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$\mathcal{N} = 2$ supergravity and supercurrents

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Abstract

We address the problem of classifying all $\mathcal{N} = 2$ supercurrent multiplets in four space-time dimensions. For this purpose we consider the minimal formulation of $\mathcal{N} = 2$ Poincaré supergravity with a tensor compensator, and derive its linearized action in terms of three $\mathcal{N} = 2$ off-shell multiplets: an unconstrained scalar superfield, a vector multiplet, and a tensor multiplet. Such an action was ruled out to exist in the past. Using the action constructed, one can derive other models for linearized $\mathcal{N} = 2$ supergravity by applying $\mathcal{N} = 2$ superfield duality transformations. The action depends parametrically on a constant real isotriplet $g^{ij} = g^{ji} \neq 0$ which originates as an expectation value of the tensor compensator. Upon reduction to $\mathcal{N} = 1$ superfields, we show that the model describes two dually equivalent formulations for the massless multiplet $(1, 3/2) \oplus (3/2, 2)$ depending on a choice of g^{ij}. In the case $g^{\underline{11}} = g^{\underline{22}} = 0$, the action describes (i) new minimal $\mathcal{N} = 1$ supergravity; and (ii) the Fradkin-Vasiliev-de Wit-van Holten gravitino multiplet. In the case $g^{\underline{12}} = 0$, on the other hand, the action describes (i) old minimal $\mathcal{N} = 1$ supergravity; and (ii) the Ogievetsky-Sokatchev gravitino multiplet.

Contents

1 Introduction

The supercurrent [1] is a supermultiplet which contains the conserved energy-momentum tensor and the conserved supersymmetry current, along with some other components including the R-current (which is not always conserved). Its fundamental significance is due to the fact that this multiplet embraces all the conserved currents associated with the super-Poincaré symmetry. In complete analogy with the energy-momentum tensor, which is the source of gravity, the supercurrent is the source of supergravity [2, 3].

The structure of supercurrent multiplets in $\mathcal{N}=1$ supersymmetric theories is fully understood. The supercurrent can be consistently derived by varying the matter action in a curved superspace with respect to the supergravity prepotentials, and then restricting to the flat superspace background. Since there exist several off-shell formulations for $\mathcal{N}=1$ supergravity (specifically, the old minimal [4], the new minimal [5] and the non-minimal [6] formulations), they lead to different supercurrent multiplets (see textbooks [7, 8] for pedagogical reviews) of which the Ferrara-Zumino multipet [1] corresponds to old minimal supergravity. The technique of deriving the supercurrent from off-shell supergravity, which was described in detail in [7, 8], can be streamlined and re-formulated as a superfield Noether procedure [9, 10]. Recently, there has been much interest in various aspects of the supercurrents emerging in gauge theories with Fayet-Iliopoulos terms and nonlinear sigma-models with non-exact Kähler forms [11, 12, 13, 14, 15, 16, 17] inspired by the work of Komargodski and Seiberg [11, 14].

Unlike the case of simple supersymmetry, the structure of supercurrents in $\mathcal{N}=2$ supersymmetric theories is much less studied. In the early papers [18, 19, 20], supercurrents were studied on a model-dependent basis. At that time it was practically impossible to derive supercurrents starting from supergravity, since only the discovery of harmonic superspace [21] made possible the construction of fully-fledged prepotential formulations for $\mathcal{N}=2$ supergravity [22, 23] (see [24] for a review.) The supergravity origin of several $\mathcal{N}=2$ supercurrent multiplets was revealed in [25].

The oldest $\mathcal{N}=2$ supercurrent multiplet was introduced by Sohnius [18] by considering the hypermultiplet. The supercurrent is described by a real scalar superfield $\mathcal{J}$. The associated trace supermultiplet is an isotriplet, $\mathcal{T}^{ij}=\mathcal{T}^{ji}$, constrained by

$$D_\alpha^{(i}\mathcal{T}^{jk)}=\bar{D}_{\dot\alpha}^{(i}\mathcal{T}^{jk)}=0\ ,\qquad (\mathcal{T}^{ij})^*=\mathcal{T}_{ij}=\varepsilon_{ik}\varepsilon_{jl}\mathcal{T}^{kl}\ . \tag{1.1}$$

Neither the supercurrent nor the multiplet of anomalies have a central charge. The constraints (1.1) are characteristic of the $\mathcal{N}=2$ linear multiplet [26, 27]. The supercurrent conservation equation is[1]

$$\frac{1}{4}\bar{D}^{ij}\mathcal{J} = \mathrm{i}\,\mathcal{T}^{ij} = -\frac{1}{4}D^{ij}\mathcal{J} \ , \tag{1.2}$$

where $D^{ij} := D^{\alpha i}D_\alpha^j = D^{ji}$ and $\bar{D}^{ij} := \bar{D}^{(i}_{\dot\alpha}\bar{D}^{j)\dot\alpha} = \bar{D}^{ji}$.

It is worth giving a couple of examples of $\mathcal{N}=2$ supersymmetric theories in which the supercurrent has the type just described. First of all, we consider the $\mathcal{N}=2$ Maxwell action [28]

$$S_{\rm V} = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\, W^2 \ , \qquad \bar{D}^{\dot\alpha}_i W = 0 \ , \tag{1.3}$$

where W is the chiral field strength of an Abelian vector multiplet. It obeys the Bianchi identity

$$D^{ij}W = \bar{D}^{ij}\bar{W} \ . \tag{1.4}$$

The field strength can be constructed in terms of Mezincescu's prepotential [29], V_{ij}, which is a real unconstrained $SU(2)$ triplet:

$$W = \bar{D}^4 D^{ij}V_{ij} \ , \qquad V_{ij} = V_{ji} \ , \qquad (V_{ij})^* = V^{ij} = \varepsilon^{ik}\varepsilon^{jl}V_{kl} \ . \tag{1.5}$$

The supercurrent for this model [19] is

$$\mathcal{J} = W\bar{W} \ , \qquad \mathcal{T}^{ij} = 0 \ . \tag{1.6}$$

A more interesting example is given by the low-energy effective action [30]

$$S = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\, F(W^I) + \int \mathrm{d}^4x\,\mathrm{d}^4\bar\theta\, \bar{F}(\bar{W}^I) \ , \tag{1.7}$$

with F a holomorphic function of n variables. For this theory, the supercurrent and the trace supermultiplet are

$$\mathcal{J} = \bar{W}^I F_I(W) + W^I \bar{F}_I(W) + \Big\{W^I F_I(W) - 2F(W) + \text{c.c.}\Big\} \ , \tag{1.8a}$$

$$\mathcal{T}^{ij} = \frac{\mathrm{i}}{4}D^{ij}\Big\{W^I F_I(W) - 2F(W)\Big\} + \text{c.c.} \tag{1.8b}$$

[1] The conservation equation (1.2) can be rewritten in a different form $\bar{D}^{ij}\hat{\mathcal{J}} = D^{ij}\hat{\mathcal{J}} = 4\hat{\mathcal{T}}^{ij}$, where $\hat{\mathcal{T}}^{ij}$ is a real isotriplet obeying the constraint $D^{(i}_\alpha \hat{\mathcal{T}}^{jk)} = \bar{D}^{(i}_{\dot\alpha}\hat{\mathcal{T}}^{jk)} = 0$. This form is obtained from (1.2) in two steps. First, one represents $\mathcal{T}^{ij} = -\mathrm{i}\,D^{ij}\Psi + \mathrm{i}\,\bar{D}^{ij}\bar{\Psi}$, for some chiral superfield Ψ. Secondly, one defines the modified supercurrent $\hat{\mathcal{J}} := \mathcal{J} + 8(\Psi + \overline{\Psi})$. It obeys the conservation equation postulated, with $\hat{\mathcal{T}}^{ij} := D^{ij}\Psi + \bar{D}^{ij}\bar{\Psi}$.

In a somewhat different form, this supercurrent was derived in [10].

Now, consider the $\mathcal{N}=2$ vector multiplet with a Fayet-Iliopoulos term

$$S_{\rm V+FI} = \frac{1}{2}\int {\rm d}^4x\,{\rm d}^4\theta\,W^2 - \int {\rm d}^4x\,{\rm d}^4\theta\,{\rm d}^4\bar\theta\,\xi^{ij}V_{ij}~, \qquad \xi^{ij} = {\rm const}~. \tag{1.9}$$

The corresponding equation of motion is $D^{ij}W = \xi^{ij}$. One can see that the only gauge invariant candidate for the supercurrent is again $\mathcal{J} = W\bar{W}$. However, the supercurrent conservation equation becomes

$$\bar{D}^{ij}\mathcal{J} = \xi^{ij}W~. \tag{1.10}$$

It is obvious that this conservation equation differs from that in eq. (1.2). The supergravity origin of this difference is very simple. As shown in [25], the conservation equation (1.2) occurs in those $\mathcal{N}=2$ supersymmetric theories which couple only to the minimal multiplet of $\mathcal{N}=2$ supergravity [26] (i.e., the Weyl multiplet [31, 32] coupled to an Abelian vector multiplet, the latter being the first supergravity compensator), and do not couple to a second supergravity compensator. In the case under consideration, the Fayet-Iliopoulos term directly couples to the second compensator in the off-shell formulation for $\mathcal{N}=2$ Poincaré supergravity proposed in [33]. Its second compensator is the improved $\mathcal{N}=2$ tensor multiplet [33, 34] which is a natural generalization of the improved $\mathcal{N}=1$ tensor multiplet [35].

The above example provides enough rational for investigating general $\mathcal{N}=2$ supercurrent multiplets. In the case of $\mathcal{N}=1$ supersymmetry, the variant supercurrents are naturally associated with linearized off-shell formulations for $\mathcal{N}=1$ supergravity. Given such a formulation, the supercurrent conservation equation can be obtained by coupling the supergravity prepotentials to external sources and then demanding the resulting action to be invariant under the linearized supergravity gauge transformations. Since the linearized off-shell $\mathcal{N}=1$ supergravity actions have been classified [36], all consistent supercurrents can be generated. This has been carried out in [15]. We wish to extend the $\mathcal{N}=1$ construction to the $\mathcal{N}=2$ case. Our approach to addressing this problem consists in constructing a linearized superfield action for $\mathcal{N}=2$ supergravity formulation proposed in [33] at the component level. Other linearized supergravity actions can be obtained from the one constructed below by superfield duality transformations. We should mention that some models for linearized $\mathcal{N}=2$ supergravity appeared in the early 1980s [37, 38] shortly before Ref. [33] appeared. We will comment on these later on.

The supergravity formulation of [33] was originally derived in components. Its reformulation in superspace is necessary for our goals. There exist two fully-fledged manifestly supersymmetric settings to describe general $\mathcal{N} = 2$ supergravity-matter systems: (i) the harmonic superspace approach [22, 23, 39] (see [24] for a review); and (ii) the projective superspace approach [40, 41, 42]. We will use both of them in the present paper.

In the projective superspace approach [40, 41, 42], the action for pure $\mathcal{N} = 2$ Poincaré supergravity with tensor compensator [33] consists of two terms

$$S_{\rm SUGRA} = S_{\rm minimal} + S_{\rm tensor} \ , \tag{1.11a}$$

$$S_{\rm minimal} = -\frac{1}{2\kappa^2}\int {\rm d}^4x\, {\rm d}^4\theta\, \mathcal{E}\, \mathcal{W}^2 = -\frac{1}{4\kappa^2}\int {\rm d}^4x\, {\rm d}^4\theta\, \mathcal{E}\, \mathcal{W}^2 + {\rm c.c.} \ , \tag{1.11b}$$

$$S_{\rm tensor} = \frac{1}{2\pi\kappa^2}\oint_C v^i {\rm d}v_i \int {\rm d}^4x\, {\rm d}^4\theta {\rm d}^4\bar\theta\, \frac{E}{S^{(2)}\breve{S}^{(2)}}\, \mathcal{G}^{(2)} \ln \frac{\mathcal{G}^{(2)}}{{\rm i}\breve{\Upsilon}^{(1)}\Upsilon^{(1)}} \ , \tag{1.11c}$$

with κ the gravitational coupling constant. Here the first term, $S_{\rm minimal}$, corresponds to the minimal supergravity multiplet [26]. It describes the coupling of the Weyl supergravity multiplet [31, 32] to an Abelian vector multiplet (with a wrong sign for the kinetic term). The Weyl multiplet is described using Howe's superspace geometry [43] (see also [44]) elaborated in detail in [41]. The vector multiplet is described by a covariantly chiral field strength $\mathcal{W}$ and its conjugate $\bar{\mathcal{W}}$,

$$\bar{\mathcal{D}}^{\dot\alpha}_i \mathcal{W} = 0 \ , \qquad \Big(\frac{1}{4}\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha + S^{ij}\Big)\mathcal{W} = \Big(\frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}{}^{(i}\bar{\mathcal{D}}^{j)\dot\alpha} + \bar{S}^{ij}\Big)\bar{\mathcal{W}} \ , \tag{1.12}$$

where S^{ij} and $\bar S^{ij}$ are special dimension-1 components of the torsion, see [41] for more details. Finally, the superfield $\mathcal{E}$ in (1.11b) denotes the chiral density, see [44, 45] for its definition.

