Dilaton stabilization by massive fermion matter

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The implications for the Dilaton stabilization problem are investigated when the effective potential for this field is generated by the existence of a massive fermion. The previously evaluated two loop correction for this quantity indicates that the Dilaton field tends to be fixed at a high value close to the Planck scale, in accord with the needs for predicting Einstein gravity from string theory. Moreover, the mass of the Dilaton is evaluated to be close to the Planck mass, which assures the absence of Dilaton scalar signals in modern cosmological observations. These properties arise when the fermion mass is chosen to be either at a lower bound corresponding to the top quark mass, or alternatively, at a very much higher value assumed to be in the grand unification energy range. The renormalization scale μ is chosen to be given by the Z particle mass. We also consider the case when μ is a dynamical parameter fixed by minimization of the effective potential. The results rest on the basic assumption that the higher three or more loop calculations do not drastically affect the two loop potential. Higher loop sample calculations are expected to be considered elsewhere in order to give full ground to the conclusions.

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I. INTRODUCTION

The Dilaton is an essential ingredient of superstring theory, and constitutes a scalar field partner of the graviton [1]. Therefore, the background fields associated with the vacuum state of superstring theory should involve this field in common with the metric in the basic action. This is referred to as Dilaton gravity [2, 3]. To the lowest level of approximation the Dilaton is a free and massless scalar field with a special kind of coupling to the matter fields. As a consequence of this coupling, a time varying Dilaton field determines time-dependent coupling constants. In order to overcome this difficulty the Dilaton should remain constant during the present stage of evolution of the Universe. Moreover, unless the Dilaton turns out to be very massive, its existence could lead to an observable "Fifth force" similar to the ones which are currently associated to the observations of the Dark Matter. The constraints posed by current experimental observations determine the lower bound on the mass of the Dilaton to be of the order $m < 10^{-12}$GeV [4] (but see [5] for an attempt to make a running Dilaton consistent with late time cosmology).

The Dilaton stabilization problem has been at the center of an intense research activity in recent times because of its physical relevance. It should be emphasized that the Dilaton is one of various scalar fields appearing in the formulation of superstring theory in the low-energy limit. The sizes and shapes of the extra spatial dimensions associated with superstring theory are also leading to additional scalar fields, called "moduli fields". The stabilization of such moduli fields has been the object of recent attention particularly in connection with Type IIB superstring theory. The introduction of fluxes within the compactification spaces has made it possible to stabilize various moduli fields [7]. Also, gaugino condensation [8] has been employed to stabilize the Dilaton field in the context of heterotic superstring theory [9] and in string gas cosmology [10].

It should be remarked that, since Dilaton stabilization has special relevance for late time cosmology, there is motivation for finding mechanisms which do not directly rest on the concrete assumptions defining the nature of the extra dimensions. An additional motivation to search for alternative Dilaton stabilization mechanisms comes from String Gas Cosmology (SGC). The SGC [11, 12] is a model of early universe cosmology which employs new degrees of freedom and symmetries of string theory, and couples these elements with gravity and Dilaton fields into a classical action background model. The Universe is considered to start as a compact space containing a gas of strings. Since in string theory there is a maximal temperature for a gas of closed strings, the initial state of the cosmological evolution in SGC will be a phase of almost constant temperature, the so called "Hagedorn phase". The SGC is able to define a non-singular cosmology in which there is no starting Big Bang explosion. It has been noted that the thermal fluctuations in a gas of closed strings in the Hagedorn phase can justify the scale-invariant spectrum of cosmological fluctuations observed in Nature [13, 14], with a particular prediction of a slight blue tilt for gravitational waves [15]. However, the consistency of the picture requires that the Dilaton field be fixed during the Hagedorn phase. Therefore, in the SGC theory the Dilaton needs to be fixed at very early times and at very late times.

