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M5-brane Defect and QHE in $AdS_4 \times N(1,1)/\mathcal{N}=3$ SCFT

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We study the $d=11$ gravity dual $AdS_4 \times N(1,1)$ of the $d=3$ $\mathcal{N}=3$ flavored Chern-Simons-matter (CSM) theory. In the dual gravity side, we analyze the M5-brane making the edge on the boundary of AdS_4 and derive the quantized Hall conductivity of the dual gauge theory. In the gauge theory side, this M5-brane intersects the gauge theory at the codimension-one defect.

INTRODUCTION — $D=3$ $\mathcal{N}=6$ Chern-Simons-matter theory with the levels $(k,-k)$ [40] for each node (the ABJM theory) [1] has recently been proposed as the effective theory of multiple M2-branes on the singularity of the orbifold $\mathbb{C}^4/\mathbb{Z}_k$. The ABJM theory could also be explained from the IR limit of an elliptic brane setup in Type IIB string theory. Through T-duality and M-theory lift involving a 2-torus, the elliptic brane setup can be interpreted as the M2-branes transverse to a $d=8$ cone over $S^7/\mathbb{Z}_k$. After including the backreaction of N M2-branes in the $d=11$ supergravity side, the gravity dual [3–5] beomes a solution of $d=11$ supergravity: $AdS_4 \times S^7/\mathbb{Z}_k$. Generalizing their idea to yield elliptic $\mathcal{N}=4$ SCFTs is explored in [6].

Moreover, addition of the flavor degrees of freedom [7] to the ABJM theory was investigated in [8–12] and for $\mathcal{N}=4$ SCFT's, it was investigated in [13]. In the Type IIB string theory, adding massless flavors correspond to further placing D5-branes on the elliptic brane setup and preserve $\mathcal{N}=3$ superconformal symmetry. Through T-duality and M-theory lift, the D5-branes become the Kaluza-Klein monopole. The gravity dual of these SCFT's with flavors was proposed as the $d=11$ supergravity on $AdS_4 \times \mathcal{M}_7$ where the cone over $\mathcal{M}_7$ is a $d=8$ toric hyperKähler manifold [14, 15] with special $sp(2)$ holonomy and with 3/16 supersymmetry. According to [13], here, $\mathcal{M}_7$ was proposed to be the Eschenburg space [16] parametrized by three natural numbers $(t_1,t_2,t_3)=(pN_F,qN_F,kpq)$, where p, q, and N_F correspond to the charges of NS5-branes, $(1,k)$5-branes, and D5-branes in the Type IIB elliptic brane setup, respectively. In particular, the Eschenburg space with $t_1=t_2=t_3=1$ is interesting since it is equal to the regular tri-Sasaki manifold $N(1,1)$. The metric of $N(1,1)$ can be easily written and the KK spectra of $N(1,1)$ was obtained in [17–19]. However, the dual $\mathcal{N}=3$ SCFT has been less known.

The purpose of this paper is to progress the study of the AdS/CFT correspondence between the gravity dual $AdS_4 \times N(1,1)$ and $\mathcal{N}=3$ SCFT. According to [9, 13], the flavored ABJM theory with the Chern-Simons levels $(1,-1)$ and a flavor is considered as the $\mathcal{N}=3$ SCFT dual to the $d=11$ supergravity on $AdS_4 \times N(1,1)$ by analyzing its moduli space.

