Cosmic Sum Rules

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We introduce new sum rules allowing to determine universal properties of the unknown component of the cosmic rays and show how it can be used to predict the positron fraction at energies not yet explored by current experiments and to constrain specific models.

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The data recently collected by PAMELA [1] indicate that there is a positron excess in the cosmic ray (CR) energy spectrum above 10 GeV which was already observed in 1974. Such a rising behavior does not fit previous estimates of the CR formation and propagation implying the possible existence of a direct excess of CR positron fraction of unknown origins. Interestingly PAMELA's data show a clear feature of such a positron fraction but no excess in the anti-protons one.

FERMI-LAT [2], ATIC [3], PPB-BETS [4] and HESS [5] studies of the electron-positron total spectrum up to TeV also show a hardening in the energy range from 100 GeV to 1 TeV. In the conventional approach, the injection spectrum of the $e^\pm$ is taken as a single power-law. Since diffusion and cooling are more efficient at higher energies, one would expect the spectrum to soften with the energy. The observed spectrum hardening is thus a second indication of the presence of a new unknown source of $e^\pm$ CRs.

These interesting features have drawn much attention, and many explanations have been proposed: For example, these excesses could be due to an inadequate account of the CR background in previous modeling; The presence of new astrophysical sources; They could also originate from annihilations and/or decays of dark matter. We refer to [6] for a recent review.

Whatever the origin of these excesses might be, we show that we can derive novel constraints able to shed light on the physical nature of their source and/or propagation.

We start by writing the observed flux of electrons and positrons as the sum of two contributions: A background component, $\phi^B_\pm$, due to known astrophysical sources (at least for the electrons), and an unknown component, $\phi^U_\pm$, in formulae:

$$\phi_\pm = \phi^U_\pm + \phi^B_\pm \ . \tag{1}$$

The component $\phi^U_\pm$ is the one needed to explain the features in the spectra observed by PAMELA and FERMI-LAT.

These experiments measure respectively the positron fraction and the total electron and positron fluxes as a function of the energy E of the detected $e^\pm$, i.e.:

$$P(E) = \frac{\phi_+(E)}{\phi_+(E)+\phi_-(E)} \ , \qquad F(E) = \phi_+(E) + \phi_-(E) \ . \tag{2}$$

The left-hand side of the equations above refer to the experimental measures. The contribution from the unknown source is then:

$$\phi^U_+(E) \ = \ P(E)\ F(E) - \phi^B_+(E) \ , \tag{3}$$

$$\phi^U_-(E) \ = \ F(E)\ (1-P(E)) - \phi^B_-(E) \ . \tag{4}$$

In terms of their difference and sum:

$$\begin{aligned} \phi^U_+(E) - \phi^U_-(E) \ &= \ F(E)\ (2P(E)-1) + (\phi^B_-(E) - \phi^B_+(E)) \ , \\ \phi^U_+(E) + \phi^U_-(E) \ &= \ F(E) - (\phi^B_-(E) + \phi^B_+(E)) \ . \end{aligned} \tag{5}$$

The latter equation implies $F(E) \geq \phi^B_-(E) + \phi^B_+(E)$. We model the background spectrum using $\phi^B_\pm(E) = N_B B^\pm(E)$, where N_B is a normalization coefficient such that $F(E)/(B^-(E)+B^+(E)) > N_B$ and $B^\pm(E)$ are provided using specific astrophysical models. In this paper we adopt the popular Moskalenko and Strong model [7, 8], for which N_B is less than 0.75 and $B^\pm(E))$ are given by:

$$B^+ \ = \ \frac{4.5E^{0.7}}{1+650E^{2.3}+1500E^{4.2}} \ , \tag{6}$$

$$B^- \ = \ B^-_1 + B^-_2 \ , \tag{7}$$

$$B^-_1 \ = \ \frac{0.16E^{-1.1}}{1+11E^{0.9}+3.2E^{2.15}} \ , \tag{8}$$

$$B^-_2 \ = \ \frac{0.70E^{0.7}}{1+110E^{1.5}+600E^{2.9}+580E^{4.2}} \ , \tag{9}$$

where E is measured in GeV and the Bs in $\text{GeV}^{-1}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$ units. We checked that our results remain unchanged when replacing the parameterization above with the one adopted by the Fermi Collaboration (model zero) [9, 10].