Thus, clarifying the mechanisms of Dilaton field stabilization is an important question in particle physics today. It is worth noting that the universal type of coupling of the Dilaton to the matter fields not only leads to an unwanted effect as the time-dependence of the coupling constants but it also furnishes the possibility that quantum effects due to the interaction of the Dilaton with matter might generate interesting contributions to the effective potential of the Dilaton. In a previous work published in Ref. [16], we started to explore this question. The work considered the cosmological periods when the additional spatial dimensions of superstring theory were already stabilized and the study was done in the framework of a four-dimensional field theory. The objective of study was then the interaction of the Dilaton with massive fermions. Such masses can be defined by fluxes about internal manifolds. In late time cosmology, the masses could had been generated after supersymmetry breaking. In an alternative early universe cosmology, one may consider thermally generated fermion masses. Henceforth, in Ref. [16] we started to investigate the possibility that the appearance of fermion masses could stabilize the Dilaton and also generate its mass.

In Ref. [16] we considered a simple form for the Dilaton gravity action in which a massive Dirac fermion term was added [17]. The action was chosen in the Einstein frame, which does not show any Dilaton field dependence in the kinetic terms for the fermions. On the other hand, the fermion mass becomes a function of the Dilaton, involving a universal exponential factor in Dilaton gravity [2, 3]. The chosen action described the low energy effective interaction of Super-Yang-Mills fermions with the Dilaton field in superstring theory [16]. The effective potential for the Dilaton field was evaluated up to two loop corrections in the small Dilaton field limit. A fixed value of the cosmological scale factor was assumed. The outcome of the work was, thanks to the appearing of logarithms in the loop calculations, that the Dilaton field appeared in the result in quadratic powers multiplied by the exponential factors of the field. This structure led to the possible existence of stabilizing minima of the potential in a finite range of the parameters.

Motivated by these results in Ref. [16], we here investigate some of the physical consequences of the two loop evaluation of the effective potential for the Dilaton field. The main issues are the stabilizing effect of the existence of massive matter on the mean value of the Dilaton field, and the magnitude of its mass. The potential found in Ref. [16] was a function of two parameters: the mass m of the fermion field and the dimensional regularization scale parameter μ. For the mass m two alternatives will be considered. We choose first the highest mass already manifested by a particle: the top quark mass at nearly 172 GeV. The second choice is a mass value at the GUT scale in the range of 10^{16} GeV. For each fermion mass value, two values for the μ scale parameter are selected: firstly, the scale of the mass of the Z particle (close to 91 GeV). The second selection explores the idea that the scale parameter could be dynamically defined by the string theory as one further coupling constant of the low energy theory. This assumption is motivated in the present view by that the coupling constants in string theory behave as dynamical fields. The existence of minima in the two loop action evaluated in Ref. [16] as a function of μ allows this possibility in the system under consideration. Accordingly, we first search for a value of μ determining an extremum in the effective potential for any value of the Dilaton mean field, and then substitute it into the effective action. Next, the mean Dilaton field is determined by finding the extremum in this modified action.

The results obtained indicate a surprising effect: in all the considered cases the mean value of the Dilaton field tends to be stabilized at the scale of the Planck mass. Moreover, in all cases the mass of the Dilaton field also comes out to be of the order of the Planck mass. Therefore, the results suggest that the appearance of mass for matter in the course of the evolution of the Universe can generate a strong stabilizing action on the vacuum expectation value of the Dilaton field. This action tends to stop the time evolution of the mean value, which consequently leads it to relax at high values, at the Planck scale. Further, the Dilaton mean value becomes strongly bound to the value at the minimum potential, due to the resulting large mass of the order of the Planck mass.

The paper proceeds as follows: In Section II, we rewrite the expressions for the two loop effective potential for the Dilaton, as calculated in Ref. [16]. Definitions of the parameters and the units in which the analysis is done, are given here. In Section III we present our evaluation of the effective potential and the Dilaton masses for the case in which the mass of the fermions are assumed either to have the mass of the top quark $m_{top} \simeq 172$ GeV or a mass at the high GUT unification scale $m_{GUT} = 10^{16}$ GeV. The m_{top} value was selected to represent a lower bound for the yet unknown fermion masses, and m_{GUT} representing the possible existence of fermion matter fields of the order of the unification scale. In both cases the renormalization scale is chosen to be $\mu \simeq 91$ GeV, as is usual in the Standard Model. In Section IV we assume that μ in fact is a dynamical variable to be defined by the minimum of the low energy effective action of the underlying string theory. Again the Dilaton mean fields and their masses are evaluated at the two energy scales. In the concluding Section V the results are resumed and commented.

