

Inertial forces and dissipation on accelerated boundaries

C.D. Fosco,¹ F.C. Lombardo,² and F. D. Mazzitelli²

¹*Centro Atómico Bariloche and Instituto Balseiro, Comisión Nacional de Energía Atómica*

²*Departamento de Física Juan José Giambiagi, FCEyN UBA, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*

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We study dissipative effects due to inertial forces acting on matter fields confined to accelerated boundaries in $1+1$, $2+1$, and $3+1$ dimensions. These matter fields describe the internal degrees of freedom of ‘mirrors’ and impose, on the surfaces where they are defined, boundary conditions on a fluctuating ‘vacuum’ field. We construct different models, involving either scalar or Dirac matter fields coupled to a vacuum scalar field, and use effective action techniques to calculate the strength of dissipation. In the case of massless Dirac fields, the results could be used to describe the inertial forces on an accelerated graphene sheet.

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I. INTRODUCTION

In what is one of the most startling manifestations of the quantum nature of the electromagnetic (EM) field, the Dynamical Casimir Effect (DCE) predicts that, in the presence of an accelerated mirror, the quantum vacuum evolves to an excited state (i.e., one with a non-vanishing number of photons). As a consequence, the mirror will experience a dissipative force. Both the dissipation and the number of created photons are rather small for a single accelerated mirror. However, the effect can be enhanced in electromagnetic cavities with time dependent length or, more generally, with time dependent electromagnetic properties (see the reviews [1], and references therein).

In this letter we consider an effect that, although of a different nature than the DCE, may nevertheless arise under fairly similar conditions: acceleration of the mirrors. The main difference with the DCE is that we shall consider the ‘inertial’ dissipation that results from the transfer of energy to the matter fields inside the mirror, rather than to the EM field.

Effects due to the inertial forces in accelerated media are known, and usually described in terms of induced time-dependent polarizations or currents, like in the Barnett and Stewart-Tolman effects [2]. More generally, there will be inertial forces as long as there is energy transfer from the collective coordinates of the mirrors, that specify their position and shape as a function of time, to their internal degrees of freedom.

Contrary to the vacuum friction effects considered up to now [1]-[4], the inertial effects described in this letter will exist even when the internal degrees of freedom are not coupled to the EM field. Our aim in this work is to evaluate these effects using specific models for the matter fields, allowing for mirrors with a quite general space and time dependence.

II. THE MODEL

Let us begin by considering a scalar model, described by an Euclidean action S with the structure:

$$S = S_f + S_m + S_I, \quad (1)$$

where S_f describes the vacuum field dynamics in the absence of mirrors. It will be assumed to be a free field theory in $d+1$ dimensions ($d = 1, 2$ or 3). S_m is the free part of the matter field action, i.e., it ignores the coupling to the vacuum field; this coupling is instead contained in the last term, S_I .

Both S_m and S_I have support on \mathcal{M} , the d -dimensional spacetime manifold swept by $\Sigma(\tau)$, the space occupied by the mirrors at the time τ , in the course of their time evolution.

The vacuum field φ is assumed to be a scalar, equipped with the action

$$S_f(\varphi) = \frac{1}{2} \int d^{d+1}x \partial_\mu \varphi(x) \partial_\mu \varphi(x). \quad (2)$$

In this scalar model, the matter degrees of freedom are described by ξ , also a real scalar field, but living on \mathcal{M} , a curved d -dimensional manifold. To make this more explicit, we introduce d variables: $\sigma^0, \sigma^1, \dots, \sigma^{d-1}$, such that σ^0 is the time coordinate; \mathcal{M} can then be parametrized by $d+1$ functions y^μ , $\mu = 0, 1, \dots, d$, as follows:

$$\mathcal{M}: \sigma \longrightarrow y^\mu(\sigma^0, \dots, \sigma^{d-1}) \quad (3)$$

such that $y^0 = \sigma^0 \equiv x^0 \equiv \tau$ and $y^j = y^j(\tau, \sigma^1, \dots, \sigma^{d-1})$ ($j = 1, \dots, d$), is a parametrization of $\Sigma(\tau)$ (for each τ). Note that \mathcal{M} may denote either one or more than one mirror. When dealing with more than one mirror, we shall introduce an alternative, more explicit notation, obtained by using a different parametrization for each connected piece in \mathcal{M} .