We are interested in the application of our $AdS_4/\mathcal{N}=3$ SCFT correspondence to the Quantum Hall Effect (QHE) [20] including the Fractional Quantum Hall Effect (FQHE) [21] in the condensed matter physics. Here, the Hall conductivity in the QHE is quantized in units of e^2/h and the longitudinal conductivity vanishes near the Plateau. There were holographic models describing the QHE [22]-[25] and the recent holographic constructions of the FQHE [10, 26, 27] inspired for M2-brane theories. Moreover, non-abelian fractional Hall wavefunctions were obtained from M5 theory compactification [28] (see also [29]). Remind that the Hall current in the QHE doesn't flow in the 2-dimensional space where electrons are localized but flow on an edge called the edge states [20]. According to [26], we introduce an edge M5-brane making the edge on the boundary of AdS_4 where the Hall current flows and derive the Hall conductivity of the dual flavored ABJM theory in the strong coupling region $N \gg k^5$ by analyzing the edge M5-brane. The M5-brane intersection in respect to the brane configurations of the 11-dimensional theory is also interesting since M2-branes corresponding to the $\mathcal{N}=3$ SCFT move the cone over the Eschenburg space or $N(1,1)$.

This paper is organized as follows: 1) We briefly review the $\mathcal{N}=3$ SCFT dual to $d=11$ supergravity on $AdS_4 \times N(1,1)$. 2) We give the short review for the $\mathcal{N}=3$ AdS_4/SCFT correspondence. 3) We derive the QHE from the M5-brane worldvolume action in the dual gravity background.

$d=3$ $\mathcal{N}=3$ SCFT — In the gauge theory side, we consider the $d=3$ $\mathcal{N}=3$ flavored ABJM theory with C-S levels $(1,-1)$ and a flavor proposed in the papers [8–10]. First, the ABJM theory without flavors consists of two gauge multiplets for the two copies of gauge group $U(N)_1 \times U(N)_2$ and bi-fundamental chiral multiplets (A_1,A_2) and $(\bar{B}_1,\bar{B}_2)$ in the $(\mathbf{N}_1,\bar{\mathbf{N}}_2)$ representation. The global symmetry of the ABJM theory at the classical level is baryonic $U(1)_b$ and $SU(4)_R$ R-symmetry. $D=3$ $\mathcal{N}=3$ flavored ABJM theory with C-S levels $(1,-1)$ and a flavor can be constructed by adding the D-term for the fundamental chiral-multiplets $(Q^1,\bar{\tilde{Q}}^1)$ transforming under the first gauge group as $\mathbf{N}_1$ to the ABJM theory and by modifying the superpotential. R-symmetry is now broken to $SU(2)_R$, but the baryonic $U(1)_b$ stays unchanged, and the conformal symmetry is preserved.

The moduli space of $\mathcal{N}=3$ flavored ABJM theory which has C-S levels $(1,-1)$ and a flavor is discussed in [9]. This theory has the ring of chiral operators transforming as **8** under the flavor $SU(3)$ and so has the global $SU(3)\times SU(2)_R$ symmetry. That is, the isometry of these theory is the same as that of $N(1,1)$.

$\mathcal{N}=3$ $AdS_4/SCFT$ — We briefly review the dual $d=11$ supergravity on $AdS_4\times N(1,1)$. We start with the Ricci-flat M-theory background $R^{1,2}\times\mathcal{M}_8$ without the backreaction of N M2-branes. Here, $\mathcal{M}_8$ is the cone over the Eschenburg space $S(1,1,1)(=N(1,1))$. Since the transverse geometry is $d=8$ cone over $N(1,1)$, after the backreaction of N M2-branes we have supersymmetry enhanced to a fraction 3/8 and are left with $AdS_4\times N(1,1)$ under the normalization

$$ds^2_{11D}=\tfrac{R^2}{4}ds^2_{AdS_4}+R^2ds^2_7, \quad (1)$$

$$N=\tfrac{1}{(2\pi\ell_p)^6}\int_{N(1,1)}*F_4,\ F_4=\tfrac{3}{8}R^3vol_{AdS_4}, \quad (2)$$

$$6R^6vol(N(1,1))=\frac{3}{4}\pi^4R^6=(2\pi\ell_p)^6N. \quad (3)$$

Also, we have required $R_{ab}=6g_{ab}$ for the $N(1,1)$ metric. Note that $R=2R_{AdS}$ is the radius of $N(1,1)$. This background is regarded as the gravity dual of our $\mathcal{N}=3$ flavored ABJM theory.