II. THE TWO LOOP EXPRESSION FOR THE DILATON POTENTIAL

In this section we will rewrite the result of a two loop calculation of the Dilaton potential which is determined by the vacuum fluctuations of a fermion field of mass m. This reproduces the evaluation already done in Ref. [16]. Since the result is expected to be larger for larger values of the masses, the most interesting values of m should be those corresponding to the largest masses in Nature. Recall the notation and basic conditions in the starting action in Ref. [16]:

$$S = \int d^4x\sqrt{-g(x)}\left(-\frac{1}{2\kappa^2}g^{\mu\nu}(x)\partial_\mu\phi(x)\partial_\nu\phi(x)+\overline{\Psi}(x)(i\frac{g^{\mu\nu}\gamma_\mu\overleftrightarrow{\partial}_\nu}{2}-\exp(\alpha^*\ \phi)m)\Psi(x)\right), \quad (1)$$

$$x^\mu = (x^0,x^1,x^2,x^3), \quad \overleftrightarrow{\partial}=\overrightarrow{\partial}-\overleftarrow{\partial}, \quad \{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}(x), \quad (2)$$

$$g_{\mu\nu}(x) = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \sqrt{-g(x)}=1. \quad (3)$$

Note that we considered the standard Dilaton Gravity interacting with a massive fermion action in the Einstein frame in which the metric $g_{\mu\nu}$ was approximated by the Minkowski metric in order to simplify the evaluation. Differently from the choice in Ref. [16], the gravitational constant is here explicitly introduced, and natural units are employed for the distances and mass. The parameter defining the Dilaton field dependent exponential, the Planck length $\kappa = l_P$ and mass m_P are defined by the expressions

$$\alpha^* = -\frac{3}{4}, \quad (4)$$

$$\kappa^2 = \frac{8\pi G\hbar}{c^3}, \quad (5)$$

$$\kappa = l_P = \frac{1}{m_P} = 8.10009\times 10^{-33}\ \text{cm}, \quad (6)$$

$$G = 6.67\times 10^{-8}\ \text{cm}^3\ \text{g}^{-1}\ \text{s}^{-2}, \quad (7)$$

$$\hbar = 1.05457\times 10^{-27}\ \text{cm}^2\ \text{g}\ \text{s}^{-1}, \quad (8)$$

$$c = 2.9979245800\times 10^{10}\ \text{cm}\ \text{s}^{-1}. \quad (9)$$

In the above expression for the action, the coordinates and times are measured in cm, the masses m in the natural unit cm^{-1} and the Dilaton field is dimensionless.

Starting from the classical action, we evaluated in Ref. [16] a two loop correction to the effective action, assuming a homogenous and time independent value of the Dilaton mean field ϕ as

$$\frac{\Gamma[\phi]}{V^{(4)}} = -V^{eff}(\phi), \quad (10)$$

where $V^{(4)}$ is the four dimensional volume. In this work we shall also be interested in estimating the masses of the Dilaton field. For this reason we shall consider an approximation in which a classical effective action for the Dilaton field S^{eff} is defined as the classical (tree) kinetic part of the action including the two loop evaluated effective potential for homogeneous mean fields (as evaluated in inhomogeneous field values $\phi(x)$). For homogeneous field values this action coincides with the negative of the two loop effective potential and thus its minima define the equilibrium values of the Dilaton field. In order to eliminate the explicit appearance of the gravitational constant from the expression of the action, we will absorb it by redefining the Dilaton field value and the α constant as