We assume the scalar matter field action to be given

by:

$$S_m(\xi; \mathcal{M}) = \frac{1}{2} \int d^d \sigma \sqrt{g(\sigma)} [g^{\alpha\beta}(\sigma) \partial_\alpha \xi(\sigma) \partial_\beta \xi(\sigma) + \mu^2 \xi^2(\sigma)], \quad (4)$$

where $g_{\alpha\beta}(\sigma)$ is the induced metric on \mathcal{M} :

$$g_{\alpha\beta}(\sigma) = \partial_\alpha y^\mu(\sigma) \partial_\beta y^\mu(\sigma), \quad \alpha = 0, 1, \dots, d-1, \quad (5)$$

$g(\sigma)$ its determinant, and μ the mass of the field.

For the S_I piece, on the other hand, we shall consider a term of the form:

$$S_I(\varphi, \xi; \mathcal{M}) = -i \zeta \int d^d \sigma \sqrt{g(\sigma)} \xi(\sigma) \varphi[y(\sigma)], \quad (6)$$

where ζ denotes the coupling constant.

The action S can be considered as a toy model for the interaction between the EM field (φ in the toy model) and charged degrees of freedom living on the mirrors (described by ξ), although, as it shall be evident below, it is unnecessary to have a coupling between the internal degrees of freedom and the vacuum field for dissipation to arise. It is nevertheless interesting to consider models where that coupling exists, in order to compare ‘inertial’ and DCE dissipations.

The scalar nature of the matter field is appropriate to the field configuration describing an ‘internal’ (charge) degree of freedom. We shall also consider the case of a Dirac-like matter field, but we shall not dwell with vector-like matter fields, which could account for phonon-like excitations, and certainly also exhibit inertial dissipation.

To take into account the effects of the internal degrees of freedom, we introduce $\Gamma_m(\varphi)$, the effective action due to the matter fields, defined as follows:

$$e^{-\Gamma_m(\varphi; \mathcal{M})} \equiv \int \mathcal{D}\xi e^{-S_m(\xi; \mathcal{M}) - S_I(\varphi, \xi; \mathcal{M})}, \quad (7)$$

which also encompasses the dependence on the geometry of \mathcal{M} .

Because of their quite different properties, we split Γ_m into two contributions, as follows:

$$\Gamma_m(\varphi; \mathcal{M}) = \Gamma_i(\mathcal{M}) + \Gamma_b(\varphi; \mathcal{M}) \quad (8)$$

where

$$\Gamma_i(\mathcal{M}) \equiv \Gamma_m(\varphi; \mathcal{M})|_{\varphi=0},$$

$$\Gamma_b(\varphi; \mathcal{M}) = \Gamma_m(\varphi; \mathcal{M}) - \Gamma_m(\varphi; \mathcal{M})|_{\varphi=0}.$$

Each one of these terms is relevant to different effects: $\Gamma_i(\mathcal{M})$ can induce dissipative effects, while $\Gamma_b(\varphi; \mathcal{M})$ is responsible for the emergence of boundary conditions on φ . Our focus shall be the calculation of $\Gamma_i(\mathcal{M})$, although we shall also derive the form of $\Gamma_b(\varphi; \mathcal{M})$ for the sake of completeness.

It is quite straightforward to see that:

$$\Gamma_i(\mathcal{M}) = \frac{1}{2} \text{Tr} \ln \mathcal{K} \quad (9)$$

where \mathcal{K} is the operator

$$\mathcal{K} = -\partial_\alpha [g^{1/2} g^{\alpha\beta} \partial_\beta] + g^{1/2} \mu^2 = g^{1/2} (-\Delta_{\mathcal{M}} + \mu^2), \quad (10)$$

and $\Delta_{\mathcal{M}} \equiv g^{-1/2} \partial_\alpha [g^{1/2} g^{\alpha\beta} \partial_\beta]$ is the Laplacian corresponding to the induced metric on \mathcal{M} . Note that the operator \mathcal{K} is Hermitian for the scalar product $(f_1, f_2) = \int d^d \sigma (f_1(\sigma))^* f_2(\sigma)$.

Besides, the boundary interaction term $\Gamma_b(\varphi; \mathcal{M})$, is given by:

$$\Gamma_b(\varphi; \mathcal{M}) = \frac{\zeta^2}{2} \int d^d \sigma g^{1/2}(\sigma) \int d^d \sigma' g^{1/2}(\sigma') \times \varphi[y(\sigma)] \mathcal{K}^{-1}(\sigma, \sigma') \varphi[y(\sigma')]. \quad (11)$$

Expressions (9) and (11), properly interpreted, are valid for one or more than one mirrors. If there is just one mirror, their interpretation is immediate: one needs to know a parametrization of its surface, then all the objects involved in (9) and (11) are derived from it and the free scalar field propagator. When there are $N > 1$ mirrors, we introduce a discrete index a , and use a different notation for the functions that parametrize each mirror: y_a^μ , $a = 1, \dots, N$ (which can be, for example, restrictions of the parametrization of \mathcal{M}). Then, since the action for the matter fields is localized on each mirror, we see that the effective action becomes the sum of the contributions due to each mirror:

$$\Gamma_i(\mathcal{M}) = \sum_{a=1}^N \Gamma_i(\mathcal{M}_a), \quad \Gamma_b(\varphi; \mathcal{M}) = \sum_{a=1}^N \Gamma_b(\varphi; \mathcal{M}_a), \quad (12)$$

where \mathcal{M}_a is the world-volume of each mirror.