We write the metric of $AdS_4\times N(1,1)$ [30, 31] and the background flux F_4 as

$$ds^2=\Big[\frac{dz^2+dx^pdx_p}{4z^2}+ds^2_{N(1,1)}\Big],\ F_4=6e^0e^1e^2e^3, \quad (4)$$

where $e^{\bar{p}}$ $(\bar{p}=0,1,2,3)$ is the vierbein of the AdS_4 spacetime and we set the $N(1,1)$ radius 1. Here, $ds^2_{N(1,1)}$ is the following metric on the manifold of an SO(3) bundle over $\mathbb{CP}^2$:

$$\begin{aligned}&2ds^2_{N(1,1)}\\ &=d\alpha^2+\frac{1}{4}\sin^2\alpha(\sigma_1^2+\sigma_2^2)+\frac{1}{4}\sin^2\alpha\cos^2\alpha\sigma_3^2\\ &+\frac{1}{2}\Big[(\Sigma_1-\cos\alpha\sigma_1)^2+(\Sigma_2-\cos\alpha\sigma_2)^2\\ &+(\Sigma_3-\frac{1}{2}(1+\cos^2\alpha)\sigma_3)^2\Big], \quad (5)\end{aligned}$$

$$\sigma_1=\sin\phi d\theta-\cos\phi\sin\theta d\psi, \quad (6)$$

$$\sigma_2=\cos\phi d\theta+\sin\phi\sin\theta d\psi,\ \sigma_3=d\phi+\cos\theta d\psi, \quad (7)$$

with $0\le\alpha\le\pi/2$, $0<\theta<\pi$, $0<\phi<4\pi$ and $0<\psi<2\pi$. Here, Σ_i are right-invariant 1-forms on SO(3), and σ_i are right-invariant 1-forms on SU(2). According to [31], Σ_i must be of these forms in order for the part orthogonal to $\mathbb{CP}^2$ metric to be regular. 7-dimensional metric $ds^2_{N(1,1)}$ is scaled as $R_{mn}=6g_{mn}$.

In $N(1,1)$, there are two important submanifolds S^3 at $\alpha=0$, $\Sigma_i=0$ and $S^3/\mathbb{Z}_2$ at $\alpha=0$, $\sigma_i=0$. Since both submanifolds are in $\alpha=0$ where the base $\mathbb{CP}^2$ metric vanishes, vierbeins of both submanifolds are also in the $SO(3)$ bundle direction. The position of S^3 and $S^3/\mathbb{Z}_2$ in $N(1,1)$ is investigated precisely in [30].

Lastly, we want to comment on the M-circle. As stated in [9], the position of the M-circle is the same as that in the GGPT geometry for the ABJM theory [14]. According to [9], moreover, the dilaton in the IIA supergravity (or the coefficient of the M-circle) should not be constant in the internal manifold. Thus, the variables in $S^3/\mathbb{Z}_2$ should not be identified as the M-circle and so we regard the M-circle as an angular variable in S^3 [41].

QHE from the M5-brane worldvolume action — In this section, we analyze the M5-action [33–35] on $AdS_3\times S^3$ and derive the Quantum Hall Effect (QHE). Since we use the dimensional reduction, our analysis is valid in the low energy limit and a part of $d=11$ metric reduces to an 1-form as seen in the reduction to the IIA supergravity. We also break the gauge symmetry $U(N)\times U(N)$ to the N copies of $U(1)\times U(1)$ since the QHE will be described by the $U(1)$ Chern-Simons theory.