$$\varphi = \phi/\kappa, \quad (11)$$

$$\alpha = \alpha^*\kappa = -\frac{3}{4}\kappa. \quad (12)$$

After these changes, the effective action S^{eff} can be expressed as follows

$$S^{eff}[\varphi,m,\mu] = \int d^4x\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi(x)\partial_\nu\varphi(x)-V^{eff}(\varphi)\right), \quad (13)$$

$$V^{eff}(\varphi,m,\mu)\equiv V^{eff}(\varphi,m,\mu)=V^{(1)}(\varphi,m,\mu)+V^{(2)}(\varphi,m,\mu), \quad (14)$$

where the one and two loop corrections to the potential are explicitly given by [16]

$$V^{(1)}(\varphi,m,\mu) \;=\; (\frac{m^2}{4\pi})^2 \exp(4\alpha\varphi)(\frac{3}{2}-\gamma-\log(\frac{m^2}{4\pi\mu^2})-2\alpha\varphi), \tag{15}$$

$$\begin{aligned} V^{(2)}(\varphi,m,\mu) \;=\;& \frac{1}{512\pi^4}\alpha^2 m^6 \exp(6\alpha\varphi)[4(\log(\frac{m^2}{\mu^2})+2\alpha\varphi)\times \\ & (3(\log(\frac{m^2}{\mu^2})+2\alpha\varphi)-2(5+\log(64)+3\log\pi)+6\gamma) \\ & +2(25+8\log(2)(5+\log(8))+\log(\pi)\times(20+6\log(16\pi)))- \\ & 8\gamma\,(5+\log(64)+3\log(\pi))+\pi^2+12\gamma^2], \end{aligned} \tag{16}$$

$$\gamma \;=\; 0.57721. \tag{17}$$

The action S^{eff} can be also expressed in dimensionless coordinates and parameters after defining the Dilaton field, coordinates x, mass m and scale parameter μ in terms of their dimensionless corresponding counterparts (σ, z, m^*, μ^*), as follows

$$\sigma(z) \;=\; |\alpha|\varphi(x), \tag{18}$$

$$z^\mu \;=\; m\; x^\mu, \tag{19}$$

$$m^* \;=\; |\alpha|\; m, \tag{20}$$

$$\mu^* \;=\; |\alpha|\; \mu, \tag{21}$$

which allows to write for S^{eff}

$$\begin{aligned} S^{eff}[\sigma,m^*,\mu^*] \;=\;& \frac{1}{\alpha^2 m^2}\int d^4z\left(-\frac{1}{2}g^{\mu\nu}\frac{\partial}{\partial z^\mu}\sigma(z)\frac{\partial}{\partial z^\nu}\sigma(z)-\frac{\alpha^2}{m^2}V^{eff}(\frac{\sigma}{|\alpha|},m,\mu)\right) \\ =\;& \frac{1}{m^{*2}}\int d^4z\left(-\frac{1}{2}g^{\mu\nu}\frac{\partial}{\partial z^\mu}\sigma(z)\frac{\partial}{\partial z^\nu}\sigma(z)-V_z^{eff}(\sigma,m^*,\mu^*)\right), \end{aligned} \tag{22}$$

$$V_z^{eff}(\sigma,m^*,\mu^*) \;\equiv\; \frac{\alpha^2}{m^2}V^{eff}(\frac{\sigma}{|\alpha|},m,\mu). \tag{23}$$

In the following sections we shall investigate what properties the above expressions for the effective action predict for the vacuum values of the mean Dilaton field and its excitations.