In other words, quantum effects of the matter fields are, for this model, additive with respect to the mirrors. Thus, in the rest of this paper we shall only calculate Γ_i and Γ_b for single mirrors. Before presenting results of the evaluation of those contributions for different numbers of dimensions, we check that, in the limit when the microscopic degrees of freedom do not propagate, the boundary interaction term reduces to the standard one. Indeed, keeping just the leading term in a large- μ expansion:

$$\mathcal{K}^{-1}(\sigma, \sigma') \sim \mu^{-2} g^{-1/2}(\sigma) \delta(\sigma - \sigma') \quad (13)$$

so that for the leading behaviour of the boundary interaction term we have the following expression:

$$\Gamma_b(\varphi; \mathcal{M}) \sim \frac{\lambda}{2} \int d^d \sigma g^{1/2}(\sigma) \varphi^2[y(\sigma)] \quad (14)$$

where $\lambda \equiv \left(\frac{\zeta}{\mu}\right)^2$. This is the usual δ -like potential that has been used to analyze the Casimir effect with imperfect boundary conditions [5].

III. NO INERTIAL DISSIPATION IN 1 + 1 DIMENSIONS

To gain some insight into the previous expressions, let us first evaluate the effective action for a case where the outcome will be that there will be no inertial dissipation, namely, $d = 1$. Here the vacuum field is a massless real scalar field (in 1 + 1 dimensions) while ξ is a single quantum mechanical degree of freedom. For the model we consider, ξ cannot experience inertial forces, since it is not spatial (its values belong to an internal space), and moreover it is only a function of time.

The spacetime manifold \mathcal{M} is one-dimensional, a plane curve which we parametrize as follows: $\tau \rightarrow (\tau, q(\tau))$, where τ is the time and $q(\tau)$ the position of the mirror. The S_m and S_I terms may therefore be written in the form:

$$\begin{aligned} S_m(\xi; \mathcal{M}) &= \frac{1}{2} \int d\tau \left[e^{-1}(\tau) \left(\frac{d\xi(\tau)}{d\tau} \right)^2 + \mu^2 e(\tau) \xi^2(\tau) \right] \\ S_I(\varphi, \xi; \mathcal{M}) &= -i\zeta \int d\tau e(\tau) \xi(\tau) \varphi(\tau, q(\tau)) \end{aligned} \quad (15)$$

where $e(\tau) \equiv \sqrt{g(\tau)}$, and $g(\tau) \equiv 1 + \dot{q}^2(\tau)$. Then we note that, by performing the reparametrization: $\tau \rightarrow s$, such that $ds = e(\tau)d\tau$, we get:

$$\begin{aligned} S_m(\xi; \mathcal{M}) &= \frac{1}{2} \int ds \left[\left(\frac{d\tilde{\xi}(s)}{ds} \right)^2 + \mu^2 \tilde{\xi}^2(s) \right] \\ S_I(\varphi, \xi; \mathcal{M}) &= -i\zeta \int ds \tilde{\xi}(s) \tilde{\varphi}(s, \tilde{q}(s)) \end{aligned} \quad (16)$$

where we have introduced the notations:

$$\tilde{\xi}(s) \equiv \xi[\tau(s)], \quad \tilde{\varphi}[s, \tilde{q}(s)] \equiv \varphi[\tau(s), \tilde{q}(s)], \quad (17)$$

Integrating out the matter field (we assume its path integral to be invariant under reparametrizations [12]), we see that in the effective action term Γ_i , the dependence on $e(\tau)$ (hence on $q(\tau)$) is completely erased. Thus no dissipative effects from this origin may arise in $d = 1$, as advanced.

