

# Visual Selection and Attention Shifting Based on FitzHugh-Nagumo Equations

Haili Wang<sup>1</sup>, Yuanhua Qiao<sup>1</sup>, Lijuan Duan<sup>2</sup>, Faming Fang<sup>2</sup>, Jun Miao<sup>3</sup>,  
and Bingpeng Ma<sup>3</sup>

<sup>1</sup> College of Applied Science, Beijing University of Technology, Beijing 100124, China

<sup>2</sup> College of Computer Science and Technology, Beijing University of Technology,  
Beijing 100124, China

<sup>3</sup> Key Laboratory of Intelligent Information Processing, Institute of Computing Technology,  
Chinese Academy of Sciences, Beijing 100190, China  
{qiaoyuanhua, ljduan}@bjut.edu.cn,  
{wanghaili, fmfang}@emails.bjut.edu.cn,  
{bpma, jmiao}@ict.ac.cn

**Abstract.** In this paper, we make some analysis on the FitzHugh-Nagumo model and improve it to build a neural network, and the network is used to implement visual selection and attention shifting. Each group of neurons representing one object of a visual input is synchronized; different groups of neurons representing different objects of a visual input are desynchronized. Cooperation and competition mechanism is also introduced to accelerate oscillating frequency of the salient object as well as to slow down other objects, which result in the most salient object jumping to a high frequency oscillation, while all other objects being silent. The object corresponding to high frequency oscillation is selected, then the selected object is inhibited and other neurons continue to oscillate to select the next salient object.

**Keywords:** F-N model, Neural Network, Visual Selection, Attention Shifting.

## 1 Introduction

Due to the limited processing capacity of biological system, some mechanisms have evolved in order to permit these systems to perform tasks. Visual selection and attention shifting are important mechanisms to ensure that the limited processing ability to perform tasks best. The ability to extract salient features from the images or receptive fields, and group them into objects, then select the salient object, is a fundamental task of perception. This ability is the visual selection of the visual perception. When the salient object has been selected out, because of the adaptability of the visual system, attention will shift from the salient object to the next salient object , which is the attention shifting of the visual perception . The application of neural network model based on synchronous oscillation in image processing is more and more widely [1], [2], [3], where pulse coupled neural dynamical model in image segmentation has obtained many good results. But the application in the visual selection and attention shifting is still very important for the current research and application [4], [5].

Techniques for identifying multiple objects have a certain development. For example, in 1997, Dietmar Heinke etc. proposed SAIM model [6]. In this model took the regulating effect of higher-level knowledge networks to the visual processing into account. They use the regulating effect of the knowledge networks to perform visual selecting task. In 2007, Dietmar Heinke etc. improved the original SAIM model [7], and completed the visual selection of color images. In 2006, Itti etc. They combined the top-down regulatory and bottom-up selection mechanism to perform the visual search task of natural images [8]. They used the extracting network to select feature from different scales, and the top-down regulation is mainly searching for the characteristics of the object and the background. In recent years, neural network models based on synchronous oscillation have continually been developed and built out, and significant progress has been made in image segmentation. In 2007, Liang Zhao and Fabricio A.B etc. also improved Wilson-Cowan network on the basis of predecessors [9]. Firstly they obtained very good results in the visual selection task. Then they turned its application to the visual selection and attention shifting in [10]. They interpret the visual selection and attention shifting from the dynamical mechanism. In 2009, M.G. Qu iles, D.L. Wang and L.Zhao present a neurocomputational model of object-based selection in the framework of oscillatory correlation [11]. This object selection system has been applied to nature images.

In this paper, we construct a new visual selection and attention shifting model and mainly perform the visual selection and attention shifting task when the input is gray image. Our model is based on the F-N model [1].

The rest of the paper is organized as follows. In section 2, we introduce the F-N model and our model. In section 3, the simulation experiments of the proposed model are given. In section 4, we give the conclusions.