The M5-brane is wrapped on the submanifold $AdS_3\times S^3$ parametrized by $\alpha=\Sigma_1=\Sigma_2=\Sigma_3=0$. The 6-dimensional worldvolume coordinates are parametrized by $\xi^{\hat{m}}$ $(\hat{m}=0,...,5)$: $\xi^0=t,\ \xi^1=x^1,\ \xi^2=z,\ \xi^3=\psi,\ \xi^4=\theta,\ \xi^5=\phi'=\phi/2$. The induced metric and the gauge field a on the M5-brane are given by

$$\begin{aligned}ds^2&=\frac{1}{4}ds^2_{AdS_3}\\ &+\frac{1}{4}\Big(d\theta^2+\sin^2\theta d\psi^2\Big)+\Big(d\phi'+\frac{1}{2}\cos\theta d\psi\Big)^2,\\ a&=\frac{1}{2}\cos\theta d\psi, \quad (8)\end{aligned}$$

where $\int_{S^2}F'=2\pi$ $(F'=da)$.

The 6-dimensional metric $G_{\hat{m}\hat{n}}$ contains 5-dimensional pieces G_{mn},G_{m5},G_{55}. By setting $G_{5,5}=1$ and $\xi^5=\phi'(=X^{11})$, we can represent $G_{\hat{m}\hat{n}}$ by using the 6×6 matrix

$$G_{\hat{m}\hat{n}}=\begin{pmatrix}G^{(6)}_{mn}+a_ma_n & a\\ a^\top & 1\end{pmatrix}, \quad (9)$$

where we have rewritten G_{5m} as a_m for convenience since after the dimensional reduction, a_m become 1-forms. We introduce the self-dual tensor gauge field by B_{mn} and $H_{mnr}=3\partial_{[m}B_{nr]}$. Dual of H_3 becomes $\tilde{H}^{mn}=\epsilon^{mnrls}H_{rls}/6$, where ϵ^{mnrls} is the 5-dimensional flat epsilon symbol. It is convenient to define $\mathcal{H}_3=H_3-b_3$, where b_3 is the 3-form in the 11-dimensional supergravity.

The action of the M5-brane [34, 35] is $L_1 + L_2$ where

$$L_1 = -\sqrt{-G}\sqrt{1 + z_1 + \frac{1}{2}z_1^2 - z_2}, \tag{10}$$

$$L_2 = \frac{1}{8}\epsilon_{mnrls}\frac{G^{5r}}{G^{55}}\tilde{H}^{mn}\tilde{H}^{ls}, \tag{11}$$

$$z_1 = \frac{G_{mn}\tilde{H}^{nr}G_{rl}\tilde{H}^{lm}}{2(-G_5)} \equiv \frac{\mathrm{Tr}(G\tilde{H}G\tilde{H})}{2(-G_5)}, \tag{12}$$

$$z_2 = \frac{\mathrm{Tr}(G\tilde{H}G\tilde{H}G\tilde{H}G\tilde{H})}{4(-G_5)^2}. \tag{13}$$

Here, G is the 6-dimensional determinant, G_5 is the 5-dimensional determinant written with $G_{mn} + a_m a_n$, and $G^{\hat{m}\hat{n}}$ is the inverse of $G_{\hat{m}\hat{n}}$. We have not written WZ term, since we don't need it for later analysis.

By dropping all dependence on ξ^5, we obtain the following 5-dimensional action:

$$S = -\frac{1}{(2\pi)^4}\int d^5\xi\Big(\sqrt{-G}\sqrt{1 + z_1 + \frac{z_1^2}{2} - z_2} + \frac{\epsilon_{mnlst}a^m\tilde{H}^{nl}\tilde{H}^{st}}{8(1+a^2)}\Big). \tag{14}$$