It is convenient to introduce the following parameter:

$$r_U(E) \equiv \frac{\phi^U_-(E)}{\phi^U_+(E)} = \frac{F(E)\ (1-P(E)) - \phi^B_-(E)}{P(E)\ F(E) - \phi^B_+(E)} \ . \tag{10}$$

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This equation can be rewritten as

$$R(E) \equiv \frac{F(E)}{B^-(E)} \; \frac{1-(1+r_U(E))P(E)}{1-r_U(E)\frac{\phi_+^B(E)}{\phi_-^B(E)}} = N_B \; . \qquad (11)$$

Although the sum rule $R(E)$ seems to depend on the energy it should, in fact, be a constant as it is clear from the right hand side of the previous equation. *This leads to a nontrivial constraint linking together in an explicit form the experimental results, the model of the backgrounds and the dependence on the energy of the unknown components.*

We now turn to the actual data and show in which way the sum rule (11) provides essential information on the unknown components of the CRs.

Since we use simultaneously the results of FERMI-LAT and PAMELA we are obliged to consider only the common energy range. Note that the CRs energy range below $10-20$ GeVs, where the spectrum is affected by the Sun, is outside the common range. Within the relevant but limited range of energies we will consider here it is therefore sensible to assume r_U to be nearly constant.

We find useful to plot the function $R(E)$ for different values of r_U in order to test the sum rule (11). This would imply that this function is independent on the energy. The associated constant value would then be identified with the background CRs normalization factor N_B. We report the results in Fig. 1. The straight (red) line is the $N_B = 0.75$ value which is the largest one can assume for the background not to be larger than the FERMI-LAT results. We observe that there is a clear tendency for the combined data to predict a lower value of the constant $N_B = 0.66, 0.64, 0.62, 0.58$ for increasing value of the ratio $r_U = 0, 1/2, 1, 2$. This is clear when looking, from top to bottom, at the different panels of Fig. 1. It is interesting to note that we find a plateau, in the relevant energy range, up to r_U near the value of 2 when it starts showing some deviation.

Given the large uncertainties we cannot yet provide a more solid conclusion, however we can use the derived normalization for each different ratios of the unknown components to *predict* the positron fraction at energies higher than the ones provided so far by PAMELA. In order to be able to make such a prediction we first rewrite (11) as follows:

$$P(E) = \frac{1}{1+r_U}\left(1 - \frac{\phi_-^B(E)}{F(E)}(1 - r_U\frac{\phi_+^B(E)}{\phi_-^B(E)})\right) \; , \qquad (12)$$

where we use for each r_U the estimated associated N_B. The different predictions for the positron fraction, assuming that r_U remains constant over the entire energy range, up to 1000 GeV are shown in Fig. 2. The resulting picture disfavors both,very small and very large values of r_U which, in turn, means that one expects the electron fraction to neither to be too small nor to dominate the CRs.

Using the sum rule will allow to extract vital information from the data as they become more and more