## 2 Model Description

### 2.1 Further Study about F-N Equations

The FitzHugh-Nagumo equations describe the interaction between the voltage  $V$  across the axon membrane, which is driven by an input current  $I$  and a recovery variable  $R$ . FitzHugh-Nagumo equations are given as follows:

$$\begin{cases} \frac{dV}{dt} = 10(V - \frac{V^3}{3} - \alpha R + I) \\ \frac{dR}{dt} = 0.8(-R + \beta V + 1.5) \end{cases} \quad (1)$$

Here 10 and 0.8 is the inverse of the time constant for  $V$  and  $R$ , and  $\alpha > 0$ ,  $\beta > 0$  describes the action strength for  $R$  to  $V$  and  $V$  to  $R$  respectively. The time constant for  $V$  is 12.5 times faster than that for  $R$ , which reflects the fact that activation processes in the axon are much more rapid than the recovery processes.

Let  $(V, R)$  be equilibrium point. At equilibrium the Jacobian is:

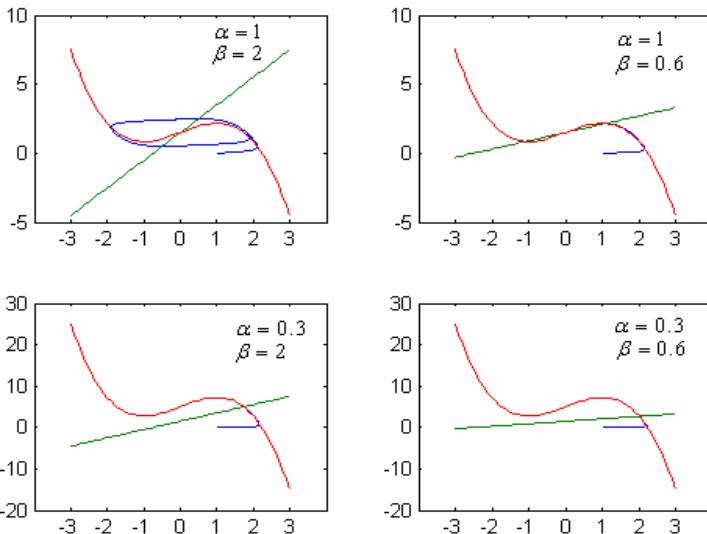
$$A = \begin{pmatrix} 10 - 10V^2 & -10\alpha \\ 0.8\beta & -0.8 \end{pmatrix} \quad (2)$$

Then the characteristic equation is:

$$\lambda^2 + (10V^2 - 9.2)\lambda + 8(V^2 + \alpha\beta - 1) = 0 \quad (3)$$

Let  $\lambda_1$  and  $\lambda_2$  be the characteristic roots of (2). Obviously we have the following conclusions:

- (1) If  $\lambda_1\lambda_2 = 8(V^2 + \alpha\beta - 1) < 0$ , then the equilibrium point  $(V, R)$  is unstable;
- (2) If  $\lambda_1\lambda_2 = 8(V^2 + \alpha\beta - 1) > 0$  and  $\lambda_1 + \lambda_2 = -(10V^2 - 9.2) > 0$  the equilibrium point  $(V, R)$  is unstable;
- (3) If  $\lambda_1\lambda_2 = 8(V^2 + \alpha\beta - 1) > 0$  and  $\lambda_1 + \lambda_2 = -(10V^2 - 9.2) < 0$ , the equilibrium point  $(V, R)$  is stable.