The second term in the supergravity action, $S_{\rm tensor}$, describes the coupling of the Weyl multiplet to an improved tensor multiplet (with wrong sign for the kinetic term). The action involves a closed contour integral over auxiliary isotwistor variables $v^i \in \mathbb{C}^2 \setminus \{0\}$. The tensor compensator is described by its field strength $\mathcal{G}^{ij}$, which appears as $\mathcal{G}^{(2)}$ and obeys covariant constraints, namely

$$\mathcal{G}^{(2)}(v) := \mathcal{G}^{ij}\, v_i v_j \ , \qquad (\mathcal{G}^{ij})^* = \mathcal{G}_{ij} \ , \qquad \mathcal{D}^{(i}_\alpha \mathcal{G}^{jk)} = \bar{\mathcal{D}}^{(i}_{\dot\alpha}\mathcal{G}^{jk)} = 0 \ . \tag{1.13}$$

The superfields $S^{(2)}$ and $\breve{S}^{(2)}$ in (1.11c) are defined as

$$S^{(2)}(v) := S^{ij}\, v_i v_j \qquad \breve{S}^{(2)}(v) := \bar{S}^{ij}\, v_i v_j \ . \tag{1.14}$$

As usual, $E = {\rm Ber}(E_M{}^A)$ denotes the full superspace density, with $E_M{}^A$ being the super-vielbein. Finally, $\Upsilon^{(1)}(v)$ denotes a weight-1 covariant arctic hypermultiplet and $\breve{\Upsilon}^{(1)}(v)$ its smile-conjugate (see [41] for the definition of smile conjugation). The superfields $\Upsilon^{(1)}$ and $\breve{\Upsilon}^{(1)}$ are purely gauge degrees of freedom, as proved in [45]. This property is analogous to that characteristic of the $\mathcal{N}=1$ improved tensor multiplet [35], that is: the corresponding action in Minkowski superpsace

$$S \propto \int \mathrm{d}^4x\,\mathrm{d}^4\theta\, G \ln\big(G/\phi\bar{\phi}\big)\ , \qquad \bar{D}^2 G = D^2 G = 0 \tag{1.15}$$

does not depend on the chiral superfield ϕ or its conjugate. The supergravity action $S_{\rm SUGRA}$ is invariant under arbitrary supergravity gauge and super-Weyl transformations (see [41] for more details).

Our goal is to linearize the action $S_{\rm SUGRA}$ around Minkowski superspace which is an exact solution of $\mathcal{N}=2$ Poincaré supergravity. The vector and the tensor compensators must be characterized by *non-vanishing* constant background values related to each other by

$$\mathcal{W}\bar{\mathcal{W}} = \sqrt{\frac{1}{2}\mathcal{G}^{ij}\mathcal{G}_{ij}}\ , \qquad \mathcal{W}\neq 0\ , \tag{1.16}$$

which is the equation of motion for the gravitational superfield. This equation has a natural counterpart at the component level [33]. In what follows, we set $\kappa=1$.

This paper is organized as follows. In section 2 we elaborate on the supergravity origin of various $\mathcal{N}=2$ supercurrent multiplets building on the results obtained in [25]. In section 3 we derive the linearized action for $\mathcal{N}=2$ supergravity. Its reduction to $\mathcal{N}=1$ superfields is carried out in section 4. Some implications of our results are discussed in section 5. Appendix A provides a brief review of the $\mathcal{N}=1$ superfield formulation [34] for the improved $\mathcal{N}=2$ tensor multiplet. Appendix B contains the technical details of the $\mathcal{N}=1$ reduction.

2 The supergravity origin of supercurrents

As shown in [25], in harmonic superspace the minimal supergravity multiplet [26] can be described by two prepotentials, $H(z,u)$ and $V_5^{++}(z,u)$. The gravitational superfield H describes the Weyl multiplet [31, 32]. It is a real unconstrained superfield with the Fourier expansion[1]

$$H(z,u) = \mathbf{H}(z) + \sum_{n=1}^{\infty} H^{(i_1\cdots i_n j_1\cdots j_n)}(z) u^+_{i_1}\cdots u^+_{i_n} u^-_{j_1}\cdots u^-_{j_n} = \breve{H}(z,u)\ . \tag{2.1}$$

[1]The harmonics $(u_i^-, u_i^+) \in SU(2)$ obey $u^{i+}u_i^- = 1$, $u_i^+ = \epsilon_{ij}u^{j+}$, and $(u^{i+})^* = u_i^-$.

The second prepotential V_5^{++} is a real analytic superfield of $U(1)$ charge $+2$,

$$D^+_\alpha V_5^{++} = \bar{D}^+_{\dot\alpha} V_5^{++} = 0 \ , \qquad \breve{V}_5^{++} = V_5^{++} \ , \tag{2.2}$$

where $D^\pm_\alpha = u^\pm_i D^i_\alpha$ and $\bar{D}^\pm_{\dot\alpha} = u^\pm_i \bar{D}^i_{\dot\alpha}$. It describes an Abelian vector multiplet which gauges the central charge that can be interpreted as the derivative in an extra bosonic coordinate x^5. The central charge gauge field can be represented by a Fourier series[2]

$$\begin{aligned} V_5^{++}(z,u) &= (D^+)^4 U^{--}(z,u) \ , \\ U^{--}(z,u) &= \mathbf{V}^{ij}(z)\, u^-_i u^-_j + \sum_{n=2}^{\infty} U^{(i_1 \cdots i_{n-1} j_1 \cdots j_{n+1})}(z)\, u^+_{i_1} \cdots u^+_{i_{n-1}} u^-_{j_1} \cdots u^-_{j_{n+1}} \ , \end{aligned} \tag{2.3}$$

with $\mathbf{V}^{ij}$ Mezincescu's prepotential (compare with eq. (1.5)).

The minimal supergravity multiplet is characterized by three types of gauge symmetries. Here we present their linearized form only; see [25] for the complete discussion. First of all, we have the so-called pre-gauge invariance

$$\begin{aligned} \delta H &= \frac{1}{4}(D^+)^2 \Omega^{--} + \frac{1}{4}(\bar{D}^+)^2 \breve{\Omega}^{--} \ , \\ \delta V_5^{++} &= -\bar{w}(D^+)^4 \Omega^{--} - w (D^+)^4 \breve{\Omega}^{--} \ , \end{aligned} \tag{2.4}$$

with the parameter $\Omega^{--}(z,u)$ an unconstrained complex harmonic superfield,

$$\Omega^{--}(z,u) = \sum_{n=1}^{\infty} \Omega^{(i_1 \cdots i_{n-1} j_1 \cdots j_{n+1})}(z)\, u^+_{i_1} \cdots u^+_{i_{n-1}} u^-_{j_1} \cdots u^-_{j_{n+1}} \ . \tag{2.5}$$

The constant w is the vacuum value of the first compensator $\mathcal{W}$ in (1.11b). In [25], the super-Weyl and local $U(1)$ gauge

$$w = i \tag{2.6}$$

was chosen. We use the same gauge in the present section, but we will keep w arbitrary in section 3.

The second gauge freedom corresponds to linearized general coordinate transformations[3]

$$\delta H = -D^{++} l^{--} \ , \qquad \delta V_5^{++} = 0 \ , \tag{2.7}$$

[2]In what follows, we use several conventions conventions for products of spinor derivatives, specifically: $D^4 := \frac{1}{48} D^{ij} D_{ij}$, $(D^+)^4 := \frac{1}{16}(D^+)^2(\bar{D}^+)^2$ and $(D^-)^4 := \frac{1}{16}(D^-)^2(\bar{D}^-)^2$.

[3]In the central basis, the harmonic derivative D^{++} is defined by its action on the harmonics as $D^{++}u^-_i = u^+_i, D^{++}u^+_i = 0$, and similarly for D^{--}.

with the gauge parameter l^{--} being an unconstrained real harmonic superfield,

$$l^{--}(z,u) = \sum_{n=1}^{\infty} l^{(i_1\cdots i_{n-1}j_1\cdots j_{n+1})}(z)\, u^+_{i_1}\cdots u^+_{i_{n-1}} u^-_{j_1}\cdots u^-_{j_{n+1}} = \breve{l}^{--}(z,u)\ . \qquad (2.8)$$

Finally, we have the vector multiplet gauge freedom

$$\delta H = 0\ , \qquad \delta V_5^{++} = -D^{++}\lambda\ , \qquad D^+_\alpha \lambda = \bar{D}^+_{\dot\alpha}\lambda = 0\ , \qquad (2.9)$$

with the gauge parameter $\lambda(z,u) = \breve{\lambda}(z,u)$ being real analytic but otherwise arbitrary.

The gauge freedom (2.7) can be used to choose the gauge condition[4]

$$D^{++}H = 0 \quad \Longleftrightarrow \quad H(z,u) = \mathbf{H}(z)\ . \qquad (2.10)$$

The surviving gauge freedom (which we will call the "supergravity gauge transformation") consists of those combined transformations (2.4) and (2.7) which preserve the above gauge condition, that is

$$\delta \mathbf{H}(z) = \frac{1}{12} D_{ij}\Omega^{ij}(z) + \frac{1}{12}\bar{D}_{ij}\bar{\Omega}^{ij}(z) \qquad (2.11)$$

where $\Omega^{ij}(z)$ is the leading component in the harmonic expansion of the parameter $\Omega^{--}(z,u)$ in (2.4). We point out that the linearized super-Weyl tensor $\mathbf{W}_{\alpha\beta}$ [32]

$$\bar{D}^k_{\dot\gamma}\mathbf{W}_{\alpha\beta} = 0\ , \qquad D^{\alpha\beta}\mathbf{W}_{\alpha\beta} = \bar{D}^{\dot\alpha\dot\beta}\bar{\mathbf{W}}_{\dot\alpha\dot\beta} \qquad (2.12)$$

can be expressed in terms of the gravitational superfield in the form [19]

$$\mathbf{W}_{\alpha\beta} := \bar{D}^4 D_{\alpha\beta}\mathbf{H}\ , \qquad D_{\alpha\beta} := D^i_\alpha D_{\beta i} \qquad (2.13)$$

and proves to be invariant under the gauge transformations (2.11).

2.1 Type-I supercurrent

Given a matter system coupled to the minimal supergravity multiplet, we define its supercurrent and multiplet of anomalies following [25]

$$\mathbb{J} = \frac{\delta S}{\delta H}\ , \qquad \mathcal{T}^{++} = \frac{\delta S}{\delta V_5^{++}} \qquad (2.14)$$

[4]It was shown for the first time in [19, 37] that the linearized $\mathcal{N}=2$ Weyl multiplet can be described by a real unconstrained prepotential $\mathbf{H}$. The origin of such a prepotential in the harmonic superspace approach to $\mathcal{N}=2$ supergravity was revealed in [46] at the linearized level, and in [25] at the fully nonlinear level.

with S being the matter action. Here the variational derivatives with respect to the supergravity prepotentials are defined, in the flat superspace limit, as follows:

$$\delta S = \int \mathrm{d}^4x\, \mathrm{d}^8\theta\, \mathrm{d}u\, \delta H\, \frac{\delta S}{\delta H} + \int \mathrm{d}\zeta^{(-4)}\, \delta V_5^{++}\, \frac{\delta S}{\delta V_5^{++}} \ , \tag{2.15}$$

where the analytic integration measure is defined by $\mathrm{d}\zeta^{(-4)} := \mathrm{d}u\,(D^-)^4$, with $\mathrm{d}u$ the usual Haar measure for $SU(2)$ (see e.g. [24] for more details). The supercurrent $\mathbb{J}$ is a real harmonic superfield, $\breve{\mathbb{J}} = \mathbb{J}$, while the multiplet of anomalies $\mathcal{T}^{++}$ is a real analytic superfield,

$$D^+_\alpha \mathcal{T}^{++} = \bar{D}^+_{\dot\alpha} \mathcal{T}^{++} = 0 \ , \qquad \breve{\mathcal{T}}^{++} = \mathcal{T}^{++} \ . \tag{2.16}$$

The action is required to be invariant under the gauge transformations (2.4). This implies that

$$\begin{aligned}
\delta_\Omega S &= \frac{1}{4}\int \mathrm{d}^4x\, \mathrm{d}^8\theta\, \mathrm{d}u\, \mathbb{J}\,(D^+)^2\Omega^{--} + \mathrm{i}\int \mathrm{d}\zeta^{(-4)}\mathcal{T}^{++}\,(D^+)^4\Omega^{--} \ + \ \text{c.c.} \\
&= \int \mathrm{d}^4x\, \mathrm{d}^8\theta\, \mathrm{d}u\, \Omega^{--}\left\{\frac{1}{4}(D^+)^2\mathbb{J} + \mathrm{i}\,\mathcal{T}^{++}\right\} \ + \ \text{c.c.} = 0
\end{aligned}$$

for arbitrary Ω^{--}. As a consequence, we get the conservation equation

$$\frac{1}{4}(D^+)^2\mathbb{J} + \mathrm{i}\,\mathcal{T}^{++} = 0 \ , \qquad \frac{1}{4}(\bar{D}^+)^2\mathbb{J} - \mathrm{i}\,\mathcal{T}^{++} = 0 \ . \tag{2.17}$$

Next, the invariance of S with respect to the l^{--} transformations, (2.7), means

$$\delta_l S = -\int \mathrm{d}^4x\, \mathrm{d}^8\theta\, \mathrm{d}u\, (D^{++}l^{--})\,\mathbb{J} = \int \mathrm{d}^4x\, \mathrm{d}^8\theta\, \mathrm{d}u\, l^{--}D^{++}\mathbb{J} = 0$$

for arbitrary l^{--}, and hence

$$D^{++}\mathbb{J} = 0 \ . \tag{2.18}$$

We see that the matter supercurrent is u-independent, $\mathbb{J} = \mathcal{J}(z)$. Finally, the action is invariant under the $U(1)$ gauge transformations (2.9),

$$\delta_\lambda S = -\int \mathrm{d}\zeta^{(-4)}\, \mathcal{T}^{++}\, D^{++}\lambda = \int \mathrm{d}\zeta^{(-4)}\, \lambda D^{++}\mathcal{T}^{++} = 0 \ , \tag{2.19}$$

and hence

$$D^{++}\mathcal{T}^{++} = 0 \ . \tag{2.20}$$

The general solution of this equation in the central frame reads

$$\mathcal{T}^{++}(z,u) = \mathcal{T}^{ij}(z)u^+_i u^+_j \ . \tag{2.21}$$

Since $\mathcal{T}^{++}(z,u)$ has to be analytic, the multiplet of anomalies $\mathcal{T}^{ij}$ satisfies eq. (1.1). The equations (2.17), (2.18) and (2.21) imply that the conservation law (1.2) holds.

If the theory possesses a restricted chiral superfield X constrained by

$$\bar{D}^i_{\dot{\alpha}} X = 0 \ , \qquad D^{ij} X = \bar{D}^{ij} \bar{X} \ , \tag{2.22}$$

then the supercurrent and the anomaly supermultiplet can be modified by adding improvement terms [47]

$$\mathcal{J} \to \mathcal{J} + i(\bar{X} - X) \ , \qquad \mathcal{T}^{ij} \to \mathcal{T}^{ij} + \frac{1}{4} D^{ij} X \tag{2.23}$$

without changing the conservation equation (1.2). This is similar to the situation in $\mathcal{N} = 1$ supersymmetric theories [1, 14, 15].