III. DILATON STABILIZATION FOR $\mu = M_Z$

Let us now investigate the extremal values of the effective potential V_z^{eff} as a function of the homogeneous values of the Dilaton field σ for the important case in which the renormalization scale μ is fixed to the mass of the Z boson

$$m_Z = 91.1876 \pm 0.0021 \text{ GeV} = 4.6211\times 10^{15}\text{ cm}^{-1}.$$

We shall search for the minima for two limiting values of the masses: that the largest fermion mass in Nature is at least of the order of the mass of the top quark

$$m_{top} \;=\; 172.0 \pm 0.9 \text{ GeV} = 8.7164\times 10^{15}\text{ cm}^{-1},$$

or at most of the order of the GUT scale

$$\begin{aligned} m_{GUT} \;&=\; 5.06773\times 10^{29}\text{cm}^{-1} \\ &\equiv\; 10^{16}\text{ GeV}. \end{aligned}$$

For this study it is helpful to express the potential in (14) in terms of the Dilaton field σ, and the new parameters M^* and X as

$$M^* \;=\; \frac{m^*}{\mu^*}, \tag{24}$$

$$X \;=\; \log\left(\frac{(m^*)^2}{(\mu^*)^2}\right) = \log\left((M^*)^2\right). \tag{25}$$

With these changes the potential V_z^{eff} becomes

$$\begin{aligned} V_z^{eff}(\sigma, M^*, X, \mu^*) &= V_z^{eff}(\sigma, m^*, \mu^*) \\ &= \mu^{*2}\frac{(M^*)^2}{(4\pi)^2}\exp(-4\sigma)(a_1+\log(4\pi)-X+2\sigma) \\ &\quad +\mu^{*4}\frac{(M^*)^4}{512\pi^4}\exp(-6\sigma)[4(X-2\sigma)(3(X-2\sigma)+a_2)+a_3], \\ a_1 &= \frac{3}{2}-\gamma, \\ a_2 &= -2(5+\log(64)+3\log\pi\ )+6\gamma, \\ a_3 &= +2(25+8\log(2)(5+\log(8))+\log(\pi)\times(20+6\log(16\pi)))- \\ &\quad 8\gamma(5+\log(64)+3\log(\pi))+\pi^2+12\gamma^2. \end{aligned} \tag{26}$$

Above, the renormalization scale is set at the mass of the Z boson, $\mu = 91.1876 \pm 0.0021$ GeV$= 4.6211 \times 10^{15}\,\mathrm{cm}^{-1}$, and the mass of the fermion is fixed at the known lower bound of the mass for the matter field, that is, the top quark mass $m_{top} = 172 \pm 0.9\,\mathrm{GeV} = 8.7164 \times 10^{15}\,\mathrm{cm}^{-1}$. Then, the parameter M^* takes the value

$$M^* = \frac{m_{top}}{\mu}.$$

After employing the equivalence factor: $1\,\mathrm{GeV} \equiv 1.6021765 \times 10^{-3}\,\mathrm{cm}^2\mathrm{g}\,\mathrm{s}^{-2}$, between the energy expressed in GeV and in *cgs* units, the values of the parameters μ^* and m^* are

$$\begin{aligned} \mu^* &= |\alpha|\ \mu = \frac{3}{4}\kappa\ \mu \\ &= 2.8073 \times 10^{-17}, \end{aligned} \tag{27}$$

$$m^* = |\alpha|\ m = \frac{3}{4}\kappa\ m \tag{28}$$

$$= 5.2958 \times 10^{-17}. \tag{29}$$

We plot the potential V_z^{eff} as a function of the value of the Dilaton field σ in Figure 1.

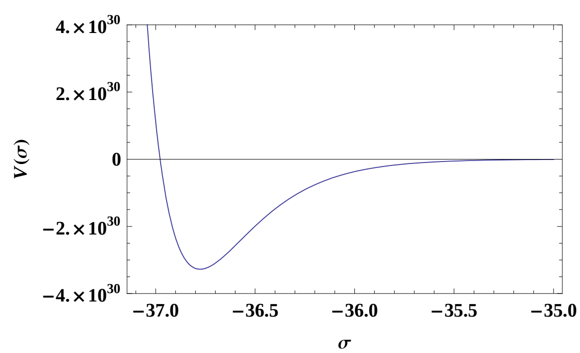

FIG. 1: The effective potential $V(\sigma)$ defined by Eq. (26) as a function of the dimensionless Dilaton field σ. The fermion mass was fixed to correspond to the top quark mass m_{top} and the renormalization scale μ is chosen to be the Z boson mass. The minimum of the potential is near the value $\sigma = -36.7765$, which indicates that field is stabilized at a high value near the Planck scale. The high values of the potential also show that the Dilaton mass takes large values near the Planck mass.