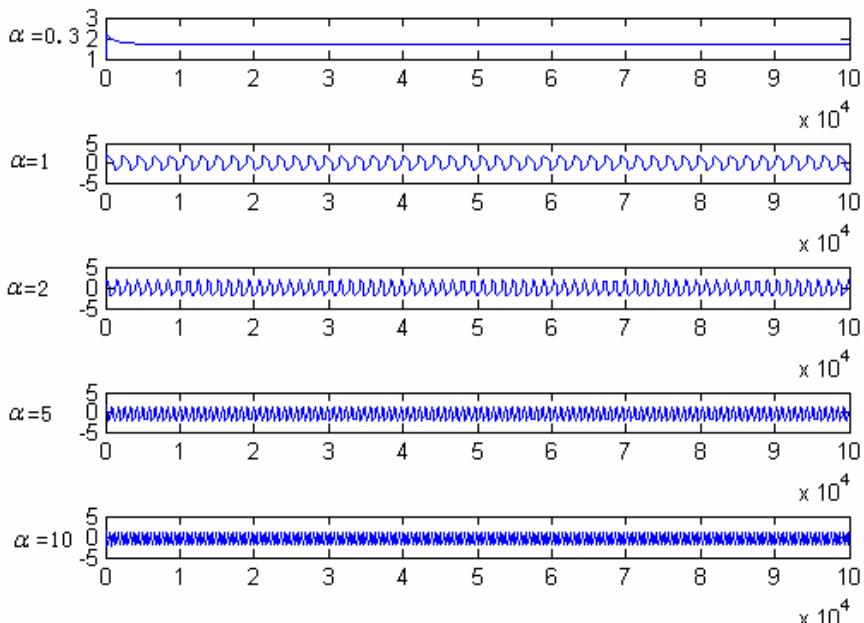
**Fig. 1.** The figure shows the phase plane of FitzHugh-Nagumo equations along with two isolines. The red curve describes  $dV/dt = 0$  and the green line describes  $dR/dt = 0$ . The blue curve shows  $R - V$  trajectory. When  $\alpha = 1$ ,  $\beta = 2$ , (1) yield only one limit cycle. Here  $I = 1.5$ .

Only when the equilibrium point  $(V, R)$  is unstable, there is a stable limit cycle around the equilibrium. When equilibrium is stable, there is no limit cycle or an unstable limit cycle, but unstable limit cycle does not exist in biological systems, it has no sense to biological systems.

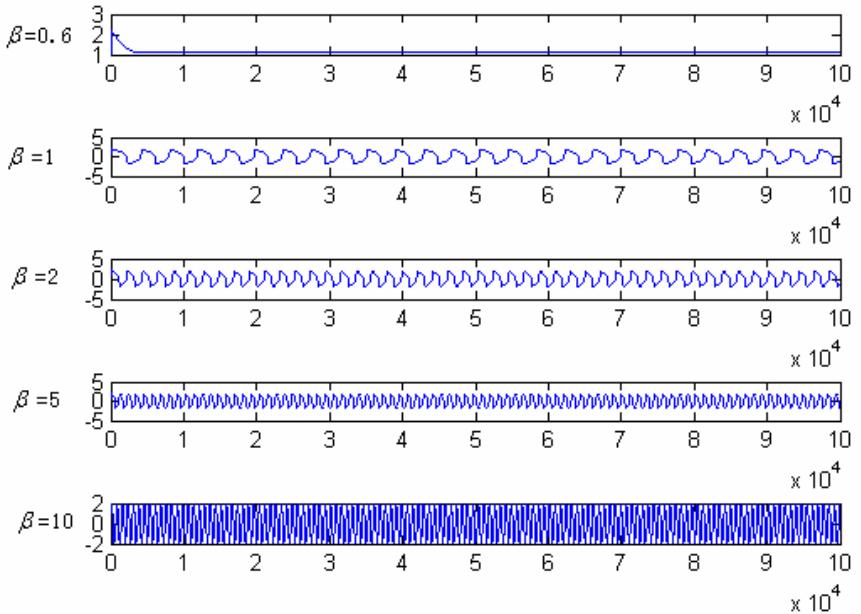
From the three conclusions above, let  $I = 1.5$ , (1) has only one stable limit cycle when  $\alpha = 1$ ,  $\beta = 2$ , no limit cycle occurs when  $\alpha = 1$ ,  $\beta = 0.6$  or  $\alpha = 0.3$ ,  $\beta = 2$  or  $\alpha = 0.3$ ,  $\beta = 0.6$ . Phase plane for (1) with isoclines are shown in Fig 1.

The spiking frequency of (1) can be controlled by changing the parameter  $\alpha$  and  $\beta$  in (1). In Fig 2 and Fig 3 we show the time series of (1) by varying  $\alpha$  and  $\beta$  respectively. From the two figures we notice that as  $\alpha$  and  $\beta$  increase the frequency of (1) increases. When  $\alpha$  or  $\beta$  takes a small value (for example the case of  $\alpha = 1$ ,  $\beta = 0.6$  or  $\alpha = 0.3$ ,  $\beta = 2$  or  $\alpha = 0.3$ ,  $\beta = 0.6$ ), (1) do not fire spikes.