Superconformal field theories can be coupled to the Weyl multiplet only, and hence for such theories

$$\mathcal{T}^{ij} = 0 \ . \tag{2.24}$$

An example of superconformal theories is the vector multiplet model (1.7) in the case that $F(W^I)$ is a homogeneous function of degree two,

$$W^I F_I(W) = 2F(W) \ . \tag{2.25}$$

Under this condition, the multiplet of anomalies is zero, in accordance with (1.8b).

Another example is the improved $\mathcal{N} = 2$ tensor multiplet model[5] [33, 34], which can be written in projective superspace [49, 50, 51] as

$$S_{\rm IT} = -\frac{1}{2\pi} \oint v^i dv_i \int \mathrm{d}^4 x \, \Delta^{(-4)} \mathcal{G}^{(2)} \ln \mathcal{G}^{(2)} \ , \qquad \mathcal{G}^{(2)} := \mathcal{G}^{ij} v_i v_j \ , \tag{2.26}$$

with the superfield $G^{ij} = G^{ji}$ describing the tensor multiplet,

$$(\mathcal{G}^{ij})^* = \mathcal{G}_{ij} \ , \qquad D^{(i}_\alpha \mathcal{G}^{jk)} = \bar{D}^{(i}_{\dot{\alpha}} \mathcal{G}^{jk)} = 0 \ . \tag{2.27}$$

The action involves the following fourth-order differential operator:

$$\Delta^{(-4)} := \frac{1}{16} \nabla^\alpha \nabla_\alpha \bar{\nabla}_{\dot{\beta}} \bar{\nabla}^{\dot{\beta}} \ , \quad \nabla_\alpha := \frac{1}{(v,u)} u_i D^i_\alpha \ , \quad \bar{\nabla}_{\dot{\beta}} := \frac{1}{(v,u)} u_i \bar{D}^i_{\dot{\beta}} \ , \tag{2.28}$$

where $(v,u) := v^i u_i$. Here u_i is a fixed isotwistor chosen to be arbitrary modulo the condition $(v,u) \neq 0$ along the integration contour. The supercurrent for the model (2.26) can be shown to be

$$\mathcal{J} = -\sqrt{\frac{1}{2} \mathcal{G}^{ij} \mathcal{G}_{ij}} \equiv -\mathcal{G} \ , \qquad \mathcal{T}^{ij} = 0 \ . \tag{2.29}$$

[5]The harmonic superspace formulation of the improved $\mathcal{N} = 2$ tensor multiplet is given in [39].

The condition that $\mathcal{T}^{ij}$ vanishes (i.e. that $D^{ij}\mathcal{J} = 0$) follows (after some algebra) from the equation of motion for the tensor multiplet, which may be written [52]

$$0 = \frac{\mathcal{G}}{2}\bar{D}_{ij}\left(\frac{\mathcal{G}^{ij}}{\mathcal{G}^2}\right) = \frac{1}{6}\frac{\bar{D}_{ij}\mathcal{G}^{ij}}{\mathcal{G}} - \frac{1}{9}\bar{D}_{\dot{\alpha}k}\mathcal{G}^{ki}\bar{D}^{\dot{\alpha}}_{\ell}\mathcal{G}^{\ell j}\frac{\mathcal{G}_{ij}}{\mathcal{G}^3} \ . \tag{2.30}$$

As a final example of a conformal current, we consider an $\mathcal{N} = 2$ superconformal model of interacting tensor and vector multiplets [52] given by

$$S = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{W}^2 - \frac{1}{2\pi}\oint v^i dv_i \int \mathrm{d}^4x\,\Delta^{(-4)}\mathcal{G}^{(2)}\ln\mathcal{G}^{(2)} \\ + \left(\lambda\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\psi\,\mathcal{W} + \text{c.c.}\right) \ , \tag{2.31}$$

where ψ is the chiral prepotential of $\mathcal{G}^{ij}$

$$\mathcal{G}^{ij} = \frac{1}{4}D^{ij}\psi + \text{c.c.} \tag{2.32}$$

and λ is a real constant. The *gauge-invariant* action (2.31) describes a superconformal massive tensor multiplet (or equivalently, a massive vector multiplet). The interaction term may equally well be written in projective superspace

$$\frac{\lambda}{2\pi}\oint v^i dv_i \int \mathrm{d}^4x\,\Delta^{(-4)}\mathcal{G}^{(2)}\,\mathcal{V} \ , \tag{2.33}$$

where $\mathcal{V}(v^i)$ is the tropical prepotential for the vector multiplet.[6] The field strength $\mathcal{W}$ is given in terms of $\mathcal{V}$ as

$$\mathcal{W} = \frac{1}{8\pi}\oint v^i dv_i\,\bar{\nabla}^2\mathcal{V} \ , \tag{2.34}$$

with $\bar{\nabla}_{\dot{\beta}}$ defined in (2.28). In either form the term is topological (i.e. it is independent of the supergravity prepotential) and so the supercurrent is simply the sum of the two free supercurrents

$$\mathcal{J} = \mathcal{W}\bar{\mathcal{W}} - \sqrt{\frac{1}{2}\mathcal{G}^{ij}\mathcal{G}_{ij}} \ , \qquad \mathcal{T}^{ij} = 0 \ . \tag{2.35}$$

Demonstrating that $D^{ij}\mathcal{J}$ vanishes requires the equations of motion[7]

$$\frac{\mathcal{G}}{2}\bar{D}_{ij}\left(\frac{\mathcal{G}^{ij}}{\mathcal{G}^2}\right) = -4\lambda\mathcal{W} \ , \tag{2.36a}$$

$$D^{ij}\mathcal{W} = -4\lambda\mathcal{G}^{ij} \ . \tag{2.36b}$$

[6]The tropical prepotential $\mathcal{V}(v^i)$ is a homogeneous function of v^i of degree zero.

[7]The combination appearing on the left side of (2.36a) must be reduced chiral for the equation to be sensible. This feature was discussed in [33] and elaborated upon in [52].

2.2 Type-II supercurrent

Many non-superconformal theories must couple to a second supergravity compensator. Suppose the latter is an ω-hypermultiplet [21], that is a real unconstrained analytic superfield $\omega(z,u)$. It proves to be inert under the Ω-transformations (2.4). In the linearized approximation, it changes under the l-transformations as follows[8]

$$\delta_l \omega = -(D^+)^4 D^{--} l^{--} . \tag{2.37}$$

This transformation law implies that in the present case, the equation (2.18) must be modified as follows [25]:

$$D^{++}\mathbb{J} + D^{--}\mathcal{X}^{(+4)} = 0 , \tag{2.38}$$

where we have introduced

$$\mathcal{X}^{(+4)} := \frac{\delta S}{\delta \omega} = \breve{\mathcal{X}}^{(+4)} , \qquad D^+_\alpha \mathcal{X}^{(+4)} = \bar{D}^+_{\dot\alpha} \mathcal{X}^{(+4)} = 0 . \tag{2.39}$$

We see that the supercurrent $\mathbb{J}$ becomes u-dependent. As to the equation (2.17), it remains intact.

In practice, the superfield $\mathcal{X}^{(+4)}$ is often characterized by the additional property:

$$D^{++}\mathcal{X}^{(4)} = 0 \quad \Longrightarrow \quad (D^{++})^3 \mathbb{J} = 0 . \tag{2.40}$$

These conditions and the conservation equation (2.38) imply that

$$\mathbb{J}(u) = \mathcal{J} - 2\mathcal{X}^{ijkl} u^+_i u^+_j u^-_k u^-_l , \qquad \mathcal{X}^{(+4)}(u) = \mathcal{X}^{ijkl} u^+_i u^+_j u^+_k u^+_l , \tag{2.41}$$

where $\mathcal{X}^{ijkl}$ obeys the analyticity constraints

$$D^{(i}_\alpha \mathcal{X}^{jklm)} = \bar{D}^{(i}_{\dot\alpha} \mathcal{X}^{jklm)} = 0 . \tag{2.42}$$

The conservation equation (2.17) turns into

$$\frac{1}{4}\bar{D}^{ij}\mathcal{J} = \frac{1}{20}\bar{D}_{kl}\mathcal{X}^{klij} + \mathrm{i}\,\mathcal{T}^{ij} . \tag{2.43}$$

Setting $\mathcal{T}^{ij} = 0$ gives the supercurrent multiplet introduced by Stelle [20].

As an example of theories with supercurrent (2.43), we consider the massive vector multiplet [21]

$$S^{(m)}_{\rm V} = \frac{1}{2}\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, W^2 - \frac{1}{2}m^2 \int \mathrm{d}\zeta^{(-4)}\, (V^{++})^2 . \tag{2.44}$$

[8]This rule assumes that the background value of ω is set to -2. A more general case will be discussed in Appendix C.

The chiral field strength, W, can be expressed in terms of V^{++} [24, 48] as

$$W(z) = \frac{1}{4}\int \mathrm{d}u\, (\bar{D}^-)^2\, V^{++}(z,u)\ . \tag{2.45}$$

The equation of motion is

$$\frac{1}{4}(D^+)^2 W - m^2\, V^{++} = 0 \quad \Longrightarrow \quad D^{++}V^{++} = 0\ . \tag{2.46}$$

Here it is worth remembering that the field strength obeys the Bianchi identity $(D^+)^2\, W = (\bar{D}^+)^2 \bar{W}$. The supercurrent multiplet is

$$\mathbb{J} = W\bar{W} - \frac{1}{2}m^2\, V^{++}(D^{--})^2 V^{++}\ , \tag{2.47a}$$

$$\mathcal{X}^{(4)} = \frac{1}{2}m^2\, (V^{++})^2\ , \qquad \mathcal{T}^{++} = 0\ . \tag{2.47b}$$

The massless case ($m = 0$) is both gauge invariant and superconformal.

Another example is the massive tensor multiplet [19]

$$S^{(m)}_{\mathrm{T}} = \frac{1}{2}\int \mathrm{d}\zeta^{(-4)}\, (G^{++})^2 - \frac{1}{4}m^2 \Big\{ \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \Psi^2 + \text{c.c.} \Big\}\ . \tag{2.48}$$

The field strength

$$G^{++}(z,u) := G^{ij}(z) u^+_i u^+_j\ , \qquad G^{ij} = \frac{1}{8}D^{ij}\Psi + \frac{1}{8}\bar{D}^{ij}\bar{\Psi}\ , \qquad \bar{D}^i_{\dot\alpha}\Psi = 0\ . \tag{2.49}$$

This action generates the following equation of motion

$$\frac{1}{4}(\bar{D}^-)^2 G^{++} - m^2\, \Psi = 0 \tag{2.50}$$

and its conjugate. The supercurrent multiplet is

$$\mathbb{J} = m^2 \Psi\bar{\Psi} - \frac{1}{2}G^{++}(D^{--})^2 G^{++}\ , \tag{2.51a}$$

$$\mathcal{X}^{(4)} = \frac{1}{2}(G^{++})^2\ , \qquad \mathcal{T}^{++} = 0\ . \tag{2.51b}$$

The massless case ($m = 0$) is gauge invariant but *not* superconformal.

2.3 Type-III supercurrent

We now turn to studying the supercurrent corresponding to the off-shell formulation for $\mathcal{N} = 2$ Poincaré supergravity proposed in [33]. At the linearized level, the tensor compensator $\mathbf{G}^{ij}$ can be represented as above,

$$\mathbf{G}^{ij} = \frac{1}{4}D^{ij}\boldsymbol{\Psi} + \frac{1}{4}\bar{D}^{ij}\bar{\boldsymbol{\Psi}}\ , \qquad \bar{D}^i_{\dot\alpha}\boldsymbol{\Psi} = 0\ . \tag{2.52}$$

It can be shown that the chiral prepotential $\mathbf{\Psi}$ is inert under the l-transformations (2.7). On the other hand, the Ω-gauge symmetry (2.4) acts on $\mathbf{\Psi}$ as

$$\delta_\Omega \mathbf{\Psi} = \frac{1}{3} g_{ij} \bar{D}^4 \bar{\Omega}^{ij} \ , \tag{2.53}$$

where g_{ij} denotes the expectation value of the tensor compensator. This transformation law allows us to read off the supercurrent conservation equation:

$$\frac{1}{4} \bar{D}^{ij} \mathcal{J} + g^{ij} \mathcal{Y} = \mathrm{i}\, \mathcal{T}^{ij} \ , \tag{2.54}$$

where we have denoted

$$\mathcal{Y} := \frac{\delta S}{\delta \mathbf{\Psi}} \ , \qquad \bar{D}^i_{\dot\alpha} \mathbf{\Psi} = 0 \ . \tag{2.55}$$

Since the chiral prepotential $\mathbf{\Psi}$ in (2.52) is defined modulo gauge transformations generated by a restricted chiral superfield, the multiplet $\mathcal{Y}$ is a restricted chiral superfield,

$$D^{ij} \mathcal{Y} = \bar{D}^{ij} \bar{\mathcal{Y}} \ . \tag{2.56}$$

If the theory possesses a composite tensor multiplet L^{ij} such that

$$(L^{ij})^* = L_{ij} \ , \qquad D^{(i}_\alpha L^{jk)} = \bar{D}^{(i}_{\dot\alpha} L^{jk)} = 0 \ , \tag{2.57}$$

then $\mathcal{J}$ and $\mathcal{Y}$ can be modified by adding improvement terms

$$\mathcal{J} \to \mathcal{J} + g_{ij} L^{ij} \ , \qquad \mathcal{Y} \to \mathcal{Y} - \frac{1}{12} \bar{D}_{ij} L^{ij} \tag{2.58}$$

without changing the conservation equation (2.54). This can be compared with eq. (2.23) which describes the structure of improvement terms for the type-I supercurrent.