The plot clearly shows that the minimum of the potential for the considered physical conditions, is close to the value $\sigma_{\min} = -36.7765$. Therefore, the value of the Dilaton mean field at the minimum in natural units (cm^{-1}) is given by

$$\varphi_{\min} = \frac{4}{3}\frac{\sigma_{\min}}{l_P}. \tag{30}$$

Thus, the Dilaton mean field is near values of the order of the Planck mass

$$m_P = 1.23455 \times 10^{32}\ \mathrm{cm}^{-1},$$

assuming the existence of the top quark fluctuations in the physical vacuum.

The mass of the Dilaton, thanks to the approximate structure assumed for the classical effective action in (22), is simply given as the squared root of the second derivative over σ of the curve depicted in Figure 1, at the point of the minimum and the chosen values for m^* and μ^*. Evaluating this quantity gives for the Dilaton mass in the present case the value

$$\begin{aligned} m^{top}_{Dilaton}(Z) &= \sqrt{m^2 \frac{d^2}{d\sigma^2}[V^{eff}_z(\sigma, m^*, \mu^*)]_{\sigma=\sigma_{\min}}} \\ &= 9.0408 \times 10^{30}\ \text{cm}^{-1}. \end{aligned}$$

We can then conclude that the presence of the top quark fluctuations in the physical vacuum, as reflected in the two loop calculation, tends to stabilize the Dilaton field to a value tightly at the Planck mass scale. Furthermore, these fluctuations also generate a large contribution to the Dilaton mass, which resulted in being close to the Planck mass. Such results suggest that the observability of the Dilaton field in string theory may be strongly suppressed at the late stages of the cosmological evolution, after the particles have been massified at their currently observed values. Since the top quark is an experimentally observed particle, the total effective action of the Dilaton should include its evaluated contribution. However, in High Energy Physics one may expect the existence and perhaps future detection of much more massive fermion particles. Therefore, we consider performing a similar evaluation in which we choose a mass at the Grand Unification scale

$$\begin{aligned} m_{GUT} &= 10^{16}\ \ GeV. \\ &\simeq 5.06773 \times 10^{29}\text{cm}^{-1}. \end{aligned} \tag{31}$$

The renormalization scale will be again fixed at the same Z boson mass. The effective potential curve for this case is plotted in Figure 2. Obviously the mean field at the stable point $\sigma_{\min}$ is lower in absolute value than at the top

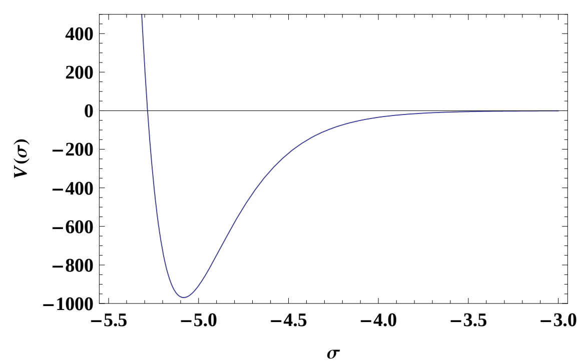

FIG. 2: The plot of the potential $V(\sigma)$ defined by (26) but now evaluated at a larger fermion mass, at the grand unification scale $m_{GUT} = 10^{16}$ GeV. The renormalization scale μ was taken as the value of the Z boson mass. Note that, as before, the Dilaton field is stabilized at a somewhat lower value, $\sigma = -5.08272$, still close to the Planck scale. Similarly, the large values of the potential show that the Dilaton mass will again be close to the Planck mass.