In our model, we take advantages of this to determine visual attention, which means that the synchronized neurons corresponding to the salient object will fire more frequently, while the neurons corresponding to the other objects will fire with low frequency or not fire.



**Fig. 2.**  $\beta = 2$ ,  $I = 1.5$ , the spike sequence when  $\alpha$  equals to 0.3, 1, 2, 5, 10 respectively



**Fig. 3.**  $\alpha = 1$ ,  $I = 1.5$ , the spike sequence when  $\beta$  equals to 0.3,1,2,5,10 respectively

## 2.2 Model Construction

Our model is a two dimensional neuron network governed by the following equations:

$$\begin{cases} \frac{dV_{i,j}}{dt} = 10((V_{i,j} + \Delta V_{i,j}) - \frac{(V_{i,j} + \Delta V_{i,j})^3}{3} - \alpha R_{i,j} + I_{i,j}) \\ \frac{dR_{i,j}}{dt} = 0.8(-(R_{i,j} + \Delta R_{i,j}) + \beta V_{i,j} + 1.5) \end{cases} \quad (4)$$

Where  $(i, j)$  indicates the  $i$  th row  $j$  th column in the image,  $1 \leq i \leq M$ ,  $1 \leq j \leq N$  ( $M$  and  $N$  indicates the size of the image). Where  $\Delta V_{i,j}$  and  $\Delta R_{i,j}$  indicate that the impact of peripheral neurons, which are defined by:

$$\begin{aligned} \Delta x_{i,j} = & \gamma_{i-1,j-1;i,j}(x_{i-1,j-1} - x_{i,j}) + \gamma_{i-1,j;i,j}(x_{i-1,j} - x_{i,j}) + \\ & \gamma_{i-1,j+1;i,j}(x_{i-1,j+1} - x_{i,j}) + \gamma_{i,j-1;i,j}(x_{i,j-1} - x_{i,j}) + \\ & \gamma_{i,j+1;i,j}(x_{i,j+1} - x_{i,j}) + \gamma_{i+1,j-1;i,j}(x_{i+1,j-1} - x_{i,j}) + \\ & \gamma_{i+1,j;i,j}(x_{i+1,j} - x_{i,j}) + \gamma_{i+1,j+1;i,j}(x_{i+1,j+1} - x_{i,j}) \end{aligned} \quad (5)$$

$$\gamma_{i,j;p,q} = \begin{cases} 1 & \text{if neuron } (i,j) \text{ is coupled to } (p,q) \\ 0 & \text{else} \end{cases} \quad (6)$$

Where  $x$  denotes  $V$  or  $R$ .

As the impact of recovery variables of each neuron to other neurons is very small, so here we make  $\Delta R_{i,j} = 0$ .

In order to achieve the behavior that the synchronized neurons corresponding to the salient object will fire more frequently, while the neurons corresponding to the other objects will fire with low frequency or not fire. Firstly, we let the neurons run with fixed parameters  $\alpha$  and  $\beta$  until the neurons corresponding to the same object synchronize, which means that the segmentation task has been performed. Secondly, after the first step, whenever any neuron fires, it will produce two types of signals to itself and other neurons: an excitatory signal to itself and neurons that fire together with it and an inhibitory signal to neurons that don't fire together with it.

Without considering the coupling terms, (4) are the same as (1). From the analysis of (1), we know that parameters  $\alpha$  and  $\beta$  can control the activities of neurons. For example, if the neuron at  $(i, j)$  fires. Then the excitatory and the inhibitory signals can be defined varying parameters  $\alpha$  and  $\beta$  as follows:

$$\alpha_{p,q}(\tau) = \alpha_{p,q}(\tau-1) + \frac{h_1(\alpha_{p,q}(\tau-1))}{M(\tau)} \sum_{i,j \in \Delta(\tau)} I_{i,j} f_1(\|V_{i,j} - V_{p,q}\|) \quad (7)$$

$$\beta_{p,q}(\tau) = \beta_{p,q}(\tau-1) + \frac{h_2(\beta_{p,q}(\tau-1))}{M(\tau)} \sum_{i,j \in \Delta(\tau)} I_{i,j} f_2(\|V_{i,j} - V_{p,q}\|) \quad (8)$$