3 The linearized $\mathcal{N} = 2$ supergravity action

Our goal is to linearize the action (1.11a)–(1.11c) around Minkowski superspace which is an exact solution of the $\mathcal{N} = 2$ Poincaré supergravity equations. We represent the compensators in the form

$$\mathcal{W} = w + \mathbf{W}, \qquad \mathcal{G}^{ij} = g^{ij} + \mathbf{G}^{ij} \tag{3.1}$$

where w and g^{ij} are constant background values satisfying the equation of motion for the gravitational superfield

$$w \bar{w} = g \ , \qquad g := \sqrt{\frac{1}{2} g^{ij} g_{ij}} \ . \tag{3.2}$$

There is no background value for the gravitational superfield. For the vector compensator, its linearized field strength $\mathbf{W}$ obeys the flat-superspace version of the equations (1.12). As to the tensor compensator, its linearized field strength $\mathbf{G}^{ij}$ obeys the flat-superspace version of the equations (1.13).

The terms involving only the linearized compensators are easy to find:

$$
\begin{aligned}
&-\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathbf{W}\mathbf{W}+\frac{1}{2\pi}\oint v^i dv_i\int \mathrm{d}^4x\,\Delta^{(-4)}\,\frac{\mathbf{G}^{(2)}\mathbf{G}^{(2)}}{2g^{(2)}}\\
&=-\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathbf{W}\mathbf{W}-\frac{i}{8\pi g}\oint v^i dv_i\int \mathrm{d}^4x\,\Delta^{(-4)}\,\frac{\mathbf{G}^{(2)}\mathbf{G}^{(2)}}{v^{\underline{1}}\,v^{\underline{2}}}\ .
\end{aligned}
\tag{3.3}
$$

Here the second term is given in the projective superspace setting. We note that

$$
S_{\mathrm{T}}=\frac{i}{4\pi}\oint v^i dv_i\int \mathrm{d}^4x\,\Delta^{(-4)}\,\frac{\mathbf{G}^{(2)}\mathbf{G}^{(2)}}{v^{\underline{1}}\,v^{\underline{2}}}
\tag{3.4}
$$

describes a free massless $\mathcal{N}=2$ tensor multiplet [49]. In appendix A we give a different derivation of the second term in (3.3), which is based on the use of $\mathcal{N}=1$ superfields.

It is possible to write the second term in (3.3) in harmonic superspace [39] as

$$
-\frac{1}{4g}\int \mathrm{d}\zeta^{(-4)}\,\mathbf{G}^{++}\mathbf{G}^{++}\ ,
\tag{3.5}
$$

or without any recourse to auxiliary space $\mathbb{C}P^1$ simply in the form [27]

$$
-\frac{1}{320g}\int \mathrm{d}^4x\,D^{(ij}\bar{D}^{kl)}(\mathbf{G}_{ij}\mathbf{G}_{kl})\ ,
\tag{3.6}
$$

where the indices $ijkl$ are totally symmetrized (with a factor of 1/4! included). We will use the harmonic form in what follows.

The linearized gravitational superfield $\mathbf{H}$ and the linearized compensators $\mathbf{W}$ and $\mathbf{G}^{ij}$ transform under the supergravity gauge transformations[1]

$$
\delta\mathbf{H}=D^{ij}\mathbf{\Omega}_{ij}+\bar{D}_{ij}\bar{\mathbf{\Omega}}^{ij}\ ,
\tag{3.7a}
$$

$$
\delta\mathbf{W}=-\bar{D}^4D^{ij}(\bar{w}\mathbf{\Omega}_{ij}+w\bar{\mathbf{\Omega}}_{ij})\ ,
\tag{3.7b}
$$

$$
\delta\mathbf{G}_{ij}=D_{ij}\bar{D}^4(\bar{\mathbf{\Omega}}^{kl}g_{kl})+\bar{D}_{ij}D^4(\mathbf{\Omega}^{kl}g_{kl})\ .
\tag{3.7c}
$$

The transformation rule for $\mathbf{W}$ can be derived from (2.4). The rule for $\mathbf{G}_{ij}$ can similarly be derived from the transformation of analytic densities considered in [25].[2]

[1]As compared with (2.11), we have rescaled the gauge parameter and switched to bold-face notation, $\Omega_{ij}\to 12\mathbf{\Omega}_{ij}$.

[2]This transformation of $\mathbf{G}_{ij}$ was also postulated in [10].

Using these transformation rules, it is possible to "complete" the pure compensator actions by adding terms involving $\mathbf{H}$ to make the entire result gauge invariant. However, it is more illuminating to first motivate the terms linear in $\mathbf{H}$. These must arise from the coupling of $\mathbf{H}$ to the supercurrent:

$$\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\mathbf{H}\mathcal{J} = \int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\mathbf{H}(\mathcal{G} - \mathcal{W}\bar{\mathcal{W}})\ , \tag{3.8}$$

where $\mathcal{G} = \sqrt{\frac{1}{2}\mathcal{G}^{ij}\mathcal{G}_{ij}}$ and $\mathcal{W}$ involve the full nonlinear superfields. Expanding to first order using (3.1) yields both the background equation of motion, eq. (3.2), as well as the second-order terms involving a single $\mathbf{H}$:

$$\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\mathbf{H}\Big(\frac{1}{2g}g_{ij}\mathbf{G}^{ij} - w\bar{\mathbf{W}} - \bar{w}\mathbf{W}\Big)\ . \tag{3.9}$$

We may then write down the linearized action as

$$S_{\mathrm{SUGRA}} = S_W + S_G + S_H\ , \tag{3.10}$$

where

$$S_W = -\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathbf{W}\mathbf{W} - \int \mathrm{d}^4x\,\mathrm{d}^8\theta\,(\bar{w}\mathbf{W}\mathbf{H} + w\bar{\mathbf{W}}\mathbf{H})\ , \tag{3.11a}$$

$$S_G = -\frac{1}{4g}\int \mathrm{d}\zeta^{(-4)}(\mathbf{G}^{++})^2 + \frac{1}{2g}\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,g_{ij}\,\mathbf{G}^{ij}\mathbf{H} \tag{3.11b}$$

are all the terms involving $\mathbf{W}$ and $\mathbf{G}^{ij}$, respectively. The remaining S_H must involve all terms second order in $\mathbf{H}$.

The explicit form of S_H can be fixed by considering the gauge transformations of the functionals S_W and S_G:

$$\delta S_W = \int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\Big(\bar{w}^2\mathbf{H}\bar{D}^4D^{ij}\mathbf{\Omega}_{ij} + w\bar{w}\mathbf{H}\bar{D}^4D^{ij}\bar{\mathbf{\Omega}}_{ij} + \text{c.c.}\Big)\ , \tag{3.12a}$$

$$\delta S_G = \frac{1}{2g}\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\Big(g_{ij}g_{jk}\mathbf{H}D^{ij}\bar{D}^4\bar{\mathbf{\Omega}}^{kl} + \text{c.c.}\Big)\ . \tag{3.12b}$$

These can be cancelled by δS_H where

$$S_H = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\Big\{w\bar{w}\mathbf{H}\left(\Box - \frac{1}{16}D^{ij}\bar{D}_{ij}\right)\mathbf{H} - \bar{w}^2\,\mathbf{H}\bar{D}^4\mathbf{H} - w^2\,\mathbf{H}D^4\mathbf{H} - \frac{1}{32g}g_{ij}g_{kl}\,\mathbf{H}D^{ij}\bar{D}^{kl}\mathbf{H}\Big\}\ . \tag{3.13}$$

The second and third terms of S_H are chosen to cancel δS_W and δS_G; a remaining term is left that can be cancelled by the first term provided the background equation of motion $w\bar{w} = g$ holds.[3]

It is natural to choose the background super-Weyl gauge so that

$$w\bar{w} = g = 1 \ . \tag{3.14}$$

In this gauge w is a pure phase which breaks the background $U(1)_R$ invariance while g^{ij} is a unit isospin vector which breaks $SU(2)_R$ to a $U(1)$ subgroup. The linearized action then takes the form

$$\begin{aligned} S_{\text{SUGRA}} &= S_W + S_G + S_H \ , && \text{(3.15a)} \\ S_W &= -\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathbf{W}\mathbf{W} - \int \mathrm{d}^4x\,\mathrm{d}^8\theta\,(\bar{w}\mathbf{W}\mathbf{H} + w\bar{\mathbf{W}}\mathbf{H}) \ , && \text{(3.15b)} \\ S_G &= -\frac{1}{4}\int \mathrm{d}\zeta^{(-4)}(\mathbf{G}^{++})^2 + \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^8\theta\, g_{ij}\,\mathbf{G}^{ij}\mathbf{H} \ , && \text{(3.15c)} \\ S_H &= \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\Big\{\mathbf{H}\Big(\Box - \frac{1}{16}D^{ij}\bar{D}_{ij}\Big)\mathbf{H} \\ &\qquad -\bar{w}^2\,\mathbf{H}\bar{D}^4\mathbf{H} - w^2\,\mathbf{H}D^4\mathbf{H} - \frac{1}{32}g_{ij}g_{kl}\,\mathbf{H}D^{ij}\bar{D}^{kl}\mathbf{H}\Big\} \ . && \text{(3.15d)} \end{aligned}$$

This linearized supergravity action is one of our main results.

4 The linearized $\mathcal{N} = 2$ supergravity action in terms of $\mathcal{N} = 1$ superfields

To better understand the physics of the linearized action, we consider its reduction to $\mathcal{N} = 1$ superfields by performing all Grassmann integrals involving $\theta_{\underline{2}}$ and $\bar{\theta}^{\underline{2}}$. This will leave manifest one supersymmetry (involving $\theta \equiv \theta_{\underline{1}}$ and $\bar{\theta} \equiv \bar{\theta}^{\underline{1}}$). The precise choice of which supersymmetry to leave manifest is not physical: different choices involve an $SU(2)_R$ rotation of the background and so involve different choices for the isospin unit vector g_{ij}.

[3] We have written the coefficient of the first term of S_H as $w\bar{w}$, but it could just as well be written g or even $w\bar{w}/3 + 2g/3$ since we are assuming the background to be on-shell.

4.1 Setup

For the pure Maxwell compensator action, this calculation is straightforward. We start from

$$-\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathbf{W}\mathbf{W}\ , \tag{4.1}$$

where $\mathbf{W}$ obeys the Bianchi identity

$$D_{ij}\mathbf{W} = \bar{D}_{ij}\bar{\mathbf{W}}\ . \tag{4.2}$$

This can be easily rewritten in terms of $\mathcal{N}=1$ superfields (making use of the Bianchi identities) as

$$-\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,W^\alpha W_\alpha - \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\chi\bar{\chi}\ , \tag{4.3}$$

where

$$\chi := \mathbf{W}|, \quad \bar{\chi} := \bar{\mathbf{W}}|\ , \tag{4.4a}$$

$$W_\alpha := -\frac{i}{2}D_\alpha{}^{\underline{2}}\mathbf{W}|, \quad \bar{W}_{\dot\alpha} := +\frac{i}{2}\bar{D}_{\dot\alpha\underline{2}}\bar{\mathbf{W}}|, \tag{4.4b}$$

are $\mathcal{N}=1$ chiral and antichiral superfields, respectively, and $|$ denotes taking $\theta_{\underline{2}} = \bar\theta^{\underline{2}} = 0$. The factor of i in (4.4b) is chosen so that (4.2) implies the $\mathcal{N}=1$ Bianchi identity

$$D^\alpha W_\alpha = \bar{D}_{\dot\alpha}\bar{W}^{\dot\alpha}\ . \tag{4.5}$$

Note that the $\mathcal{N}=1$ actions in (4.3) both have the wrong sign, implying that both $\mathcal{N}=1$ fields will play the role of compensators.

For the pure tensor compensator action written in harmonic superspace, we have

$$-\frac{1}{4}\int \mathrm{d}\zeta^{(-4)}\,(\mathbf{G}^{++})^2\ , \qquad \mathbf{G}^{++} = \mathbf{G}^{ij}u_i^+u_j^+\ , \tag{4.6}$$

where $\mathbf{G}^{++}$ obeys the constraint $D_\alpha^+\mathbf{G}^{++} = D_{\dot\alpha}^+\mathbf{G}^{++} = 0$. These constraints imply that the components $\mathbf{G}^{ij}$ consist of an $\mathcal{N}=1$ tensor multiplet G (also known as a real linear multiplet) and an $\mathcal{N}=1$ chiral scalar multiplet η:

$$\eta := \mathbf{G}_{\underline{11}}|\ , \qquad \bar\eta := \mathbf{G}_{\underline{22}}|\ , \qquad G = +2i\mathbf{G}_{\underline{12}}|\ . \tag{4.7}$$

This is easily rewritten in terms of $\mathcal{N}=1$ superfields as

$$-\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\left(\eta\bar\eta - \frac{1}{2}G^2\right)\ . \tag{4.8}$$

We note again the wrong signs, which is an indicator that these are compensators.

The remainder of the terms to be reduced to $\mathcal{N} = 1$ involve the $\mathcal{N} = 2$ superfield $\mathbf{H}$. Because of the gauge freedom for $\mathbf{H}$, most of its $\mathcal{N} = 1$ content is pure gauge. It is advantageous to eliminate as much of the gauge degrees of freedom as possible to identify the physical $\mathcal{N} = 1$ superfields. The simplest gauge choice is a Wess-Zumino gauge, where we use the $\theta^{\underline{2}}$ and $\bar{\theta}_{\underline{2}}$ components of $\mathbf{\Omega}_{ij}$ to fix the lower components of $\mathbf{H}$ to zero:

$$\mathbf{H}| = D_{\alpha}^{\underline{2}}\mathbf{H}| = D_{\dot{\alpha}\underline{2}}\mathbf{H}| = (D^{\underline{2}})^2\mathbf{H}| = (\bar{D}_{\underline{2}})^2\mathbf{H}| = 0 \ . \tag{4.9}$$

The remaining higher components of $\mathbf{H}$ are identified as[4]

$$H_{\alpha\dot{\alpha}} := \frac{1}{4}[D_{\alpha}{}^{\underline{2}}, \bar{D}_{\dot{\alpha}\underline{2}}]\mathbf{H}| \ , \tag{4.10a}$$

$$\Psi_{\alpha} := \frac{1}{8}(\bar{D}_{\underline{2}})^2 D_{\alpha}{}^{\underline{2}}\mathbf{H}| \ , \tag{4.10b}$$

$$U := \frac{1}{16}D^{\alpha\underline{2}}(\bar{D}_{\underline{2}})^2 D_{\alpha}{}^{\underline{2}}\mathbf{H}| \ . \tag{4.10c}$$

Here $H_{\alpha\dot{\alpha}}$ is the $\mathcal{N} = 1$ supergravity multiplet, Ψ_{α} is the gravitino matter multiplet associated with the second gravitino, and U is a real auxiliary superfield.