Regarding the Γ_b term, we find that:

$$\Gamma_b(\varphi; \mathcal{M}) = \frac{\zeta^2}{2} \int ds \int ds' \tilde{\varphi}(s, \tilde{q}(s)) \Delta(s-s') \tilde{\varphi}(s', \tilde{q}(s')) \quad (18)$$

with $\Delta(s-s') = e^{-\mu|s-s'|}/2\mu$. This boundary interaction term can be also written in a Fourier representation, so that it produces a frequency dependent coupling between the vacuum field and the mirror:

$$\Gamma_b(\varphi; \mathcal{M}) = \frac{\zeta^2}{2} \int \frac{d\omega}{2\pi} \frac{1}{\omega^2 + \mu^2} |\mathcal{F}(\tilde{\varphi})(\omega)|^2, \quad (19)$$

where \mathcal{F} denotes Fourier transform with respect to the s variable. We note that the outcome of this study amounts to the property that the Γ_b term is, essentially, a reparametrization invariant interaction, which is tantamount to using a relativistic invariant term. For the

non-dynamical limit, this is precisely one of the cases we have studied in [6]. Indeed, when the matter degrees of freedom do not propagate,

$$\begin{aligned} \Gamma_b(\varphi; \mathcal{M}) &\sim \frac{\lambda}{2} \int ds [\tilde{\varphi}(s, \tilde{q}(s))]^2 \\ &= \frac{\lambda}{2} \int d\tau \sqrt{1 + \dot{q}^2(\tau)} [\varphi(\tau, q(\tau))]^2, \end{aligned} \quad (20)$$

with $\lambda \equiv \frac{\zeta^2}{\mu}$, which is identical to equation (33) of [6].

In the general, nonlocal case, we can still find the effective action $\Gamma[q(\tau)]$, for a single mirror, obtained by performing now the functional integration over the vacuum field:

$$e^{-\Gamma[q(\tau)]} = \int \mathcal{D}\varphi e^{-S_f(\varphi) - \Gamma_b(\varphi; \mathcal{M})}. \quad (21)$$

To second order in the mirror departure from its average position, we get:

$$\Gamma[q(\tau)] = - \int_{-\infty}^{+\infty} d\tau e(\tau) \int_{-\infty}^{+\infty} d\tau' e(\tau') q(\tau) F_s(\tau - \tau') q(\tau') \quad (22)$$

where:

$$F_s(\tau - \tau') = \int \frac{d\omega}{2\pi} e^{i\omega(\tau - \tau')} \tilde{F}_s(\omega), \quad \tilde{F}_s(\omega) = \tilde{F}(\omega) - \tilde{F}(0), \quad (23)$$

and:

$$\tilde{F}(\omega) = \frac{1}{4} \int \frac{d\nu}{2\pi} \left[\frac{(\nu + \omega)^2 + \mu^2}{\zeta} + \frac{1}{2|\nu + \omega|} \right]^{-1} |\nu|. \quad (24)$$

Although the integral cannot be evaluated exactly, it is clear that its UV behaviour is much improved with respect to the one corresponding to a local Γ_b , like in [6]. The physical reason for that is that the nonlocal Γ_b introduces a kind of cutoff frequency (of order μ) above which the mirror becomes ineffective to impose the boundary conditions.

IV. 2 + 1 DIMENSIONS: INERTIAL DISSIPATION ON A MOVING STRING

The situation is more interesting regarding dissipation when $d = 2$, since the space dependence of the matter field allow for the action of inertial forces. Besides, if $\mu = 0$, it is possible to derive a closed-form expression for $\Gamma_m(\mathcal{M})$. Indeed, this object is formally identical to the effective action for a massless real scalar field (ξ) in two dimensions, which can be found exactly, and it is nontrivial only if R , the scalar curvature of \mathcal{M} is different from 0. Thus, before writing the explicit form of the result, we find conditions for R to be different from 0.

Using \mathbf{y} to denote the two spatial components of the parametric form of \mathcal{M} , namely, $\mathbf{y} = \mathbf{y}(\sigma^0, \sigma^1)$ ($\sigma^0 \equiv \tau$,

$\sigma^1 \equiv \sigma$), we find, after a quite straightforward calculation, that:

$$R = \frac{2}{g^2} A \quad (25)$$

where g is the determinant of the induced metric, and:

$$\begin{aligned} A = & (\partial_1 y^1)^2 \partial_0^2 y^2 \partial_1^2 y^2 - \partial_1 y^1 \partial_1^2 y^1 \partial_0^2 y^2 \partial_1 y^2 \\ & - \partial_0^2 y^1 \partial_1 y^1 \partial_1 y^2 \partial_1^2 y^2 + (\partial_1 y^2)^2 \partial_0^2 y^1 \partial_1^2 y^1 \\ & - (\partial_1 y^1)^2 (\partial_0 \partial_1 y^2)^2 - (\partial_1 y^2)^2 (\partial_0 \partial_1 y^1)^2 \\ & + 2 \partial_1 y^1 \partial_1 y^2 \partial_0 \partial_1 y^1 \partial_0 \partial_1 y^2, \end{aligned} \quad (26)$$