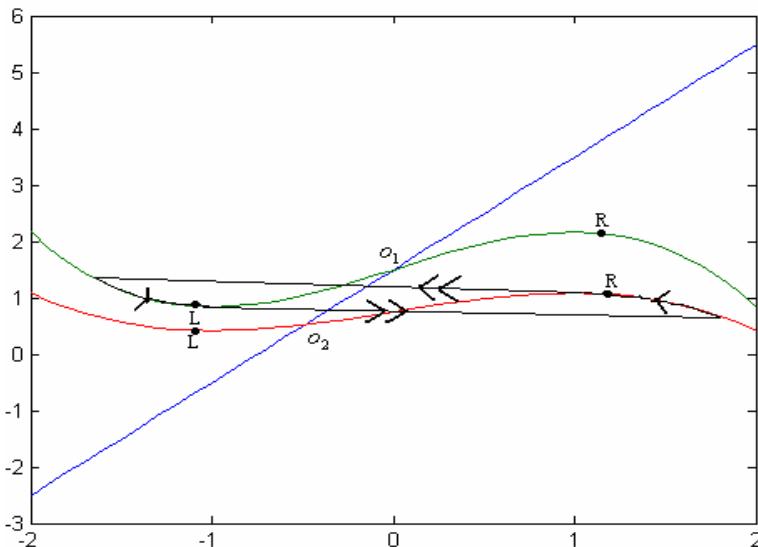
$$\text{Where } h_1(\alpha) = \begin{cases} \theta_1 & \alpha < \theta_\alpha \\ \theta_2 & \alpha \geq \theta_\alpha \end{cases}, \quad h_2(\beta) = \begin{cases} \theta_1 & \beta \geq \theta_\beta \\ \theta_2 & \beta < \theta_\beta \end{cases}, \quad \theta_1 > \theta_2 > 0.$$

$$f_1(x) = a_1 x + b_1 \quad (a_1 > 0, b_1 < 0), \quad f_2(x) = a_2 x + b_2 \quad (a_2 < 0, b_2 > 0).$$

Where  $(p, q)$  indicates the  $p$ th row  $q$ th column in the image, and  $\tau$  is a time instant with at least one firing neuron,  $M(\tau)$  is the number of neurons at the firing state at  $\tau$ , and  $\Delta(\tau)$  is the set of neurons at the firing state at  $\tau$ . By setting  $a_1 > 0, b_1 < 0$  and  $a_2 < 0, b_2 > 0$  and functions  $h_1, h_2$ , defines that each firing neuron  $(i, j)$  send excitatory or inhibitory signals to another neuron  $(p, q)$ .

From (7) and (8) we can find that when any neuron at  $(i, j)$  fires, the parameter  $\alpha$  corresponding to it will increase. If  $\alpha$  increases, the fixed point of the system will move downward along the isoclines  $dR/dt = 0$ . In other words, the intersection of the isoclines  $dV/dt = 0$  and  $dR/dt = 0$  moves downward along the isoclines  $dR/dt = 0$ . As is shown in the Fig 4, when  $\alpha = 1$ , the fixed point is  $o_1$ , and the isoclines  $dV/dt = 0$  is the green curve, and when  $\alpha = 2$ , the fixed point is  $o_2$ , and the isoclines

$dV/dt = 0$  is the red curve. The part with single arrow in the figure is the slow jumping area, and that with double arrow is the quick jump area. Here we call the left part from the point  $L$  of the curve left branch, and the right part from the point  $R$  right branch. If the neuron hasn't fired, the value of parameter  $\alpha$  corresponding to it is smaller. The  $R-V$  trajectory move downward along the left branch of the green curve. When it moves to the point  $L$  of the green curve, it will jump to the right branch from the left branch. The jumping will make the value of the parameter  $\alpha$  increase, and the isoclines  $dV/dt = 0$  move downwardly to the red curve. The  $R-V$  trajectory jumps to the right branch of the red curve, then move upward along the red curve. When it moves to the point  $R$ , it will jump down to the left branch from the right branch. The jumping will make the value of the parameter  $\alpha$  decrease, and the isoclines  $dV/dt = 0$  move upward to the green curve again. In this way, the firing rate of the neuron increases. Conversely, if the value of the parameter  $\alpha$  decreases, it would make neuron fires slower. For the parameter  $\beta$ , the influence of it to the fire rate is contrary.