This action is equal to the following action [35] with the Lagrange multiplier

$$S_{5d} = -\frac{1}{(2\pi)^4}\int d^5\xi\Big(\sqrt{-\det(G_{mn} + 2\pi\mathcal{F}_{mn})} + \int\Big(2\pi\mathcal{H}\wedge\mathcal{F} - \frac{(2\pi)^2}{2}a\wedge\mathcal{F}\wedge\mathcal{F}\Big)\Big), \tag{15}$$

where $\mathcal{F}_2$ is the worldvolume 2-form field strength assumed to be the 2-form on AdS_3. Note that the EOM of $\mathcal{F}$ gives $\tilde{\mathcal{H}}$ as the following function of $\mathcal{F}$ and a:

$$\tilde{\mathcal{H}} = \frac{2\pi}{2}\sqrt{-\det(G_{\mu\nu} + 2\pi\mathcal{F}_{\mu\nu})}\mathcal{F}^{\mu\nu} + 2\pi\hat{*}a\wedge\mathcal{F}, \tag{16}$$

where $\hat{*}$ is the 5-dimensional flat epsilon symbol.

By integration by parts and by integrating $F' = da$, the 5-dimensional action (15) reduces to the DBI+Chern-Simons action

$$\begin{aligned} S_{5d} &= -\frac{1}{(2\pi)^4}\int d^5\xi\sqrt{-\det(G_{mn} + 2\pi\mathcal{F}_{mn})} + \frac{1}{4\pi}\int A\wedge F \\ &= -\frac{1}{(4\pi)^4}\int dx^0dx^1dz\frac{1}{z^3}\cdot \\ &\quad\cdot\sqrt{1 + 32\pi^2(z^4F_{x^1z}^2 - z^4F_{x^0z}^2 - z^4F_{x^0x^1}^2)} + \frac{1}{4\pi}\int A\wedge dA. \end{aligned} \tag{17}$$

We shortly review how to derive the Hall conductivity from (17). We add the following boundary term [36] at $z = \epsilon$:

$$S_{bdy} = \frac{1}{4\pi}\int_{z=\epsilon} dx^0dx^1A_{x^0}A_{x^1}. \tag{18}$$

If the condition $\partial_{x^0}A_{x^1} = 0$ is satisfied, then, the full action is invariant under the gauge transformation $\delta A = d\chi$ as follows:

$$\begin{aligned} &\delta\frac{1}{4\pi}\Big(\int A\wedge dA + \int dx^0dx^1A_{x^0}A_{x^1}\Big) \\ &= \frac{1}{2\pi}\int dx^0dx^1\delta A_{x^0}A_{x^1} \\ &= -\frac{1}{2\pi}\int dx^0dx^1\chi\partial_{x^0}A_{x^1} = 0. \end{aligned} \tag{19}$$

This condition means that we can still fix the worldvolume electric flux $F_{x^0x^1} \neq 0$. Remind also that the on-shell variation of the Chern-Simons term and the boundary term have only one of the two variables $\delta A_{x^0}, \delta A_{x^1}$: A_{x^0} and A_{x^1} are considered as a pair of canonical variables with respect to z. To compute the Hall conductivity, moreover, the following boundary conditions should be satisfied:

$$\delta A_{x^0}|_{bdy} = 0 \quad \text{and} \quad \delta A_{x^1}|_{bdy} \quad \text{free}. \tag{20}$$

The Hall conductivity can be derived from the continuity equation

$$J_{x^2} = -\left(\frac{\partial\rho}{\partial x^0} + \frac{\partial j_{x^1}}{\partial x^1}\right). \tag{21}$$

To compute j_{x^1} and ρ holographically, we need the EOM of (17) and the relation given in [4, 5].

The equation of motion for (17) are

$$\partial_{x^1}\left(\frac{\alpha zF_{x^1z}}{\sqrt{D}}\right) - \partial_{x^0}\left(\frac{\alpha zF_{x^0z}}{\sqrt{D}}\right) + \frac{1}{2\pi}F_{x^0x^1} = 0,$$

$$\partial_z\left(\frac{\alpha zF_{x^1z}}{\sqrt{D}}\right) + \partial_{x^0}\left(\frac{\alpha zF_{x^0x^1}}{\sqrt{D}}\right) + \frac{1}{2\pi}F_{x^0z} = 0, \tag{22}$$

