N3(I).NE.5))GO TO 3
E1(I) = E(1, N1(I))
E2(I) = E(N2(I), NN)
CONTINUE
CALCULATION OF LINEAR PART OF OUTPUT SIGNAL
E5 = E(1, NN)
I1F = -I1F
GO TO 5
E12 = A2
E22 = A1
CALCULATION OF GAIN AT INPUT OF OP AMP ... INPUT
NODE OF FILTER
DO 9 I = 1, N
IF((N3(I).NE.4).OR.(N3(I).NE.5))GO TO 9
E3(I) = E(1, N1(I))
CONTINUE
I1F = -I1F*2
GO TO 5
CALCULATION OF GAIN AT OUTPUT NODE OF FILTER
...DISTORTION GENERATOR
DO 7 I = 1, N
IF((N3(I).NE.4).OR.(N3(I).NE.5))GO TO 7
E4(I) = E(N2(I), NN)
CONTINUE
CALCULATION OF TT[T13(S   ).T£5(   )] FOR ALL
DISTORTION GENERATORS
DO 10 I = 1, N
IF((N3(I).NE.4).OR.(N3(I).NE.5))GO TO 10
KP = KP + E1(I)*E1(I)*E2(I)*E2(I)*E3(I)*E4(I)
CONTINUE
CALCULATION OF THIRD ORDER KERNEL
KEQ3 = -A3*E11*E11*E12*KP
CALCULATION OF K (   ) OF EQUATION(14)
```

```
XKIM = CABS (E5 + (0.75, 0.0)*Z*Z*KEQ3)
I11 = I11 + 1
J11(I11) = JF
Y(I11) = XKIM
I1F = JF
CONTINUE
GRAPH PLOTTING
IF(I11.GT.8C)N1N = 80
IF(I11.LE.8C)N1N = I11
DO 12 I = 1, N1N
P(I, 1) = J = 2, N1N
DO 13 J = 2, N1N
P(1, J) = "
SELECTION OF MAXIMUM AND MINIMUM VALUES
JMAX = J11(1)
YMAX = Y(1)
JMIN = J11(1)
YMIN = Y(1)
DO 11 I = 1, I11
IF(J11(I).GT.JMAX) JMAX = J11 (I)
IF(J11(I).LT.JMIN) JMIN = J11(I)
IF(Y(I).GT.YMAX) YMAX = Y(I)
IF(Y(I).LT.YMIN) YMIN = Y(I)
CONTINUE
SCALING OF CALCULATED VALUES TO FIT THEM
EFFECTIVELY IN AVALABLE SPACE
KXX = N1N - 1
KYY = N1N - 1
DO 14 K = 1, I11
I = (J11(K) - JMIN)*FLOAT(KXX)/(FLOAT(JMAX - JMIN))
J = (Y(K) - YMIN)*FLOAT(KYY)/(YMAX - YMIN)
P(I, J) = '*'
CONTINUE
RESULT WRITING IN TABULAR FORM
WRITE (6, 15) (J11(I), Y(I), I = 1, I11)
FORMAT (5X, 'FREQUENCY = ', I4, 5X,' K (   ) = ', E10.5)
RESULT WRITING IN GRAPHICAL FORM
DO 16 J = 1, N1N
WRITE (6, 17) (P(I, J), I = 1, N1N)
FORMAT (10A8)
STOP
END

FUNCTION SUBPROGRAM TO CALCULATE GAIN OF
A BRANCH
COMPLEX FUNCTION D(I1, X)
```

```
COMMON N1(25), N2(25), N/BL1/B(25)
COMPLEX D1, X, B
DIMENSION N5(25)
D1 = CMPLX (0.0, 0.0)
DO 1 L = 1, N
IF(I1.EQ.N1(L))GO TO 2
IF(I1.EQ.N2(L))GO TO 3
GO TO 1
N5(L) = N2(L)
GO TO 4
N5(L) = N1(L)
IF(L.EQ.1)GO TO 5
DO 6 J = 1, L - 1
IF(N5(J).EQ.N5(L))GO TO 1
CONTINUE
D1 = D1 + E(1)
CONTINUE
D = X/D1
RETURN
END