As usual when imposing Wess-Zumino gauge, a residual gauge transformation remains. In the case under consideration, that invariance is

$$\delta H_{\alpha\dot{\alpha}} = D_{\alpha}\bar{L}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}}L_{\alpha} \ , \tag{4.11a}$$

$$\delta\Psi_{\alpha} = D_{\alpha}\Omega + \Lambda_{\alpha} \ , \qquad \bar{D}_{\dot{\alpha}}\Lambda_{\alpha} = 0 \ , \tag{4.11b}$$

$$\delta U = \Phi + \bar{\Phi} - \frac{i}{2}\partial^{\dot{\alpha}\alpha}(\bar{D}_{\dot{\alpha}}L_{\alpha} + D_{\alpha}\bar{L}_{\dot{\alpha}}) \ , \qquad \bar{D}_{\dot{\alpha}}\Phi = 0 \ , \tag{4.11c}$$

where Φ and Λ_{α} are chiral and L_{α} and Ω are unconstrained complex superfields. These gauge parameters may be defined in terms of complicated spinorial derivatives of $\mathbf{\Omega}_{ij}$ and $\bar{\mathbf{\Omega}}^{ij}$. The details are given in Appendix B.1.

Because the $\mathcal{N} = 2$ compensators transform under $\mathbf{\Omega}_{ij}$, their $\mathcal{N} = 1$ descendants should transform under the residual transformations (4.11). For the Maxwell multiplet compensators, one finds

$$\delta\chi = -w\Phi - \frac{1}{4}w\bar{D}^2 D^{\alpha}L_{\alpha} \ , \tag{4.12a}$$

$$\delta W_{\alpha} = \frac{1}{4}\bar{D}^2 D_{\alpha}\left(i\bar{w}\Omega - iw\bar{\Omega}\right) \ . \tag{4.12b}$$

[4] The precise definitions of the higher components of $\mathbf{H}$ are ambiguous up to terms that vanish in Wess-Zumino gauge.

Note that χ transforms as a chiral compensator for the $\mathcal{N}=1$ supergravity sector while W_α transforms as a chiral spinor compensator for the gravitino multiplet Ψ_α.

The tensor sector is more intricate:

$$\delta G = 2i\delta\mathbf{G}_{\underline{12}}| = \frac{i}{2}g_{\underline{12}}(D^\alpha\bar{D}^2L_\alpha + \bar{D}_{\dot\alpha}D^2L^{\dot\alpha}) - ig_{\underline{11}}D^\alpha\Lambda_\alpha + ig_{\underline{22}}\bar{D}_{\dot\alpha}\bar{\Lambda}^{\dot\alpha} \ , \quad (4.13a)$$

$$\delta\eta = \delta\mathbf{G}_{\underline{11}}| = -g_{\underline{12}}\bar{D}^2\bar{\Omega} + g_{\underline{11}}\Phi \ , \quad (4.13b)$$

$$\delta\bar{\eta} = \delta\mathbf{G}_{\underline{22}}| = +g_{\underline{12}}D^2\Omega + g_{\underline{22}}\Phi \ . \quad (4.13c)$$

The linear superfield G transforms as a linear compensator for supergravity only if $g_{\underline{12}}$ is nonzero and as a linear compensator for the gravitino only if $g_{\underline{11}}$ (and its conjugate $g_{\underline{22}}$) are nonzero. Similarly, η is a chiral compensator for the gravitino only if $g_{\underline{12}}$ is nonzero.

The vacuum value of g_{ij} not only breaks $SU(2)_R$ to some $U(1)$ subgroup but also strongly affects the form of the $\mathcal{N}=1$ supergravity sector, as the structure of the supergravity section depends greatly on these parameters. We report the relevant formulae for arbitrary g_{ij} in Appendix B.2; here, we will analyze in detail only the two interesting simple cases. The first, which we call case I, involves taking $g_{\underline{11}} = g_{\underline{22}} = 0$; the second, case II, is $g_{\underline{12}} = 0$.

4.2 Case I: New minimal supergravity

In this case, $g_{\underline{11}} = g_{\underline{22}} = 0$ and $g_{\underline{12}} = \pm i$. This choice is invariant under the diagonal $U(1)$ subgroup of $SU(2)_R$ and so this $U(1)$ should be manifest in the vacuum of the $\mathcal{N}=1$ reduction. Moreover, G is a linear compensator for supergravity with no compensating transformation for the gravitino. Similarly, η is solely a compensator for the gravitino. This strongly implies that the supergravity reduction of this model should resemble new minimal supergravity [5].

Performing the reduction with only $g_{\underline{12}} = g_{\underline{21}}$ nonzero gives $S = S_W + S_G + S_H$

with

$$S_W = -\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W^\alpha W_\alpha - \int \mathrm{d}^4x\, \mathrm{d}^4\theta \Bigg\{ \chi\bar\chi + 2i\bar w \Psi^\alpha W_\alpha - 2iw\bar\Psi_{\dot\alpha}\bar W^{\dot\alpha}$$

$$+U(w\bar\chi + \bar w\chi) - \frac{i}{2}(\bar w\chi - w\bar\chi)\partial_{\alpha\dot\alpha}H^{\dot\alpha\alpha}\Bigg\} \ , \tag{4.14a}$$

$$S_G = \frac{1}{4}\int \mathrm{d}^4x\, \mathrm{d}^4\theta \Bigg\{ G^2 + 2ig_{\underline{12}}UG - \frac{i}{2}g_{\underline{12}}[D_\alpha, \bar D_{\dot\alpha}]H^{\dot\alpha\alpha}G$$

$$-2\eta\bar\eta - 2g_{\underline{12}}\big(\Psi^\alpha D_\alpha\eta - \bar\Psi_{\dot\alpha}\bar D^{\dot\alpha}\bar\eta\big)\Bigg\} \ , \tag{4.14b}$$

$$S_H = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \Bigg\{ -\frac{1}{16}H^{\dot\alpha\alpha}D^\beta\bar D^2 D_\beta H_{\alpha\dot\alpha} - \frac{1}{4}(\partial_{\alpha\dot\alpha}H^{\dot\alpha\alpha})^2 + \frac{1}{32}([D_\alpha, \bar D_{\dot\alpha}]H^{\dot\alpha\alpha})^2$$

$$-\Psi^\alpha\bar D_{\dot\alpha}D_\alpha\bar\Psi^{\dot\alpha} - \frac{\bar w^2}{4}\Psi^\alpha\bar D^2\Psi_\alpha - \frac{w^2}{4}\bar\Psi_{\dot\alpha}D^2\bar\Psi^{\dot\alpha}$$

$$-\frac{1}{4}U[D_\alpha, \bar D_{\dot\alpha}]H^{\dot\alpha\alpha} - \frac{1}{2}U^2\Bigg\} \ . \tag{4.14c}$$

Because U appears in the action without derivatives, it plays the role of an $\mathcal{N}=1$ auxiliary superfield. If integrated out, the action becomes the sum of two decoupled sectors $S = S_{\mathrm{SG}} + S_\Psi$. The $\mathcal{N}=1$ supergravity sector is contained in S_{SG}:

$$S_{\mathrm{SG}} = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \Bigg\{ -\frac{1}{16}H^{\dot\alpha\alpha}D^\beta\bar D^2 D_\beta H_{\alpha\dot\alpha} - \frac{1}{4}(\partial_{\alpha\dot\alpha}H^{\dot\alpha\alpha})^2 + \frac{1}{16}([D_\alpha, \bar D_{\dot\alpha}]H^{\dot\alpha\alpha})^2$$

$$-\frac{1}{2}L\,[D_\alpha, \bar D_{\dot\alpha}]H^{\dot\alpha\alpha} + \frac{3}{2}L^2\Bigg\} \ , \tag{4.15}$$

where we have rescaled the linear compensator G to $L = ig_{\underline{12}}G/2$ to make contact with the conventional normalization of new minimal supergravity (see, e.g., [8] for a review). This action has the gauge invariance

$$\delta H_{\alpha\dot\alpha} = D_\alpha\bar L_{\dot\alpha} - \bar D_{\dot\alpha}L_\alpha \ , \tag{4.16a}$$

$$\delta L = \frac{1}{4}D^\alpha\bar D^2 L_\alpha + \frac{1}{4}\bar D_{\dot\alpha}D^2 L^{\dot\alpha} \ . \tag{4.16b}$$

The gravitino sector is

$$S_\Psi = \int \mathrm{d}^4x\,\mathrm{d}^4\theta \Big\{ -\Psi^\alpha \bar{D}_{\dot\alpha} D_\alpha \bar\Psi^{\dot\alpha} - \frac{\bar w^2}{4}\Psi^\alpha \bar D^2 \Psi_\alpha - \frac{w^2}{4}\bar\Psi_{\dot\alpha} D^2 \bar\Psi^{\dot\alpha}$$
$$-\frac{g_{\underline{12}}}{2}\big(\Psi^\alpha D_\alpha \eta - \bar\Psi_{\dot\alpha}\bar D^{\dot\alpha}\bar\eta\big) - \frac{1}{2}\eta\bar\eta - 2i\bar w \Psi^\alpha W_\alpha + 2iw\bar\Psi_{\dot\alpha}\bar W^{\dot\alpha}\Big\}$$
$$-\int \mathrm{d}^4x\,\mathrm{d}^2\theta\, W^\alpha W_\alpha \tag{4.17}$$

and involves two compensators – the gaugino field strength W_α and the chiral superfield η – as well as the phase w and the imaginary constant $g_{\underline{12}} = \pm i$. If we make the choice $w = -i$ and perform the field redefinition

$$\phi := -g_{\underline{12}}\,\eta\ , \qquad \bar\phi := +g_{\underline{12}}\,\bar\eta\ , \tag{4.18}$$

we end up with the massless gravitino action [55]

$$S_\Psi = \int \mathrm{d}^4x\,\mathrm{d}^4\theta \Big\{ -\Psi^\alpha \bar{D}_{\dot\alpha} D_\alpha \bar\Psi^{\dot\alpha} + \frac{1}{4}\Psi^\alpha \bar D^2 \Psi_\alpha + \frac{1}{4}\bar\Psi_{\dot\alpha} D^2 \bar\Psi^{\dot\alpha}$$
$$+\frac{1}{2}\Psi^\alpha D_\alpha\phi + \frac{1}{2}\bar\Psi_{\dot\alpha}\bar D^{\dot\alpha}\bar\phi - \frac{1}{2}\phi\bar\phi + 2\Psi^\alpha W_\alpha + 2\bar\Psi_{\dot\alpha}\bar W^{\dot\alpha}\Big\}$$
$$-\int \mathrm{d}^4x\,\mathrm{d}^2\theta\, W^\alpha W_\alpha\ . \tag{4.19}$$

This action is invariant under

$$\delta\Psi_\alpha = D_\alpha\Omega + \Lambda_\alpha\ , \qquad \bar D_{\dot\alpha}\Lambda_\alpha = 0\ , \tag{4.20}$$
$$\delta W_\alpha = -\frac{1}{4}\bar D^2 D_\alpha\left(\Omega + \bar\Omega\right)\ , \tag{4.21}$$
$$\delta\phi = -\bar D^2\bar\Omega\ , \tag{4.22}$$

with Λ_α chiral and Ω complex unconstrained. It is possible to remove one or both of the compensators by exhausting some of the gauge freedom. The gravitino model (4.19) can be shown to be equivalent to the Fradkin-Vasiliev-de Wit-van Holten formulation [53, 54] for a massless gravitino multiplet, derived originally in components. The above realization, eq. (4.19), for the gravitino action is reviewed in textbooks [7, 8].

Note that the chiral compensator χ has completely dropped out of the action, appearing neither in S_{SG} nor S_Ψ. It is a pure gauge degree of freedom, corresponding to the gauge parameter Φ.

4.3 Case II: Old minimal supergravity

For the second case, we have $g_{\underline{11}} = \gamma$ and $g_{\underline{22}} = \bar{\gamma}$ for some γ such that $\gamma\bar{\gamma} = 1$, while $g_{\underline{12}} = 0$. A particular choice of γ breaks the diagonal $U(1)$ subgroup of $SU(2)_R$ (while maintaining some other $U(1)$ subgroup) and so no manifest $U(1)_R$-symmetry exists in the vacuum of II. For this choice, the linear compensator G compensates only for the gravitino.