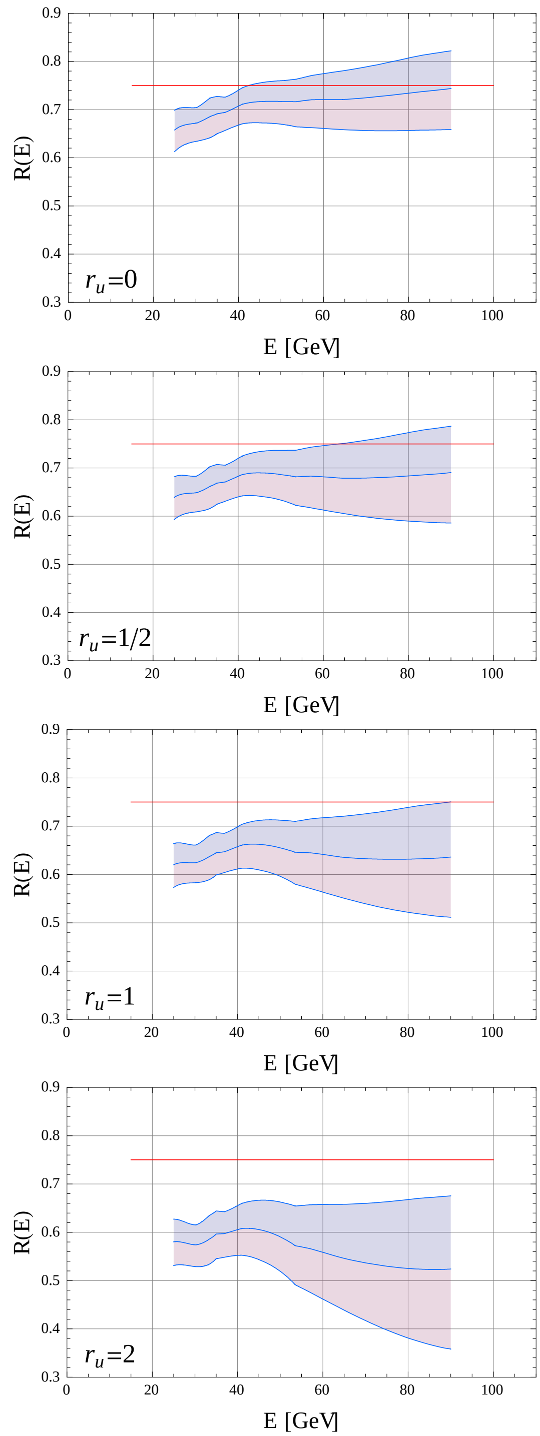

FIG. 1: Ratio $R(E)$ as a function of the energy E of electrons and positrons anf for vales of $r_U = 0, 1/2, 1, 2$, from top to bottom. The shaded region accounts for the error in PAMELA and FERMI-LAT. Secondaries are estimated according to the expressions in [7, 8].

accurate. The special case $r_U = 1$ is an automatic prediction of a great deal of models for dark matter which assume that charge symmetry holds both in the produc-

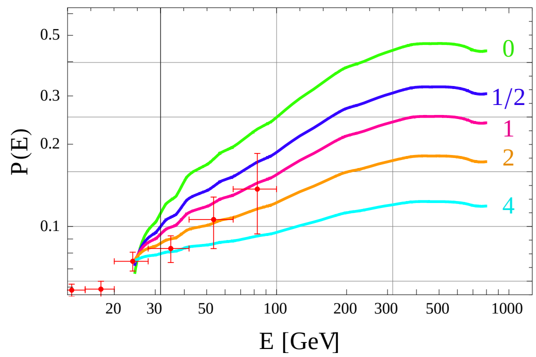

FIG. 2: Model independent prediction for the positron fraction $P(E)$ as a function of the energy E of electrons and positrons anf for vales of $r_U = 0, 1/2, 1, 2, 4$, from top to bottom. Secondaries are estimated according to the expressions in [7, 8] reported in the main text and the derived N_B values: $N_B = 0.66, 0.64, 0.62, 0.58, 0.50$.

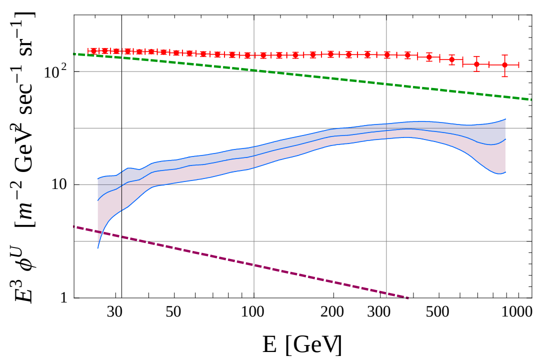

FIG. 3: We display $\phi^U(E) = \phi^U_{\pm}(E)$ corresponding to the central shaded region for the whole energy range of the FERMI-LAT experiment. For the reader's convenience we also show as the upper (red) error bars the actual FERMI-LAT data, while the background flux ϕ^B_- is reported as the upper (green) dashed line and ϕ^B_+ is the lower (magenta) dashed one.

tion and propagation of the unknown component of the CRs.

It is, therefore clear, that our formalism is more universal given that we have made *no assumption* in the derivation of the sum rules above. It is however, useful to use this model assumption to derive the further constraint:

$$\phi^U_{\pm}(E) = \frac{F(E) - (\phi^B_-(E) + \phi^B_+(E))}{2} , \qquad (13)$$

for which we now use $N_B = 0.65$. In Fig. 3 we display $\phi^U(E) = \phi^U_{\pm}(E)$ corresponding to the central shaded region for the whole energy range of the FERMI-LAT experiment. For the reader's convenience we also show as the upper (red) bars the actual FERMI-LAT data while the background flux ϕ^B_- is reported as the upper (green) dashed line and ϕ^B_+ is the lower (magenta) dashed one. Therefore our results test any model of the unknown component assuming charge symmetry for the resulting CRs.

The sum rules introduced here are general and can be extended also to the proton and antiproton fraction. We have shown that current data still allow for order one ratios of the electron to positron fraction of the unknown components of the associated CRs but disfavor electron to positron fractions smaller than one half and larger than four.

The current model independent analysis of the combined PAMELA and FERMI-LAT data shows that we will be able to deduce, thanks to the new constraints, vital information on the nature of the source and the propagation of the CRs. We have also demonstrated that the typical oversimplifying model assumption used so far constitutes a small portion of the allowed models still left unconstrained by the present data. Finally our sum rules can be easily used to constrain any specific model while we were able to predict, in a model independent way, the positron fraction at energies higher than the ones explored so far.

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