$$\partial_z\left(\frac{\alpha zF_{x^0z}}{\sqrt{D}}\right) + \partial_{x^1}\left(\frac{\alpha zF_{x^0x^1}}{\sqrt{D}}\right) + \frac{1}{2\pi}F_{x^1z} = 0, \tag{23}$$

where $D = 1 + 32\pi^2(z^4F_{x^1z}^2 - z^4F_{x^0z}^2 - z^4F_{x^0x^1}^2)$ and α is a constant. The boundary currents are given by

$$\rho = -\frac{\alpha zF_{x^0z}}{\sqrt{D}} + \frac{1}{2\pi}A_{x^1}, \quad j_{x^1} = \frac{\alpha zF_{x^1z}}{\sqrt{D}}. \tag{24}$$

By substituting (24) and (23) into (21), we obtain the Hall conductivity:

$$j_{x^2} = -\frac{1}{2\pi}\partial_{x^1}A_{x^0} = \frac{1}{2\pi}E_{x^1}, \tag{25}$$

$$\to \sigma_{x^1x^2} = \frac{e^2}{h},\ \nu = 1, \tag{26}$$

where we recovered the Planck unit $\hbar = h/2\pi$ and the electric charge e. Here, $\nu = 1$ is the filling fraction. Thus, the Hall conductivity is quantized in terms of the C-S level of the dual theory $k = 1$ and there are no other contributions (see also [10]).

It will also be interesting to derive conductivities of the boundary liquid in particular in the finite temperature [37] since for theories with gapless excitation, the conductivities in the boundary fluid will not behave like normal Fermi liquid. However, the full EOM of (17) seems not to be solved analytically and the finite temperature GKPW relation of the DBI-CS theories on AdS_3 seems to be complicated.

SUMMARY — In this paper, we studied the $AdS_4 \times N(1,1)/\mathcal{N} = 3$ SCFT correspondence confirmed by the analysis of the moduli space [9, 13].

We introduced the edge M5-brane making the edge state on the boundary in the dual gravity side. By analyzing the edge M5-brane, we derived the QHE of the $d = 3$ flavored ABJM theory with the C-S levels $(1, -1)$. We found that the Hall conductivity is quantized such as $\sigma_{xy} = e^2/h$, where the filling fraction is 1. The same method can also be applied for the gravity dual $AdS_4 \times S^7/\mathbb{Z}_k$ of the ABJM theory with C-S levels $(k, -k)$. Thus, we introduce the M5-brane wrapped on a 3-cycle $S^3/\mathbb{Z}_k$ and can derive the fractionally quantized Hall conductivity $\sigma_{xy} = e^2/kh$. This result was derived in [26] and confirmed the dimensional reduction. The different point is that S^3 (and $S^3/\mathbb{Z}_2$) in $N(1,1)$ are in trivial Homology class and $S^3/\mathbb{Z}_k$ is the 3-cycle on $S^7/\mathbb{Z}_k$. So, the M5-brane wrapped on the 3-cycle $S^3/\mathbb{Z}_k$ is the fractional M5-brane [32].

According to [32], it will be interesting to discuss the domain walls [38] in our setup that changes the rank of one gauge group by 1 (see also the paper [39]). However, there seems to be only one sphere S^3 and the information of which rank the M5-brane can change seems to be lost. Moreover, $N(1,1)$ background which can be described by $d = 8$ instanton solution contains another KK-monopole that makes the analysis complicated. In the presence of this monopole, we cannot conclude that our edge M5-brane changes the rank of the gauge group on one side of this M5-brane.

The M5-brane wrapping on $S^3/\mathbb{Z}_2$ parametrized by $\alpha = \sigma_1 = \sigma_2 = \sigma_3 = 0$ is also interesting. As pointed out in [10], the effect of the $\mathbb{Z}_2$ Wilson line on $S^3/\mathbb{Z}_2$ should be considered and so using the M5-brane without the manifest $d = 6$ covariance seems to be not appropriate. We leave analysis of this M5-brane for future work.

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