Performing the $\mathcal{N} = 1$ reduction gives the same S_W as in (4.14a), while S_G and S_H are altered:

$$S_G = \frac{1}{4}\int \mathrm{d}^4x\, \mathrm{d}^4\theta \left\{ G^2 - 2\eta\bar{\eta} + i(\bar{\gamma}\eta - \gamma\bar{\eta})\partial_{\alpha\dot{\alpha}}H^{\dot{\alpha}\alpha} \right. \\ \left. +2U\left(\gamma\bar{\eta} + \bar{\gamma}\eta\right) - 2i\gamma\Psi^\alpha D_\alpha G + 2i\bar{\gamma}\bar{\Psi}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}G \right\} , \tag{4.23a}$$

$$S_H = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \left\{ -\frac{1}{16}H^{\dot{\alpha}\alpha}D^\beta\bar{D}^2D_\beta H_{\alpha\dot{\alpha}} - \frac{1}{4}(\partial_{\alpha\dot{\alpha}}H^{\dot{\alpha}\alpha})^2 + \frac{1}{64}([D_\alpha, \bar{D}_{\dot{\alpha}}]H^{\dot{\alpha}\alpha})^2 \right. \\ -\Psi^\alpha\bar{D}_{\dot{\alpha}}D_\alpha\bar{\Psi}^{\dot{\alpha}} - \frac{w^2}{4}\bar{\Psi}_{\dot{\alpha}}D^2\bar{\Psi}^{\dot{\alpha}} - \frac{\bar{w}^2}{4}\Psi^\alpha\bar{D}^2\Psi_\alpha - \frac{1}{4}\left(\gamma D^\alpha\Psi_\alpha - \bar{\gamma}\bar{D}_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}}\right)^2 \\ \left. -\frac{1}{8}U[D_\alpha, \bar{D}_{\dot{\alpha}}]H^{\dot{\alpha}\alpha} - \frac{3}{4}U^2 \right\} . \tag{4.23b}$$

Integrating out U, we find again the sum of two actions: $S = S_{\mathrm{SG}} + S_\Psi$. In this case, the supergravity action S_{SG} is that for old minimal supergravity [4]

$$S_{\mathrm{SG}} = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \left\{ -\frac{1}{16}H^{\dot{\alpha}\alpha}D^\beta\bar{D}^2D_\beta H_{\alpha\dot{\alpha}} - \frac{1}{4}(\partial_{\alpha\dot{\alpha}}H^{\dot{\alpha}\alpha})^2 + \frac{1}{48}([D_\alpha, \bar{D}_{\dot{\alpha}}]H^{\dot{\alpha}\alpha})^2 \right. \\ \left. +i(\partial_{\alpha\dot{\alpha}}H^{\dot{\alpha}\alpha})(\sigma - \bar{\sigma}) - 3\sigma\bar{\sigma} \right\} , \tag{4.24}$$

where the chiral compensator σ is defined by

$$\sigma := \frac{1}{3}\bar{w}\chi + \frac{1}{3}\bar{\gamma}\eta . \tag{4.25}$$

The action is gauge invariant under

$$\delta H_{\alpha\dot{\alpha}} = D_\alpha\bar{L}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}}L_\alpha , \tag{4.26a}$$

$$\delta\sigma = -\frac{1}{12}\bar{D}^2D^\alpha L_\alpha . \tag{4.26b}$$

Note that σ is the particular combination of χ and η which does not transform under the Φ gauge transformation. The other linearly independent combination of χ and η – which does depend on Φ – has dropped out of the action.

The gravitino action is

$$\begin{aligned}
S_\Psi = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \Bigg\{ & -\Psi^\alpha \bar{D}_{\dot\alpha} D_\alpha \bar{\Psi}^{\dot\alpha} - \frac{w^2}{4} \bar{\Psi}_{\dot\alpha} D^2 \bar{\Psi}^{\dot\alpha} - \frac{\bar{w}^2}{4} \Psi^\alpha \bar{D}^2 \Psi_\alpha \\
& - \frac{1}{4}\left(\gamma D^\alpha \Psi_\alpha - \bar\gamma \bar{D}_{\dot\alpha} \bar{\Psi}^{\dot\alpha}\right)^2 - 2i\bar{w}\Psi^\alpha W_\alpha + 2iw\Psi_{\dot\alpha}\bar{W}^{\dot\alpha} \\
& - \frac{i}{2}\gamma \Psi^\alpha D_\alpha G + \frac{i}{2}\bar\gamma \bar{\Psi}_{\dot\alpha}\bar{D}^{\dot\alpha} G + \frac{1}{4} G^2 \Bigg\} \\
- \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W^\alpha W_\alpha & \qquad (4.27)
\end{aligned}$$

and involves again two compensators – W_α and the linear superfield G – as well as the constant phase factors w and γ. If we make the choices $w = \gamma = -i$, we find the following gravitino action [34]:

$$\begin{aligned}
S_\Psi = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \Bigg\{ & -\Psi^\alpha \bar{D}_{\dot\alpha} D_\alpha \bar{\Psi}^{\dot\alpha} + \frac{1}{4} \bar{\Psi}_{\dot\alpha} D^2 \bar{\Psi}^{\dot\alpha} + \frac{1}{4} \Psi^\alpha \bar{D}^2 \Psi_\alpha \\
& + \frac{1}{4}\left(D^\alpha \Psi_\alpha + \bar{D}_{\dot\alpha} \bar{\Psi}^{\dot\alpha}\right)^2 + 2\Psi^\alpha W_\alpha + 2\Psi_{\dot\alpha}\bar{W}^{\dot\alpha} \\
& - \frac{1}{2}\Psi^\alpha D_\alpha G - \frac{1}{2}\bar{\Psi}_{\dot\alpha}\bar{D}^{\dot\alpha} G + \frac{1}{4} G^2 \Bigg\} \\
- \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W^\alpha W_\alpha\ . & \qquad (4.28)
\end{aligned}$$

Its gauge invariance is

$$\delta\Psi_\alpha = D_\alpha \Omega + \Lambda_\alpha\ , \qquad (4.29a)$$

$$\delta W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha \left(\Omega + \bar\Omega\right)\ , \qquad (4.29b)$$

$$\delta G = -\left(D^\alpha \Lambda_\alpha + \bar{D}_{\dot\alpha}\bar\Lambda^{\dot\alpha}\right)\ . \qquad (4.29c)$$

The gravitino actions (4.19) and (4.28) are dual to each other [34]. This duality is an example of the Legendre transformation between the tensor and the chiral multiplets.

As before, it is possible to remove the compensators algebraically by exhausting some of the gauge freedom. In the gauge $W_\alpha = G = 0$, (4.28) reduces to the gravitino action discovered by Ogievetsky and Sokatchev [56]. In accordance with the above discussion, it can be considered to be dual to the Fradkin-Vasiliev-de Wit-van Holten gravitino action.

5 Discussion

We have succeeded in constructing the linearized $\mathcal{N} = 2$ supergravity action involving vector and tensor compensators along with the scalar supergravity prepotential H. Such an action was ruled out in [38] where the spinor prepotential Ψ^i_α was used for the $\mathcal{N} = 2$ supergravity sector. It was argued in [25] that parametrizations involving Ψ^i_α and those involving H are merely different choices for an underlying gauge symmetry of a harmonic prepotential; when a theory is written with the spinor prepotential, it ought then to appear only in the combination

$$H = D^\alpha_i \Psi^i_\alpha + \bar{D}^i_{\dot{\alpha}} \bar{\Psi}^{\dot{\alpha}}_i \tag{5.1}$$

In [38] it was argued that while this holds for most terms in the linearized action, the coupling of the tensor compensator to Ψ^i_α requires a different form

$$\int \mathrm{d}^4x \, \mathrm{d}^8\theta \, \mathbf{G}_{ij} D^{\alpha i} \Psi^j_\alpha + \text{c.c.} \tag{5.2}$$

where Ψ^i_α appears explicitly. This is necessary *only* if the vacuum is required to respect $SU(2)_R$ invariance. Several years later, when the minimal formulation for $\mathcal{N} = 2$ Poincaré supergravity with a tensor compensator was finally constructed [33], it became clear that the compensator's field strength $\mathcal{G}_{ij}$ itself, rather than its chiral prepotential, must gain a vacuum value, and so the theory will necessarily break $SU(2)_R$ in the appearance of that vacuum value g_{ij}. This allows the parametrization independent coupling

$$\int \mathrm{d}^4x \, \mathrm{d}^8\theta \, g^{ij} \mathbf{G}_{ij} \mathbf{H} \tag{5.3}$$

which we have used in this paper and which follows from linearizing the supercurrent. One may still choose to parametrize $\mathbf{H}$ via (5.1), but this is not a necessity.

The $\mathcal{N} = 1$ reduction of the theory is especially interesting. Once the $\mathcal{N} = 1$ auxiliary superfield U is integrated out (so that the second supersymmetry is realized only on shell), the two simple choices for g_{ij} reduce to decoupled actions of supergravity and the gravitino multiplet. The first choice, with $g_{\underline{11}} = g_{\underline{22}} = 0$, corresponds to new minimal supergravity, where the $\mathcal{N} = 1$ conformal supergravity prepotential couples to a linear compensator. The second choice, with $g_{\underline{12}} = 0$, corresponds to old minimal supergravity with a chiral compensator. As is well known, these two theories are dual to each other, but here we see them appear as different vacua of a single theory. The same curious features apply to the gravitino sector. In the first case, we have essentially the Fradkin-Vasiliev-de Wit-van Holten formulation with a chiral compensator, while in the second case, the Ogievetsky-Sokatchev formulation with a linear compensator. These gravitino models are well known to be dual to each other, but here we see them arise as physically equivalent theories, simply with different vacua.[1] More intriguing still, the chiral and linear compensators are traded between the supergravity and gravitino sectors when the background is changed by an isospin rotation.

Our model for the linearized $\mathcal{N}=2$ supergravity (3.15a)–(3.15d) admits several dual formulations obtained by applying superfield Legendre transformations. Within the harmonic superspace approach [21, 24], the tensor compensator can be dualized into a q^+-hypermultiplet or into an ω-hypermultiplet using the procedure described in [39, 24]. For this the tensor multiplet part (3.15c) of the supergravity action should be rewritten in the form

$$S_G = -\frac{1}{4}\int \mathrm{d}\zeta^{(-4)}\,\mathbf{G}^{++}\Big\{\mathbf{G}^{++} - 6g^{--}(D^+)^4\mathbf{H} - 4D^{++}\boldsymbol{\omega}\Big\}\ . \tag{5.4}$$

Here $\boldsymbol{\omega}$ is an unconstrained analytic superfield acting as a Lagrange multiplier to enforce the constraint $D^{++}\mathbf{G}^{++}=0$. One may instead integrate out $\mathbf{G}^{++}$ to arrive at a dual action in terms of $\boldsymbol{\omega}$:

$$S_{\mathrm{SUGRA}} = S_W + S_\omega + S_H\ , \tag{5.5a}$$

$$S_W = -\frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathbf{W}\mathbf{W} - \int \mathrm{d}^4x\,\mathrm{d}^8\theta\,(\bar{w}\mathbf{W}\mathbf{H} + w\bar{\mathbf{W}}\mathbf{H})\ , \tag{5.5b}$$

$$S_\omega = \int \mathrm{d}\zeta^{-4}D^{++}\boldsymbol{\omega}D^{++}\boldsymbol{\omega} - 6\int \mathrm{d}u\,\mathrm{d}^4x\,\mathrm{d}^8\theta\,\omega^{-+}\mathbf{H}\boldsymbol{\omega}\ , \tag{5.5c}$$

$$\begin{aligned}S_H = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\Big\{&\mathbf{H}\Big(\Box - \frac{1}{10}D^{ij}\bar{D}_{ij}\Big)\mathbf{H}\\ &- \bar{w}^2\,\mathbf{H}\bar{D}^4\mathbf{H} - w^2\,\mathbf{H}D^4\mathbf{H} + \frac{1}{40}\omega_{ij}\omega_{kl}\,\mathbf{H}D^{ij}\bar{D}^{kl}\mathbf{H}\Big\}\end{aligned} \tag{5.5d}$$

where ω^{ij} is a unit isospin vector and we work in the gauge where $w\bar{w} = \omega^{ij}\omega_{ij}/2 = 1$. Because the full nonlinear coupling of an ω-hypermultiplet to supergravity is known, one may compare the result from the duality transformation to the result from linearizing the ω-hypermultiplet action explicitly. The details are given in Appendix C.

In the projective superspace approach [49, 50, 51], on the other hand, the tensor compensator can be dualized into a polar hypermultiplet (following the terminology of [57]). The dual formulations thus derived may lead to new variant $\mathcal{N}=2$ supercurrent multiplets.

[1]This behavior was briefly speculated about in the closing of [33].

Building on the linearized $\mathcal{N} = 2$ supergravity action (3.15a)–(3.15d) and its dual versions, an open problem is to construct their massive extensions. It would be interesting to understand how the known off-shell realizations for massive $\mathcal{N} = 1$ gravitino and supergravity multiplets are imbedded in such $\mathcal{N} = 2$ models [59, 60, 61, 62, 63, 64, 65].

One can consistently incorporate a cosmological term into the minimal $\mathcal{N} = 2$ supergravity with tensor compensator [33]. In superspace, its description is achieved by replacing the action (1.11c) with the following [42]:

$$S^{(m)}_{\text{tensor}} = \frac{1}{2\pi\kappa^2} \oint_C v^i \mathrm{d}v_i \int \mathrm{d}^4x\, \mathrm{d}^4\theta \mathrm{d}^4\bar{\theta}\, \frac{E}{S^{(2)}\breve{S}^{(2)}}\, \mathcal{G}^{(2)} \Big\{ \ln \frac{\mathcal{G}^{(2)}}{\mathrm{i}\breve{\Upsilon}^{(1)}\Upsilon^{(1)}} - m\mathcal{V} \Big\} \ , \tag{5.6}$$

Here m is the cosmological constant, and $\mathcal{V}(v^i)$ is the weight-zero tropical gauge prepotential for the vector compensator. The equation of motion for the vector compensator in this theory is

$$\Big(\frac{1}{4}\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha + S^{ij}\Big)\mathcal{W} + m\mathcal{G}^{ij} = 0 \ . \tag{5.7}$$

The equation for the gravitational field (1.16) does not change, for the vector-tensor coupling in (5.6) is topological. In a super-Weyl and local $U(1)$ gauge $\mathcal{W} = 1$, the above equation reduces to $S^{ij} + m\mathcal{G}^{ij} = 0$, and thus $\bar{S}^{ij} = S^{ij}$. A maximally symmetric solution in $\mathcal{N} = 2$ supergravity with cosmological term is $\mathcal{N} = 2$ anti-de Sitter superspace for which the covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}_i)$ are characterized by the algebra (see [58] for more details):

$$\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = 4\boldsymbol{S}^{ij} M_{\alpha\beta} + 2\varepsilon_{\alpha\beta}\varepsilon^{ij}\boldsymbol{S}^{kl}J_{kl} \ , \qquad \{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot{\beta}}_j\} = -2\mathrm{i}\delta^i_j(\sigma^c)_\alpha{}^{\dot{\beta}}\mathcal{D}_c \ , \tag{5.8a}$$

$$[\mathcal{D}_a, \mathcal{D}^j_\beta] = \frac{\mathrm{i}}{2}(\sigma_a)_{\beta\dot{\gamma}}\boldsymbol{S}^{jk}\bar{\mathcal{D}}^{\dot{\gamma}}_k \ , \qquad [\mathcal{D}_a, \mathcal{D}_b] = -\boldsymbol{S}^2 M_{ab} \ , \tag{5.8b}$$

with $\boldsymbol{S}^2 := \frac{1}{2}\boldsymbol{S}^{ij}\boldsymbol{S}_{ij}$, and $M_{\alpha\beta}$ and J_{kl} the Lorentz and $SU(2)$ generators, respectively. Here the covariantly constant torsion $\boldsymbol{S}^{ij} = \bar{\boldsymbol{S}}^{ij}$ is the background value of $\mathcal{S}^{ij}$. It would be interesting to construct a linearized $\mathcal{N} = 2$ supergravity action around the anti-de Stitter background.