quark scale. However, the variation is not so large and the existence of massive fermions at the GUT scale also fixes the Dilaton mean field value around the Planck scale. The second derivative of the potential determines a Dilaton mass of the value

$$\begin{aligned} m^{GUT}_{Dilaton}(Z) &= \sqrt{m^2 \frac{d^2}{d\sigma^2}[V^{eff}_z(\sigma, m^*, \mu^*)]_{\sigma=\sigma_{\min}}} \\ &= 3.0206 \times 10^{30}\ \text{cm}^{-1}. \end{aligned}$$

Henceforth, the evaluations done in this section, suggest that whatever the values of the masses for the physical fermions are, in the range from m_{top} to m_{GUT}, the Dilaton mean field in the vacuum might be stabilized at the Planck scale and moreover, show an experimentally inaccessible high value of its mass. It should be underlined, that the full justification of this conclusion rests on the applicability of the evaluated two loop contribution to the effective action. A factor that could help to support this conclusion is that the numerical coefficients in higher loop contributions are expected to decrease with the order of the loop. Therefore, a direct evaluation of three loop corrections seems to be the most appropriate way of checking the present two loop results.

IV. ASSUMING μ AS A DYNAMICAL PARAMETER

Nowadays, there is a quite general belief that the physical coupling constants in the low energy effective theories of string theory are in fact not constants, but dynamically fixed fields of the underlying superstring theory. Adopting this view, after we had obtained the two loop terms for fermion contributions to the effective action of the Dilaton in Ref. [16], we noticed that the potential can show minima with respect to the dimensional regularization scale parameter μ. Therefore, the possibility appeared to explore the consequences of assuming the scale μ to be a dynamical quantity so that scale variations would exhibit a minimum at an equilibrium vacuum state in the effective potential. For this purpose let us consider V_z^{eff} as given by (26) but as a function of the parameters (σ, m^*, X) in place of (σ, M^*, X) as employed in the definition (26), now expressing M^* as a function of m^* through (24). Then one may note that the dependence of the potential on the scale μ is exclusively through the parameter X defined in (25) as $X = \log(\frac{(m^*)^2}{(\mu^*)^2})$. Thus, an extremum of the action over μ becomes also an extremum over the parameter X. Imposing the vanishing of the derivative over X of the potential,

$$\frac{d}{dX} V_z^{eff}(\sigma, m^*, X) = 0,$$

leads to an explicit expression of X in terms of the Dilaton fields as

$$X(\sigma) = \frac{1}{24}\left(\frac{32\pi^2}{m^{*2}}\exp(2\sigma) + 32\sigma - a_2\right). \tag{32}$$

Substitution of this formula into the expression for the potential gives its dependence on σ, after the extremum condition over X is already imposed

$$\begin{aligned} V_z^{eff}(\sigma, m^*, X(\sigma)) \; &= \; \frac{m^{*2}}{(4\pi)^2}\exp(-4\sigma)(a_1 + \log(4\pi) - X(\sigma) + 2\sigma\;) + \\ &+ \frac{(m^*)^4}{512\pi^4}\exp(-6\sigma)\;[4(X(\sigma) - 2\sigma)\;(3(X(\sigma) - 2\sigma) + a_2\;) + a_3\;]. \end{aligned} \tag{33}$$

The dependence of the potential on the Dilaton field σ is shown in Figures 3 and 4, each of them considered respectively for the two representative values of the fermion masses m_{top} and m_{GUT}.

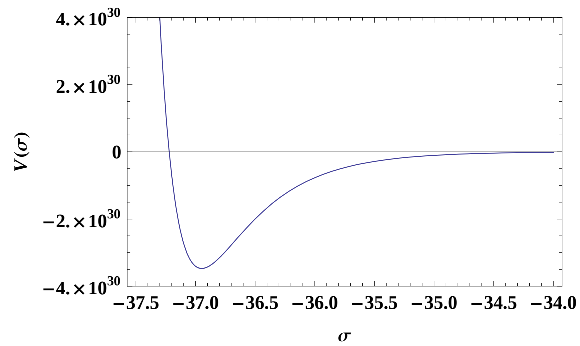

FIG. 3: The plot of the potential $V(\sigma)$ defined by the formula (33). The fermion mass is chosen at the top quark mass value and the renormalization scale is considered a dynamical quantity as defined by the extremum of the effective potential. Therefore, the formula for X in (25), expressing the extremal condition for the renormalization scale μ has been substituted in the effective potential. The results repeat the main indications of the previous section: the Dilaton field and its mass both get values near the Planck scale.