**Fig. 4.** The trajectory charts When the parameter  $\alpha$  changes. The blue curve is  $dR/dt = 0$ . The green curve is  $dV/dt = 0$  when  $\alpha = 1$ . The red curve is  $dV/dt = 0$  when  $\alpha = 2$ . Here  $o_1$  and  $o_2$  are the fixed points.  $I = 1.5$ ,  $\beta = 2$ .

When the system is running, we can just control the changes of the parameters  $\alpha$  and  $\beta$  to make that the increasing speed of  $\alpha$  is faster than the decreasing speed of  $\beta$ , when the neurons jump. In this way, the most salient object will jump to a high frequency periodic oscillating phase, while all other objects will be quite silent. The attention will focus on the most salient object. After receiving attention, this object is inhibited in order to permit other objects to become salient.

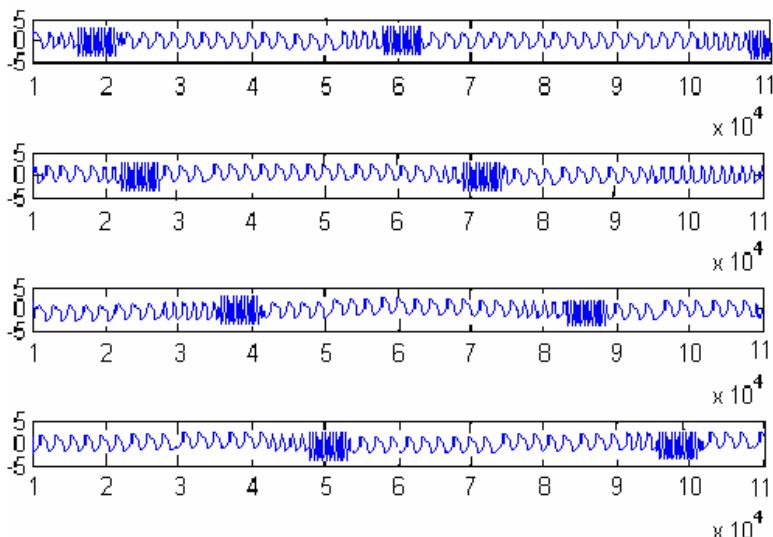
### 3 Computer Simulations

This section presents the simulation results performed on the image in Fig5. In the simulation of this paper, the following parameters are held constant at:  $a_1 = 2$ ,  $b_1 = -4$ ,  $a_2 = -4$ ,  $b_2 = 2$ ,  $\theta_1 = 0.1$ ,  $\theta_2 = 0.01$ ,  $\theta_\alpha = 0.8$ ,  $\theta_\beta = 4$ .



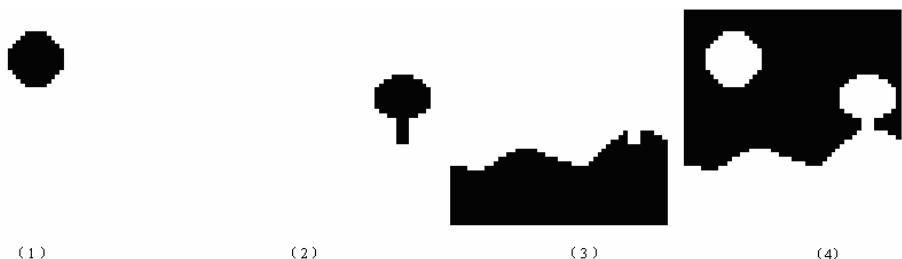
**Fig. 5.** Artificial image with four objects [11]

The image used in our experiment is a grayscale image consisting of four goals (Fig 5), in which the corresponding pixel values of the sun is the largest, followed by the tree hill and sky. Figure 6 shows the spike sequences of the four neuron groups corresponding to the four objects in Fig 5. In our simulations only the intensity of pixels are used as input. It means that the object with the highest intensity receives the attention earlier than other objects with lower intensity.