FUNCTION SUBPROGRAM TO CALCULATE RESULTANT
GAIN BETWEEN TWO UNCONNECTED NODES
COMPLEX FUNCTION E(I1, I2)
COMMON N1(25), N2(25), N/BL2/C(25), NN
DIMENSION NIJ(25, 25)
COMPLEX Q, Q1(25), Q2, C, Q3
Q = CMPLX (1.0, 0.0)
DATA Q1/25*(1.0, 0.0)/, NIJ/625*0/
Q2 = CMPLX (0.0, 0.0)
Q3 = CMPLX (0.0, 0.0)
CALCULATION OF M OF EQUATION (10)
DO 1 L1 = 1, N
IF((N1(L1).GE.I1).AND.(N2(L1).LE.I2).AND.(N2(L1).GT.
N1(L1)))G = G*C(L1)
CONTINUE
CALCULATION OF EP 1 OF EQUATION(11)
DO 2 L1 = 1, N
DO 2 J = 1, N
IF (L1.EQ.J)GO TO 2
DO 3 K = 1, N
IF((N1(K).EQ.L1).AND.(N2(K).EQ.J))GO TO 4
CONTINUE
GO TO 2
DO 5 L = 1, N
IF((N1(L).EQ.J).AND.(N2(L).EQ.L1))GO TO 6
```

```
IF((N1(L).EQ.J).AND.(N2(L).LT.L1).AND.(N2(L).GT.J))GO TO 7
CONTINUE
GO TO 2
Q1(L1) = (C(K)*C(L))/(2.0,0.0)
IF(L1.GT.J)NIJ(L1,J) = 1
GO TO 8
Q1(L1) = C(K)*C(L)
DO 9 M = 1, N
IF ((N1(M).EQ.N2(L)).AND.(N2(M).EQ.(N2(L) + 1)))GO TO 11
CONTINUE
GO TO 8
Q1(L1) = Q1(L1)*C(M)
L = L + 1
IF(N2(M).LT.L1)GO TO 10
NIJ(L1, J) = 1
Q2 = Q2 + Q1 (L1)
CONTINUE
CALCULATION OF EP 2 OF EQUATION(11)
DO 12 L1 = 1, N
DO 12 J = 1, N
IF (NIJ(L1, J).NE.1) GO TO 12
DO 12 II = 1, N
DO 12 JJ = 1, N
IF((NIJ(II, JJ).EQ.1).AND.(L1.NE.II).AND.(L1.NE.JJ)
.AND.(J.NE.II).AND.(J.NE.JJ))Q3 = Q3 + Q1(L1)*Q1(II)
CONTINUE
CALCULATION OF M OF EQUATION (10)
E = Q/(1 - Q2 + Q3)
RETURN
END
```

APPENDIX-II INPUT DATA AND RESULTS OF PROGRAM "DISTORTION ANALYSIS"

Data File

```
1  8 5 5000.0 4.0E − 04 −8.0E − 03 950 1050 1 8, 4
2  1 2 1 3.5E03
3  2 3 2 1.0E − 06
4  3.4 5 0.0
5  4 5.1 200.0
6  5 3 1 5.0E03
7  5 2 2 1.0E − 06
8  3 2 2 1.0E − 06
9  2 0 1 5.0
```

Results

FREQUENCY = 950	|Keq(iw)| = .22493E − 01
FREQUENCY = 960	|Keq(iw)| = .44784E + 00
FREQUENCY = 970	|Keq(iw)| = .55471E + 00
FREQUENCY = 980	|Keq(iw)| = .71649E + 00
FREQUENCY = 990	|Keq(iw)| = .96816E + 00
FREQUENCY = 1000	|Keq(iw)| = .12979E + 01
FREQUENCY = 1010	|Keq(iw)| = .13920E + 01
FREQUENCY = 1020	|Keq(iw)| = .11026E + 01
FREQUENCY = 1030	|Keq(iw)| = .81600E + 00
FREQUENCY = 1040	|Keq(iw)| = .62703E + 00
FREQUENCY = 1050	|Keq(iw)| = .50397E + 00