where y^1 and y^2 are the two spatial components of \mathbf{y} . A simpler expression for R may be written when the surface can be written as a Monge patch (which is then necessarily open), that is, when \mathcal{M} can be parametrized as:

$$y^0 = \tau, \quad y^1 = \sigma, \quad y^2 = y(\tau, \sigma). \quad (27)$$

Note that $y(\tau, \sigma)$ describes a moving open string in $2+1$ dimensions. In this case we have

$$R = \frac{2}{g^2} F, \quad (28)$$

with

$$F = \left| \begin{array}{cc} \partial_\tau^2 y & \partial_\tau \partial_\sigma y \\ \partial_\sigma \partial_\tau y & \partial_\sigma^2 y \end{array} \right|, \quad g = 1 + (\partial_\tau y)^2 + (\partial_\sigma y)^2. \quad (29)$$

A necessary and sufficient condition for F to vanish can be obtained:

$$F = 0 \iff \exists \alpha \in \mathbb{R} / (\partial_\tau + \alpha \partial_\sigma) y(\tau, \sigma) = 0. \quad (30)$$

This includes (since α can also be 0 or ∞), the cases of an F which depends only on τ or only on σ . We may write then all solutions to the condition above, for R to be zero (and Γ_m trivial) as follows:

$$y = f(\tau - \alpha^{-1} \sigma), \quad (31)$$

(for any α). Thus, there will be no effect if the evolution of the mirror has the form of a packet that evolves undisturbed, with any speed (eventually zero). That is to say, they are solutions of the wave equation with wave speed α (arbitrary), and with a definite chirality: namely, they should either move to the left or to the right, but cannot be a combination of both. The absence of dissipative effects for this kind of evolutions is certainly related to the Lorentz invariance of the model.

On the other hand, combining both left and right movers, one certainly can have an $F \neq 0$. Thus, what is perhaps the simplest example of a configuration where $F \neq 0$ corresponds to standing waves.

We now recall the form of the (exact) effective action for a massless real scalar field in $1+1$ dimensions [7], to write:

$$\Gamma_m(\mathcal{M}) = -\frac{1}{96\pi} \int d^2 x R \frac{1}{\Delta} R. \quad (32)$$

We shall, in what follows, only consider the massless matter field case. The reason is that for a massive field the corresponding contribution is strongly suppressed, unless the curvature is big in comparison with a scale set by the mass μ . In such a case, however, the massless field case should be a good approximation. Moreover, in the massive case there are also local terms in the curvature, but their locality makes them irrelevant to dissipative effects.

Of course, inserting the expression corresponding to a Monge Patch, one gets a more explicit form. Doing that, one may look for the Γ_m that results from the lowest order contribution, in a derivative expansion of $y(\tau, \sigma)$:

$$\begin{aligned} \Gamma_m \simeq & \frac{1}{24\pi} \int d\tau d\sigma \int d\tau' d\sigma' F(\tau, \sigma) \Delta_0^{-1}(\tau - \tau', \sigma - \sigma') \\ & \times F(\tau', \sigma'). \end{aligned} \quad (33)$$

where Δ_0^{-1} is the free propagator (which has the usual logarithmic form), and F has been defined in (29).

The pole in the free propagator is responsible for the existence of a nontrivial dissipation. Indeed, we may Fourier transform, and rotate to real time, to obtain the imaginary part of Γ_m :

$$\begin{aligned} \text{Im} [\Gamma_m] \simeq & \frac{1}{96\pi} \int \frac{dk_1}{2\pi} \frac{1}{|k_1|} \left[|\tilde{F}(k_0 = |k_1|, k_1)|^2 \right. \\ & \left. + |\tilde{F}(k_0 = -|k_1|, k_1)|^2 \right], \end{aligned} \quad (34)$$

where $\tilde{F}(k_0, k_1)$ is the Fourier transform of F . Note that the imaginary part sees the arguments of this function on-shell. On the other hand, F is quadratic in y and therefore the dissipation is quartic in the deformation of the string.

V. 3+1 DIMENSIONS: INERTIAL DISSIPATION ON A MOVING MIRROR

We conclude our study of the scalar field model with the $d = 3$ case. Here, it is not possible to find an exact effective action for an arbitrary manifold \mathcal{M} (even when $\mu = 0$). The effective action can, however, be computed approximately, in an expansion in powers of the curvature. One has, up to the second order [8]:

$$\begin{aligned} \Gamma_i(\mathcal{M}) \simeq & -\frac{1}{64 \times 2^{3/2}} \int d^3 \sigma \sqrt{g(\sigma)} \left[a_1 R_{\alpha\beta} (-\Delta)^{-\frac{1}{2}} R_{\alpha\beta} \right. \\ & \left. + a_2 R (-\Delta)^{-\frac{1}{2}} R \right] + \Gamma_{local}, \end{aligned} \quad (35)$$

where $a_1 = -1$ and $a_2 = 1/8$. Γ_{local} is a divergent contribution constructed with the induced metric and its derivatives, and we will assume that these divergences are absorbed into appropriate counterterms in the classical action. Since local terms are not relevant for the dissipative effects considered here, we will omit Γ_{local} in what follows.