The most relevant issue in these pictures is, that they show that the extremum value of the mean field is at the Planck scale in both cases. That is, also in the situation when the renormalization scale parameter is assumed to be dynamically fixed, the vacuum value of the Dilaton field receives high values consistent with the expectation that this parameter is currently unobservable. The Dilaton mass can be found by calculating the second derivative of the potential with respect to σ, but maintaining m^* and X constant (note that this is not the second derivative of the curve shown in Figure 3, since it assumes X to be dependent of σ for the plot). The results for the two fermion masses are

$$m^{top}_{Dilaton}(dyn.) = \sqrt{m^2 \frac{d^2}{d\sigma^2}[V_z^{eff}(\sigma, m^*, \mu^*)]_{\sigma=\sigma_{\min}}}$$
$$\simeq 5.84262 \times 10^{31} \quad \mathrm{cm}^{-1},$$
$$m^{GUT}_{Dilaton}(dyn.) = \sqrt{m^2 \frac{d^2}{d\sigma^2}[V_z^{eff}(\sigma, m^*, \mu^*)]_{\sigma=\sigma_{\min}}}$$
$$\simeq 1.80502 \times 10^{32} \quad \mathrm{cm}^{-1}.$$

As these numbers indicate, the Dilaton masses are also at the currently unobservable Planck scale.

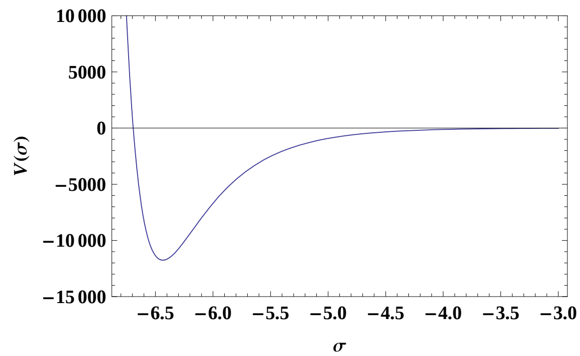

FIG. 4: A similar plot of the potential $V(\sigma)$ defined by the formula (33), but in which the fermion mass is given by the GUT unification value $m_{GUT} = 10^{16}$ GeV and the renormalization scale is considered as a dynamical quantity to be defined by the extremum of the effective potential. Therefore, the formula for X in (25), expressing the extremal condition for the regularization parameter μ has been substituted into the effective potential. The results repeat the main indications of the previous section: the Dilaton field and its mass get values near the Planck scale.

V. CONCLUSIONS

The predictions for the Dilaton stability determined by the previously evaluated two loop corrections to the Dilaton field interacting with a massive fermion field are investigated here. The fermion field mass values are considered in two cases: the top quark mass representing the lower bound of all existing but yet unknown fermion masses in Nature, and the energy scale of the grand unification theories of order 10^{16} GeV. The renormalization scale parameter was also chosen in two options: a value coinciding with the Z boson mass, and alternatively a variable μ, a dynamical quantity to be fixed at its value determining a minimum of the effective potential. In all the above described situations, the results interestingly indicate that the Dilaton mean field results stabilize at the very high values required by its role in allowing gravity to have its observed properties. Furthermore, the Dilaton field is also found to be strongly stabilized around these mean values, by showing a large mass near the Planck mass. Therefore, this work identifies a clear explanation for the lack of observable consequences of the Dilaton field in superstring theory. Moreover, one understands its dynamical fixation to the high values required to predict the observable Einstein nature of gravity predicted by the same theory. It should be remarked that the conclusions arising rest on the assumption that higher than two loops contributions will not alter the results. The higher loop evaluations required for the verification of this condition are expected to be addressed in further extensions of the work.

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