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A The improved $\mathcal{N} = 2$ tensor multiplet in $\mathcal{N} = 1$ superspace

Historically, the first formulation for the improved $\mathcal{N} = 2$ tensor multiplet was given using $\mathcal{N} = 1$ superfields [34], and the component ($\mathcal{N} = 0$) formulation of [33] appeared shortly after.[1] Here we briefly review some aspects of the $\mathcal{N} = 1$ formulation [34].

The manifestly $\mathcal{N} = 2$ superconformal action for the improved tensor multiplet model (2.26) can be rewritten in terms of $\mathcal{N} = 1$ superfields. The resulting action [34] is

$$S_{\rm IT} = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\, L_{\rm IT}\ , \qquad L_{\rm IT} = \sqrt{\mathsf{G}^2 + 4\varphi\bar\varphi} - \mathsf{G}\ \ln\left(\mathsf{G} + \sqrt{\mathsf{G}^2 + 4\varphi\bar\varphi}\right)\ . \quad \text{(A.1)}$$

Here the chiral scalar φ and real linear $\mathcal{G}$ superfields are related to $\mathcal{G}_{ij}$ as follows:

$$\varphi := \mathcal{G}_{\underline{11}}|\ , \qquad \mathsf{G} := 2i\,\mathcal{G}_{\underline{12}}|\ . \qquad \text{(A.2)}$$

A short calculation gives

$$\frac{\partial^2 L_{\rm IT}}{\partial\varphi\partial\bar\varphi} = -\frac{\partial^2 L_{\rm IT}}{\partial \mathsf{G}^2} = \frac{1}{\sqrt{\mathsf{G}^2 + 4\varphi\bar\varphi}} = \frac{1}{2\sqrt{\frac{1}{2}\mathcal{G}^{ij}\mathcal{G}_{ij}}}\Bigg|\ . \qquad \text{(A.3)}$$

This result immediately allows us to construct a linearized action of the model, $S^{(2)}$, around a constant background g^{ij},

$$\mathcal{G}^{ij} = g^{ij} + \mathbf{G}^{ij}\ , \qquad g^{ij} = \text{const}\ . \qquad \text{(A.4)}$$

The linearized action is

$$S^{(2)} = \frac{1}{2g}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\left\{\Phi\bar\Phi - \frac{1}{2}G^2\right\} \equiv \frac{1}{2g}S_{\rm T}\ , \qquad g := \sqrt{\frac{1}{2}g^{ij}g_{ij}}\ , \qquad \text{(A.5)}$$

where the chiral scalar Φ and real linear G superfields are defined by

$$\Phi = \mathbf{G}_{\underline{11}}|\ , \qquad G := 2i\,\mathbf{G}_{\underline{12}}|\ . \qquad \text{(A.6)}$$

Here $S_{\rm T}$ denotes the action for a massless $\mathcal{N} = 2$ tensor multiplet.

[1]Ref. [34] was submitted to the journal Nuclear Physics B one day earlier than [33].

B Details of the $\mathcal{N}=1$ reduction

We present here briefly the details for the $\mathcal{N}=1$ reduction. In the first subsection, we give the explicit form of the $\mathcal{N}=1$ gauge transformations in terms of the $\mathcal{N}=2$ gauge parameter. In the second, we give the form of the $\mathcal{N}=1$ reduction for arbitrary values of the isospin vector g_{ij}.

B.1 Derivation of the $\mathcal{N}=1$ gauge transformations

We relabel our two different Grassmann derivatives as

$$D_\alpha^{\underline{1}} \to D_\alpha, \quad D_\alpha^{\underline{2}} \to \nabla_\alpha \tag{B.1}$$

and similarly for their conjugates. Then the gauge transformation of $\mathbf{H}$ is written

$$\delta\mathbf{H} = D^2\mathbf{\Omega}_{\underline{11}} + 2D^\alpha\nabla_\alpha\mathbf{\Omega}_{\underline{12}} + \nabla^2\mathbf{\Omega}_{\underline{22}} + \bar{D}^2\bar{\mathbf{\Omega}}^{\underline{11}} + 2\bar{D}_{\dot\alpha}\bar{\nabla}^{\dot\alpha}\bar{\mathbf{\Omega}}^{\underline{12}} + \bar{\nabla}^2\bar{\mathbf{\Omega}}^{\underline{22}} \ . \tag{B.2}$$

From this form it is clear that the higher $\theta_{\underline{2}}$ components of $\mathbf{\Omega}_{\underline{22}}$ and $\bar{\mathbf{\Omega}}^{\underline{22}}$ are available to eliminate the lower $\theta_{\underline{2}}$ components of $\mathbf{H}$. In particular, it is possible to choose the Wess-Zumino gauge (4.9). The residual components of $\mathbf{H}$ may be taken as in (4.10).

Because $\mathbf{\Omega}_{ij}$ has not been entirely fixed, the $\mathcal{N}=1$ superfields must possess residual $\mathcal{N}=1$ gauge transformations. These may be derived by direct computation, but we may first motivate their form by considering the Noether coupling of $\mathbf{H}$ to a conserved current $\mathcal{J}$. Rewriting this coupling in $\mathcal{N}=1$ language yields

$$\int \mathrm{d}^4x\,\mathrm{d}^8\theta\,\mathbf{H}\mathcal{J} = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\left(H^{\dot\alpha\alpha}J_{\alpha\dot\alpha} + \Psi^\alpha J_\alpha + \bar\Psi_{\dot\alpha}J^{\dot\alpha} + \hat{U}J\right) \ , \tag{B.3}$$

where the $\mathcal{N}=1$ currents are

$$J_{\alpha\dot\alpha} = \frac{1}{4}[\nabla_\alpha, \bar\nabla_{\dot\alpha}]\mathcal{J}| - \frac{1}{12}[D_\alpha, \bar{D}_{\dot\alpha}]\mathcal{J}| \ , \tag{B.4a}$$

$$J_\alpha = \nabla_\alpha\mathcal{J}| \ , \tag{B.4b}$$

$$J = \mathcal{J}| \tag{B.4c}$$

and we have defined the combination

$$\hat{U} := U + \frac{1}{12}[D^\alpha, \bar{D}^{\dot\alpha}]H_{\alpha\dot\alpha} \ . \tag{B.5}$$

The $\mathcal{N}=2$ conservation condition is

$$D^{ij}\mathcal{J} = \bar{D}_{ij}\mathcal{J} = 0 \ . \tag{B.6}$$

and implies for the $\mathcal{N}=1$ currents [25]

$$D^\alpha J_{\alpha\dot\alpha} = \bar D^{\dot\alpha} J_{\alpha\dot\alpha} = 0 \ , \tag{B.7a}$$

$$D^\alpha J_\alpha = \bar D^2 J_\alpha = \bar D_{\dot\alpha} J^{\dot\alpha} = D^2 J^{\dot\alpha} = 0 \ . \tag{B.7b}$$

$$\bar D^2 J = D^2 J = 0 \ . \tag{B.7c}$$

These $\mathcal{N}=1$ conservation equations imply the corresponding gauge invariances (4.11).

It is a straightforward exercise to derive these gauge superfields in terms of the $\mathcal{N}=2$ gauge parameter $\mathbf{\Omega}_{ij}$. First, the maintenance of Wess-Zumino gauge fixes certain higher components of $\mathbf{\Omega}_{\underline{22}}$ and $\bar{\mathbf{\Omega}}^{\underline{22}}$:

$$\nabla^2 \mathbf{\Omega}_{\underline{22}} + \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{22}} = -D^2 \mathbf{\Omega}_{\underline{11}} - \bar D^2 \bar{\mathbf{\Omega}}^{\underline{11}} - 2D^\alpha \nabla_\alpha \mathbf{\Omega}_{\underline{12}} - 2\bar D_{\dot\alpha} \bar\nabla^{\dot\alpha} \bar{\mathbf{\Omega}}^{\underline{12}} \ , \tag{B.8a}$$

$$\nabla_\alpha \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{22}} = -D^2 \nabla_\alpha \mathbf{\Omega}_{\underline{11}} - \bar D^2 \nabla_\alpha \bar{\mathbf{\Omega}}^{\underline{11}} + D_\alpha \nabla^2 \mathbf{\Omega}_{\underline{12}} - 2\nabla_\alpha \bar D_{\dot\beta} \bar\nabla^{\dot\beta} \bar{\mathbf{\Omega}}^{\underline{12}} \ , \tag{B.8b}$$

$$\nabla^2 \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{22}} = -D^2 \nabla^2 \mathbf{\Omega}_{\underline{11}} - \bar D^2 \nabla^2 \bar{\mathbf{\Omega}}^{\underline{11}} - 2\bar D_{\dot\alpha} \nabla^2 \bar\nabla^{\dot\alpha} \bar{\mathbf{\Omega}}^{\underline{12}} \ . \tag{B.8c}$$

where each equation should be understood as projected to $\theta_{\underline{2}} = 0$. These ensure that Wess-Zumino gauge is maintained by an otherwise arbitrary gauge transformation,

$$\delta \mathbf{H}| = \nabla_\alpha \delta \mathbf{H}| = \nabla^2 \delta \mathbf{H}| = 0 \ . \tag{B.9}$$

Calculating $\delta H_{\alpha\dot\alpha} = \frac{1}{4}[\nabla_\alpha, \bar\nabla_{\dot\alpha}]\delta\mathbf{H}$ and imposing (B.8), one finds

$$\begin{aligned} L_\alpha = &-\frac{1}{2}\nabla_\alpha \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{12}} - \frac{1}{2}\bar D^{\dot\beta}[\nabla_\alpha, \bar\nabla_{\dot\beta}]\bar{\mathbf{\Omega}}^{\underline{11}} + \frac{1}{4}D_\alpha \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{22}} + \frac{1}{2}D_\alpha \bar D_{\dot\beta} \bar\nabla^{\dot\beta} \bar{\mathbf{\Omega}}^{\underline{12}} \\ &- \frac{1}{4}D_\alpha \nabla^2 \mathbf{\Omega}_{\underline{22}} + \frac{1}{4}D^2 \nabla_\alpha \mathbf{\Omega}_{\underline{12}} \ . \end{aligned} \tag{B.10}$$

Similarly, one may calculate $\delta\Psi_\alpha$ and show that

$$\Omega = -\frac{1}{8}\bar\nabla^2 \nabla^2 \mathbf{\Omega}_{\underline{12}} - \frac{1}{4}D^\beta \bar\nabla^2 \nabla_\beta \mathbf{\Omega}_{\underline{11}} + \frac{1}{8}\bar D^2 \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{12}} \ , \tag{B.11}$$

$$\Lambda_\alpha = \frac{1}{8}\bar D^2 \left(\bar\nabla^2 \nabla_\alpha \bar{\mathbf{\Omega}}^{\underline{11}} - D_\alpha \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{12}}\right) \ . \tag{B.12}$$

For the $\mathcal{N}=1$ auxiliary superfield U, one finds

$$\delta U = -\frac{i}{2}\partial^{\dot\alpha\alpha}\big(\bar D_{\dot\alpha} L_\alpha + D_\alpha L_{\dot\alpha}\big) + \Phi + \bar\Phi \ , \tag{B.13}$$

where

$$\Phi = \frac{1}{16}\bar D^2 \left(\nabla^2 \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{11}} - 8\Box \bar{\mathbf{\Omega}}^{\underline{11}} + 4i\partial^{\dot\alpha\alpha} D_\alpha \bar\nabla_{\dot\alpha} \bar{\mathbf{\Omega}}^{\underline{12}} + \frac{1}{2}D^2(\nabla^2 \mathbf{\Omega}_{\underline{22}} - \bar\nabla^2 \bar{\mathbf{\Omega}}^{\underline{22}})\right) \ . \tag{B.14}$$

Using

$$\delta\left([D_\beta, D_{\dot\beta}]H^{\dot\beta\beta}\right) = 6i\partial^{\dot\alpha\alpha}\big(D_\alpha L_{\dot\alpha} + \bar D_{\dot\alpha} L_\alpha\big) + 2D^2(\bar D_{\dot\alpha} L^{\dot\alpha}) + 2\bar D^2(D^\alpha L_\alpha) \tag{B.15}$$

it follows that $\delta\hat U = \hat\Phi + \bar{\hat\Phi}$, where

$$\hat\Phi = \Phi + \frac{1}{6}\bar D^2 D^\alpha L_\alpha\ . \tag{B.16}$$

B.2 $\mathcal{N} = 1$ reduction for arbitrary g_{ij}

To present the $\mathcal{N} = 1$ action involving an arbitrary isospin unit vector g_{ij}, we make the following identifications:

$$g_{\underline{11}} = x\gamma, \quad g_{\underline{12}} = g_{\underline{21}} = iy, \quad g_{\underline{22}} = x\bar\gamma \tag{B.17}$$

$$\gamma\bar\gamma = 1, \quad x^2 + y^2 = 1 \tag{B.18}$$

where x and y are real parameters and γ is a complex phase. The constraint on x and y follows from the normalization condition $g^{ij}g_{ij} = 2$.