The nonlocal kernel $(-\Delta)^{-\frac{1}{2}}$ can be formally defined using the integral representation [9]

$$(-\Delta)^{-\frac{1}{2}} = \frac{2}{\pi} \int_0^{+\infty} dm \frac{1}{-\Delta + m^2}, \quad (36)$$

in terms of the two-point function of a massive scalar field $(-\Delta + m^2)^{-1}$.

Consistently with the expansion of the effective action in powers of the curvature, we shall analyze the nonlocal part of $\Gamma_i(\mathcal{M})$ under the assumption that the mirror is almost flat, and that the time dependence of its surface is smooth. This means small departures from a flat surface almost flat manifold \mathcal{M} . To implement this in a quite straightforward way, we first assume that the parametrization used is of the form:

$$y^\mu = \begin{cases} \sigma^\mu & \text{for } \mu = 0, 1, 2 \\ y(\sigma^0, \sigma^1, \sigma^2) & \text{for } \mu = 3. \end{cases} \quad (37)$$

Then it follows that $g_{\alpha\beta} = \delta_{\alpha\beta} + h_{\alpha\beta}$, with $h_{\alpha\beta} \equiv \partial_\alpha y \partial_\beta y$. We then perform an expansion in the deformation of the manifold, i.e., around the $\delta_{\alpha\beta}$ metric: $g_{\alpha\beta} = \delta_{\alpha\beta} + h_{\alpha\beta}$, assuming that $h_{\alpha\beta}$ is small. To find the lowest non-trivial contribution in an expansion in h , we note that:

$$\Gamma_i(\mathcal{M}) \simeq \frac{1}{64 \times 2^{3/2}} \int d^3\sigma R_{\alpha\beta} (-\Delta_0)^{-\frac{1}{2}} \times [\delta_{\alpha\rho} \delta_{\beta\sigma} - \frac{1}{8} \delta_{\alpha\beta} \delta_{\rho\sigma}] R_{\rho\sigma}, \quad (38)$$

where $R_{\alpha\beta}$ is the Ricci tensor expanded to its lowest non-trivial order in $h_{\alpha\beta}$ and $(-\Delta_0)^{-\frac{1}{2}}$ is the nonlocal kernel for a flat spacetime.

To linear order in $h_{\alpha\beta}$ we have

$$R_{\alpha\beta} \simeq \frac{1}{2} (\partial_\alpha \partial_\beta h_{\gamma\gamma} + \partial_\gamma \partial_\gamma h_{\alpha\beta} - \partial_\alpha \partial_\gamma h_{\gamma\beta} - \partial_\beta \partial_\gamma h_{\gamma\alpha}) \quad (39)$$

Using this equation, it is straightforward to write the Ricci tensor in terms of the function $y(\sigma)$. For instance we have

$$R_{00} \simeq \partial_0 \partial_1 y \partial_0 \partial_1 y + \partial_0 \partial_2 y \partial_0 \partial_2 y - \partial_0 \partial_0 y \partial_1 \partial_1 y - \partial_0 \partial_0 y \partial_2 \partial_2 y, \quad (40)$$

and similar expressions for the other components.

As in the two-dimensional case, one can readily show that all components of the Ricci tensor vanish when the evolution of the mirror is described by a packet that moves with constant velocity in any direction, i.e. $y = f(\sigma^0 - \alpha^{-1} \sigma^i)$. Therefore, the manifold \mathcal{M} is flat for this kind of evolutions, since in $d = 3$ the Riemann tensor is entirely determined by the Ricci tensor. Once more, simple evolutions that produce dissipative effects are standing waves.