**Fig. 6.** The spike sequences of the four neuron groups corresponding to the four objects in Fig 5. From the top to bottom they are: sun, tree, hill, sky in turn.

From Fig 6 we can observe that, the firing rate increases gradually after the synchronization of neurons corresponding to the sun object, and finally reaches the maximum. At the same time, the firing rates of neurons corresponding to the other objects are at relatively low level. It explains that the sun is noticed firstly. After that the firing rate of neurons corresponding to the sun object will soon reduce to a lower lever, while the firing rate of neurons corresponding to the tree object will increase to the maximum, but the firing rate of the other two objects remain at relatively low level. It shows that the tree object is noticed, which implements the attention shift from one object to another object. Finally, tree and hill object will be noticed in turn (Fig 7).



**Fig. 7.** Fig (1)-(4) are the objects selected out in turn

## 4 Conclusion

This paper presents a visual selection and attention shifting model based on FitzHugh-Nagumo equations. This system can be seen as a part of visual attention system, which is responsible for selecting the most salient object from an input image and shifting attention from one object to another. The proposed model includes not only cooperation but also competition mechanism.

Computer simulations were performed in order to check our model's viability as a selection and shifting mechanism and the results show that it is a promising system. As a future work we intend to create a new system applied for natural image. In addition, we will also combine top-down and bottom-up attentions, adding the effects of priori knowledge to the visual selection.

## Acknowledgements

This research is partially sponsored by Beijing Municipal Foundation for Excellent Talents(No.20061D0501500211), Beijing Municipal Education Committee (No.KM200610005012), Natural Science Foundation of China (Nos.60673091, 60702031 and 60970087), Hi-Tech Research and Development Program of China (No.2006AA01Z122), Natural Science Foundation of Beijing (Nos.4072023 and 4102013) and National Basic Research Program of China (Nos.2007CB311100 and 2009CB320902).

## References

1. Wang, D.L.: Object selection based on oscillatory correlation. *Neural Networks* 12, 579–592 (1999)
2. Olshausen, B., Field, D.: Emergence of simple-cell receptive field properties by learning a sparse code for natural images. *Nature* 381, 607–609 (1996)
3. Campbell, S.R., Wang, D.L.: Synchronization and desynchronization in a network of locally coupled wilson-cowan oscillators. *IEEE Transactions on Neural Networks* 7, 541–554 (1996)
4. Navalpakkam, V., Itti, L.: An Integrated Model of Top-down and Bottom-up Attention for Optimizing Detection Speed. In: Proceedings of the 2006 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (2006)
5. Deco, G.: A Neurodynamical Model of Visual Attention: Feedback Enhancement of Spatial Resolution in a Hierarchical System. *Journal of Computational Neuroscience* 10, 231–253 (2001)
6. Heinke, D., Humphreys, G.W.: SAIM: A Model of Visual Attention and Neglect. In: Gerstner, W., Hasler, M., Germond, A., Nicoud, J.-D. (eds.) ICANN 1997. LNCS, vol. 1327, pp. 913–918. Springer, Heidelberg (1997)
7. Heinke, D.: Selective Attention for Identification Model: Simulating visual neglect. *Computer Vision and Image Understanding* 100, 172–197 (2005)
8. Itti, L., Koch, C.: Computational modelling of visual attention. *Nature Reviews Neuroscience* 2, 194–203 (2001)
9. Zhao, L., Breve, F.A.: Visual Selection and Shifting Mechanisms Based on a Network of Chaotic Wilson-Cowan Oscillators. In: Third International Conference on Natural Computation (2007)
10. Zhao, L., Macau, E.E.N., Omar, N.: Scene segmentation of the chaotic oscillator Network. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering* 10(7), 1697–1708 (2000)
11. Quiles, M.G., Wang, D.L., Zhao, L., Romero, R.A.F., Huang, D.: An Oscillatory Correlation Model of Object-based Attention. In: Proc. IJCNN (2009)
12. Zhao, L.: A Dynamically Coupled Chaotic Oscillatory Correlation Network. In: Proc. VI Brazilian Symposium on Neural Networks (2000)