We avoid giving the intermediate results for the actions S_W, S_G, and S_H as we do in the two special cases, but present merely the final form of the action once all terms are collected together. We collect first all terms quadratic in $\mathcal{N} = 1$ components of $\mathbf{H}$:

$$\begin{aligned} S_{HH} = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\bigg\{ & -\frac{1}{16}H^{\dot\alpha\alpha}D^\beta\bar D^2 D_\beta H_{\alpha\dot\alpha} - \frac{1}{4}(\partial_{\alpha\dot\alpha}H^{\dot\alpha\alpha})^2 \\ & + \frac{1}{64}(1+y^2)([D_\alpha, \bar D_{\dot\alpha}]H^{\dot\alpha\alpha})^2 - \frac{1}{8}(1+y^2)U[D_\beta, \bar D_{\dot\beta}]H^{\dot\beta\beta} \\ & + \frac{ixy\gamma}{8}D^\alpha\Psi_\alpha[D_\beta, \bar D_{\dot\beta}]H^{\dot\beta\beta} - \frac{ixy\bar\gamma}{8}\bar D_{\dot\alpha}\bar\Psi^{\dot\alpha}[D_\beta, \bar D_{\dot\beta}]H^{\dot\beta\beta} \\ & - \Psi^\alpha\bar D_{\dot\alpha}D_\alpha\bar\Psi^{\dot\alpha} - \frac{1}{4}w^2\bar\Psi_{\dot\alpha}D^2\bar\Psi^{\dot\alpha} - \frac{1}{4}\bar w^2\Psi^\alpha\bar D^2\Psi_\alpha \\ & - \frac{1}{4}x^2(\gamma D^\alpha\Psi_\alpha - \bar\gamma\bar D_{\dot\alpha}\bar\Psi^{\dot\alpha})^2 + U\left(-\frac{ixy\gamma}{2}D^\alpha\Psi_\alpha + \frac{ixy\bar\gamma}{2}\bar D_{\dot\alpha}\bar\Psi^{\dot\alpha}\right) \\ & - \frac{1}{2}\left(1 + \frac{1}{2}x^2\right)U^2\bigg\} \end{aligned} \tag{B.19}$$

Note that when either x or y vanishes (i.e. the two cases we have discussed in detail), the gravitino superfield Ψ_α decouples from the $\mathcal{N} = 1$ supergravity prepotential $H^{\dot\alpha\alpha}$. However, we see that for the more general case, the action is more intricate in structure.

Next we present all terms involving both a supergravity field and a compensator:

$$
\begin{aligned}
S_{HC} = \int \mathrm{d}^4x\, \mathrm{d}^4\theta \bigg\{ & \frac{ix}{4} (\bar{\gamma}\eta - \gamma\bar{\eta})\, \partial_{\alpha\dot{\alpha}} H^{\dot{\alpha}\alpha} + \frac{y}{8} G\, [D_\alpha, \bar{D}_{\dot{\alpha}}] H^{\dot{\alpha}\alpha} \\
& + \frac{i}{2}(x\gamma G + y\eta) D^\alpha \Psi_\alpha - \frac{i}{2}(x\bar{\gamma} G + y\bar{\eta}) \bar{D}_{\dot{\alpha}} \bar{\Psi}^{\dot{\alpha}} - 2i\bar{w}\Psi^\alpha W_\alpha + 2iw\bar{\Psi}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \\
& + U \left(\frac{1}{2} x(\gamma\bar{\eta} + \bar{\gamma}\eta) - \frac{1}{2} yG - \bar{w}\chi - w\bar{\chi} \right) \bigg\} \, .
\end{aligned} \tag{B.20}
$$

We see that when both x and y are nonzero, the $\mathcal{N} = 1$ supergravity prepotential $H^{\dot{\alpha}\alpha}$ and the gravitino superfield Ψ^α share the same compensators.

The terms involving just the compensators are the same for all cases:

$$
S_{CC} = -\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W^\alpha W_\alpha + \int \mathrm{d}^4x\, \mathrm{d}^4\theta \left(\frac{1}{4} G^2 - \frac{1}{2}\eta\bar{\eta} - \chi\bar{\chi} \right) \, . \tag{B.21}
$$

One may proceed as in the body of the paper to integrate out U; however, because of the nontrivial couplings between the $\mathcal{N} = 1$ supergravity prepotential $H_{\alpha\dot{\alpha}}$, the gravitino superfield Ψ_α, and their compensators, the generic action does not take a particularly clean final form.

C The linearized hypermultiplet action

The conventional formulation for a compensator hypermultiplet in harmonic superspace [21, 24] involves a doublet q_a^+ of analytic superfields with an action

$$
S = \frac{1}{2} \int \mathrm{d}\zeta^{(-4)}\, q^{a+} \nabla^{++} q_a^+ \tag{C.1}
$$

where ∇^{++} is the curved space generalization of D^{++}. In terms of the supergravity prepotential H, ∇^{++} may be written

$$
\nabla^{++}\Psi^{(n)} = D^{++}\Psi^{(n)} + (D^+)^4 (H D^{--}\Psi^{(n)}) \, , \qquad D_{\dot{\alpha}}^+ \Psi^{(n)} = \bar{D}_{\dot{\alpha}}^+ \Psi^{(n)} = 0 \, , \tag{C.2}
$$

where $\Psi^{(n)}$ is an arbitrary analytic superfield of $U(1)$ charge n.

This q^+ action can be rewritten involving a real hypermultiplet ω via the change of variables

$$
q_a^+ = u_a^+ \omega + u_a^- f^{++} \tag{C.3}
$$

where ω and f^{++} are analytic superfields. In the resulting action, one may integrate out f^{++} to end up with

$$S = \int \mathrm{d}\zeta^{(-4)} \left\{ \frac{1}{2}\nabla^{++}\omega\nabla^{++}\omega + \frac{1}{2}\omega^2 \left(H^{(+4)} - \frac{1}{2}\nabla^{++}\Gamma^{++} - \frac{1}{4}\Gamma^{++}\Gamma^{++} \right) \right\} \tag{C.4}$$

Here $H^{(+4)} = (D^+)^4 H$ and $\Gamma^{++} = (D^+)^4(D^{--}H)$.

The supercurrent for this action in a Minkowski background can be found by considering the terms first order in the prepotential H. One finds

$$\delta S = \frac{1}{2} \int \mathrm{d}^4x\, \mathrm{d}^8\theta\, \mathrm{d}u\, H \left(D^{++}\omega D^{--}\omega - \omega D^{--}D^{++}\omega + \omega^2 \right) \tag{C.5}$$

which is valid for any gauge choice of H and so the supercurrent is

$$\mathcal{J} = \frac{1}{2} \left(D^{++}\omega D^{--}\omega - \omega D^{--}D^{++}\omega + \omega^2 \right) \tag{C.6}$$

It is straightforward to check that this current obeys both $(D^+)^2\mathcal{J} = 0$ and $D^{++}\mathcal{J} = 0$ when the ω hypermultiplet is on shell.

The full linearized action in this case is also quite easy to find since ω is an unconstrained analytic superfield and so all H-dependence appears explicitly in the covariant derivative and connection terms. Linearizing ω about an on-shell background gives (in the central basis)

$$\omega = \omega_0 + \omega^{ij}u_i^- u_j^+ + \boldsymbol{\omega} \tag{C.7}$$

where ω_0 and ω^{ij} are constants and $\boldsymbol{\omega}$ is unconstrained. It is convenient to write the second term in this expression as ω^{-+}.

The linearized action has the form

$$S = \frac{1}{2} \int \mathrm{d}\zeta^{(-4)} (D^{++}\boldsymbol{\omega})^2 + \int \mathrm{d}u\, \mathrm{d}^4x\, \mathrm{d}^8\theta \left\{ \mathbf{H}(\omega_0\boldsymbol{\omega} - 3\omega^{-+}\boldsymbol{\omega}) \right.$$
$$\left. + \frac{\omega_{ij}\omega_{kl}}{160}\mathbf{H}D^{ij}\bar{D}^{kl}\mathbf{H} - \frac{\omega^{kl}\omega_{kl}}{480}\mathbf{H}D^{ij}\bar{D}_{ij}\mathbf{H} \right\} \tag{C.8}$$

where we have chosen $\mathbf{H}$ to be harmonic-independent. $\boldsymbol{\omega}$ varies under the supergravity gauge transformation as an analytic density of weight 1/2 [25]:

$$\delta\boldsymbol{\omega} = (D^+)^4 D^{--}(l^{--}\omega) - \frac{1}{2}\omega(D^+)^4 D^{--} l^{--} \tag{C.9}$$

However, in order for the gauge choice for $\mathbf{H}$ to be maintained, every $\boldsymbol{\Omega}_{ij}$-transformation must be accompanied by a certain l-transformation – specifically,

$$l^{--} = -(D^-)^2\boldsymbol{\Omega}^{-+} + 2D^{\alpha -}D^+_\alpha\boldsymbol{\Omega}^{--} \tag{C.10}$$

where $\mathbf{\Omega}^{-\pm} = \mathbf{\Omega}^{ij} u_i^- u_j^\pm$. It follows that for this restricted class of gauge transformations (which is the class that concerns us here)

$$\delta\boldsymbol{\omega} = (D^+)^4 (D^-)^2 \left(\frac{1}{2}\omega_0 \mathbf{\Omega}^{--} + \frac{1}{2}\omega^{-+}\mathbf{\Omega}^{--} - \omega^{--}\mathbf{\Omega}^{-+} \right) \qquad \text{(C.11)}$$

While the generic background involves both an isosinglet ω_0 and an isotriplet ω^{ij}, the theory dual to the improved tensor compensator should possess only an isotriplet proportional to g^{ij}, and so we will consider the case where ω_0 vanishes.

Now we may perform a duality transformation from the improved tensor multiplet. Recall that we derived the form of the linearized action for the improved tensor multiplet compensator only when coupled to the Maxwell compensator. This meant that certain of the terms quadratic in $\mathbf{H}$ were necessarily of ambiguous origin: they could have arisen from either the improved tensor or the Maxwell action. Indeed, the first set of terms in (3.13) which were written with the coefficient $w\bar{w}$ could just have well been written with the coefficient g, since the action was derived under the assumption that $g = w\bar{w}$. In order for the duality transformation to reproduce (C.8), we will need to include one such term $g\mathbf{H}D^{ij}\bar{D}_{ij}\mathbf{H}$, with a constant coefficient λ to be determined:

$$S_{\mathrm{IT}} = -\frac{1}{4g}\int \mathrm{d}\zeta^{(-4)} (\mathbf{G}^{++})^2 + \int \mathrm{d}^4x\, \mathrm{d}^8\theta \left\{ \frac{1}{2g} g_{ij}\, \mathbf{G}^{ij}\mathbf{H} - \frac{1}{64g} g_{ij} g_{kl}\, \mathbf{H} D^{ij} \bar{D}^{kl}\mathbf{H} - \frac{\lambda g}{32}\mathbf{H}D^{ij}\bar{D}_{ij}\mathbf{H} \right\} \qquad \text{(C.12)}$$

We emphasize that this action is not by itself gauge-invariant, although it does possess the property that its gauge variation is an $SU(2)$ invariant (in the sense that it involves g_{ij} only in the invariant combination g) which can be cancelled by including the gauge variation of the linearized Maxwell action under the assumption that $g = w\bar{w}$.

To perform the duality transformation, we rewrite the first two terms in analytic superspace and introduce a Lagrange multiplier field $\boldsymbol{\omega}$

$$\int \mathrm{d}\zeta^{(-4)} \left\{ -\frac{1}{4g}\mathbf{G}^{++}\mathbf{G}^{++} + \frac{3}{2g} g^{--}\, \mathbf{G}^{++} (D^+)^4 \mathbf{H} + g^{-1/2}\mathbf{G}^{++} D^{++}\boldsymbol{\omega} \right\}, \qquad \text{(C.13)}$$

which is an unconstrained analytic superfield enforcing the constraint $D^{++}\mathbf{G}^{++} = 0$. In order for this action to be gauge invariant (up to terms independent of the specific $SU(2)$ gauge choice of g_{ij}), $\boldsymbol{\omega}$ must transform as

$$\delta\boldsymbol{\omega} = g^{-1/2} (D^+)^4 (D^-)^2 \left(\frac{1}{2}\Omega^{--} g^{-+} - \Omega^{-+} g^{--} \right) \qquad \text{(C.14)}$$

Comparing this result to (C.11), we may tentatively identify $g^{ij}/\sqrt{g}$ with ω^{ij} provided we work in the case where $\omega_0 = 0$.

The dual action is found by integrating out $\mathbf{G}^{++}$:

$$\tilde{S}_{\mathrm{IT}} = \int \mathrm{d}\zeta^{-4} D^{++}\boldsymbol{\omega} D^{++}\boldsymbol{\omega} + \int \mathrm{d}u\, \mathrm{d}^4x\, \mathrm{d}^8\theta \left\{ \frac{9}{4g}(g^{--})^2 \mathbf{H}(D^+)^4\mathbf{H} - \frac{6g^{-+}}{\sqrt{g}}\mathbf{H}\boldsymbol{\omega} \right\}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^8\theta \left(-\frac{1}{64g} g_{ij}g_{kl}\, \mathbf{H}D^{ij}\bar{D}^{kl}\mathbf{H} - \frac{\lambda g}{32}\mathbf{H}D^{ij}\bar{D}_{ij}\mathbf{H} \right) \qquad \text{(C.15)}$$

The harmonic integral in the second term can be done, yielding

$$\tilde{S}_{\mathrm{IT}} = \int \mathrm{d}\zeta^{-4} D^{++}\boldsymbol{\omega} D^{++}\boldsymbol{\omega} - \int \mathrm{d}u\, \mathrm{d}^4x\, \mathrm{d}^8\theta\, \frac{6g^{-+}}{\sqrt{g}}\mathbf{H}\boldsymbol{\Omega}$$
$$+ \int \mathrm{d}^4x\, \mathrm{d}^8\theta \left\{ \frac{1}{80g}\mathbf{H}D^{ij}\bar{D}^{kl}\mathbf{H}g_{ij}g_{kl} - \frac{g}{32}\left(\frac{3}{5} + \lambda\right)\mathbf{H}D_{ij}\bar{D}^{ij}\mathbf{H} \right\} \qquad \text{(C.16)}$$

Comparing this to the linearized action for the ω-hypermultiplet in the case where ω_0 vanishes (C.8), we find agreement up to an overall factor of 2 provided $\lambda = -1/3$ and $\omega^{ij} = g^{ij}/\sqrt{g}$.

In the supergravity formulation with the ω hypermultiplet, we have several regimes to choose from for the background value. As we have just shown, the case where ω_0 vanishes is dual to a theory with an improved tensor compensator. However, we may also choose the isotriplet ω_{ij} to vanish; this would lead to a type-II supercurrent as discussed in the main body of the paper.

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