The dissipative effects induced by the matter degrees of freedom are related to the imaginary part of the effective action in Minkowski spacetime. Rotating to real time,

and introducing Fourier transforms for the flat propagator and the Ricci tensor we obtain

$$\text{Im} [\Gamma_i] \simeq \text{Im} \left[\frac{1}{32\pi 2^{3/2}} \int_0^\infty dm \int \frac{d^3 k}{(2\pi)^3} \times \frac{\tilde{R}_{\alpha\beta}(k) \tilde{R}_{\rho\sigma}(-k) [\eta^{\alpha\rho} \eta^{\beta\sigma} - \frac{1}{8} \eta^{\alpha\beta} \eta^{\rho\sigma}]}{(-k_0^2 + k_1^2 + k_2^2 + m^2 - i\epsilon)} \right] \quad (41)$$

Introducing the notation

$$|\tilde{B}(k)|^2 = \tilde{R}_{\alpha\beta}(k) \tilde{R}_{\rho\sigma}(-k) [\eta^{\alpha\rho} \eta^{\beta\sigma} - \frac{1}{8} \eta^{\alpha\beta} \eta^{\rho\sigma}], \quad (42)$$

and performing the integration over k_0 , we find

$$\text{Im} [\Gamma_i] \simeq \frac{1}{2^{3/2} 64} \int_0^\infty dm \int \frac{d^2 k}{(2\pi)^2} \times \frac{|\tilde{B}(k_0 = \omega_{\mathbf{k}}, \mathbf{k})|^2 + |\tilde{B}(k_0 = -\omega_{\mathbf{k}}, \mathbf{k})|^2}{\omega_{\mathbf{k}}} \quad (43)$$

with $\omega_{\mathbf{k}} = \sqrt{k_1^2 + k_2^2 + m^2}$. The structure of this result is similar to that of the two-dimensional case (34), except for the additional integral in the parameter m , used to have a suitable integral representation of the nonlocal kernel $(-\Delta_0)^{-\frac{1}{2}}$.

It is worth mentioning that a case in which the matter field is a massless Dirac field ψ (in 2+1 dimensions) leads to very similar results. Assuming the matrices γ^a ($a = 1, 2, 3$) for this field to be in a reducible representation of Clifford's algebra, chosen in such a way that the parity anomaly is cancelled, the matter field action S_m is given by:

$$S_m(\bar{\psi}, \psi; \mathcal{M}) = \int d^3\sigma \sqrt{g(\sigma)} \bar{\psi}(\sigma) D \psi(\sigma) \quad (44)$$

where $D = \gamma^a e_a^\mu(\sigma) D_\mu$, e_a^μ is the dreibein and $D_\mu = \partial_\mu - \frac{i}{4} \omega_{b\mu}^a \sigma_{ab}$. Here, $\omega_{b\mu}^a$ denotes the spin connection (determined by the dreibein and Christoffel symbol) and $\sigma_{ab} \equiv \frac{i}{2} [\gamma_a, \gamma_b]$. The result for Γ_i may also be read from [8], and it is given by (35) with the only difference being in the numerical values of the coefficients a_1 and a_2 , that we will not need in what follows. Thus, all the estimates below, as well as the conditions for no dissipation hold true for this model as well. Note that the Dirac field model might be relevant to models describing graphene surfaces [10].

We can obtain an estimation of $\text{Im} [\Gamma_i]$ for the scalar and Dirac fields, by assuming that the dynamics of the mirror is described by a standing wave of the form

$$y(\sigma^0, \sigma^1, \sigma^2) = y_0 \cos(\Omega \sigma^0) \cos(\sigma^1/L), \quad (45)$$

where we have denoted by y_0 the amplitude of the oscillations, Ω its frequency and L the typical distance between nodes in the direction of σ^1 . For simplicity, consider evolutions of the mirror that are invariant along σ^2 . From (39) and (40), we can calculate the Ricci tensor. For

example, we can estimate the Fourier transform of (40), which can be written as

$$\begin{aligned} \tilde{R}_{00}(k) = & -\frac{4\pi^3 y_0^2 \Omega^2}{L^2} \delta(k_2) [\delta(k_1) (\delta(k_0 - 2\Omega) \\ & + \delta(k_0 + 2\Omega)) + \delta(k_0) (\delta(k_1 - 2/L) + \delta(k_1 + 2/L))]. \end{aligned}$$

Similar expressions can be found for the 11-component of the Ricci tensor, and it is easy to see that $R_{10} = R_{01} = 0$.

In order to have a rough estimation of the magnitude of $\text{Im}[\Gamma_i]$ when the mirror's dynamics is approximated by Eq.(45), we need to perform integrations in Eq.(43). After integrating in k_1 and k_2 , we get

$$\text{Im}[\Gamma_i] \sim \frac{y_0^4 \Omega^3 T \Sigma}{L^4}, \quad (46)$$

where T is the total time during which the mirror is moving, and Σ denotes the total surface of the (unperturbed) mirror. The presence of these coefficients is a byproduct of having considered a delocalized standing wave as the one in (45). It is worth noting that, for this particular motion of the mirror, there is no threshold for the dissipation effects.

Restoring \hbar and the propagation velocity (v_F) of the matter fields inside the layer, the estimated magnitude of $\text{Im}[\Gamma_i]$ becomes

$$\frac{\text{Im}[\Gamma_i]}{T \Sigma} \sim \frac{\hbar y_0^4 \Omega^3}{v_F^2 L^4}. \quad (47)$$

We may compare this result with the one obtained for one perfect mirror oscillating according with Eq. (45). In this case, the DCE produces an imaginary part in the effective action as long as $\Omega > c/L$ [11]:

$$\frac{\text{Im}[\Gamma^{\text{DCE}}]}{T \Sigma} \sim \frac{\hbar y_0^2 \Omega^5 (1 - \frac{c^2}{L^2 \Omega^2})^{5/2}}{c^4}. \quad (48)$$

When Ω is above threshold, the ratio between both effects is approximately given by

$$\frac{\text{Im}[\Gamma_i]}{\text{Im}[\Gamma^{\text{DCE}}]} \sim \left(\frac{y_0}{L}\right)^2 \left(\frac{c}{v_F}\right)^2 \left(\frac{c}{\Omega L}\right)^2 \left(1 - \frac{c^2}{L^2 \Omega^2}\right)^{-5/2}. \quad (49)$$

This ratio may be bigger or smaller than one depending on the value of the different parameters. It is interesting to remark that while the inertial dissipation is quartic in the amplitude y_0 , the dissipation associated to the DCE is quadratic. Moreover, for the particular motion (45), the inertial effects do not have a frequency threshold, while the DCE does.

In Eq. (49) we compared the inertial dissipation with the DCE for a perfect mirror. Note, however, that, being independent of the coupling to the vacuum field, the inertial effect may be more relevant than the DCE in cases where that coupling is rather weak, namely, when the mirrors are far from perfect.

It is also worth noting that the result mentioned above, that a deformation traveling undistorted (with a constant direction) does not produce inertial dissipation, may be relevant to the DCE. Indeed, in the case of having more than one mirror with such deformation, the configuration is free of inertial dissipation, while the DCE does not vanish.

Finally, we wish to point out that we have considered here the effect due to inertial forces just for one matter field; in a real medium many different contributions can appear, coming from the excitation of different internal degrees of freedom. All of them will contribute to the imaginary part, with different parameters and kinematical factors.

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- [1] V.V. Dodonov in *Modern Nonlinear Optics*, Advances in Chemical Physics Series, ed. by M.W. Evans (Wiley, New York 2001), Vol. 119, p. 309; A. Lambrecht, J.Opt. B: Quantum Semiclass. Opt. **7**, S3 (2005); V.V. Dodonov, J. Phys.: Conf. Ser. **161**, 012027 (2009); D. A. R. Dalvit, P. A. Maia Neto and F. D. Mazzitelli, arXiv:1006.4790, to appear in *Lecture Notes in Physics*, Volume on Casimir Physics, ed. by D.A.R. Dalvit, P. Milonni, D. Roberts, and F.da Rosa.
- [2] S. J. Barnett, Phys. Rev. **6**, 239 (1915); R. C. Tolman and T. D. Stewart, Phys. Rev. **8**, 2 (1916); S. J. Barnett,

Rev. Mod. Phys. **7**, 2, 129 (1935).

- [3] M. Kardar and R. Golestanian, Rev. Mod. Phys. **71**, 1233 (1999).
- [4] J.B. Pendry, J. Phys.:Condens. Matter **9**, 10301 (1997); A. Manjavacas and F.J. Garcia de Abajo, Phys. Rev. Lett. **105**, 113601 (2010).
- [5] M. Bordag, M. Hennig and D. Robaschik, J. Phys. A:Math. Gen **25**, 4483 (1992); K. Milton, J. Phys. A: Math. Gen **37**, 6391 (2004); K. A. Milton and J. Wagner, J. Phys. A **41**, 155402 (2008); C. D. Fosco, F. C. Lombardo and F. D. Mazz-

- itelli, Phys. Rev. D **80**, 085004 (2009).
- [6] C. D. Fosco, F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. D **76**, 085007 (2007)
- [7] A. M. Polyakov, Phys. Lett. B **103**, 211 (1981).
- [8] A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B **333**, 471 (1990); I. G. Avramidi, Nucl. Phys. B **355**, 712 (1991) [Erratum-ibid. B **509**, 557 (1998)].
- [9] D. Lopez Nacir and F. D. Mazzitelli, Phys. Rev. D **75**, 024003 (2007)
- [10] M.I. Katsnelson and K.S. Novoselov, Solid State Comm. **143**, 3 (2007).
- [11] R. Golestanian and M. Kardar, Phys. Rev. A **58**,1713 (1998).
- [12] The reparametrization used, on the other hand, is always well defined since $e \neq 0$. We assume, having in mind the real time version, that the speed of the mirror is always smaller than c .