Power-Law Entropic Corrections to Newton's Law and Friedmann Equations From Entropic Force

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A possible source of black hole entropy could be the entanglement of quantum fields in and out the horizon. The entanglement entropy of the ground state obeys the area law. However, a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. Inspired by the power-law corrections to entropy and adopting the viewpoint that gravity emerges as an entropic force, we derive modified Newton's law of gravitation as well as the corrections to Friedmann equations. In a quite different approach, we obtained power-law corrected Friedmann equation by starting from the first law of thermodynamics at apparent horizon of a FRW universe, and assuming that the associated entropy with apparent horizon has a power-law corrected relation. Our study shows a consistency between the obtained results of these two different approach.

I. INTRODUCTION

Recently, Verlinde [1] demonstrated that gravity can be interpret as an entropic force caused by the changes in the information associated with the positions of material bodies. In his new proposal, Verlinde obtained successfully the Newton's law of gravitation, the Poisson's equation and Einstein field equations by employing the holographic principle as well as the equipartition law of energy. As soon as Verlinde presented his idea, many relevant works about entropic force appeared. For example, Friedmann equations from entropic force have been derived in Refs. [2, 3]. The Newtonian gravity [4], the holographic dark energy [5] and thermodynamics of black holes [6] have been investigated by using the entropic force approach. It has been shown that uncertainty principle may arise in the entropic force paradigm [7]. Other studies on the entropic force, which raised a lot of attention recently, have been carried out in [8].

On the other hand, string theory, as well as the string inspired braneworld scenarios such as RSII model, suggest a modification of Newtons law of gravitation at small distance scales [9, 10]. In addition, there have been considerable works on quantum corrections of some basic physical laws. The loop quantum corrections to the Newton and Coulomb potential have been considered in some references (see [11] and references therein). Also, corrections to Friedmann equations from loop quantum gravity has been studied in [12].

Inspired by the Verlinde's argument and considering the quantum corrections to area law of the black hole entropy, one is able to derive some physical equations with correction terms. For example, modified Newton's law of gravitation has been studied in [13], while entropic corrections to Coulomb's law have been investigated in [14]. In addition, modified Friedmann equations have also been constructed in [15, 16]. In all these cases [13–16] the corrected entropy has the logarithmic term which arises from the inclusion of quantum effects, motivated from the loop quantum gravity and is due to the thermal equilibrium fluctuations and quantum fluctuations [17].

In this paper we would like to consider the effects of the power-law correction terms to the entropy on the laws of gravitation. The power-law corrections to entropy appear in dealing with the entanglement of quantum fields in and out the horizon [18]. Indeed, it has been shown that the origin of black hole entropy may lie in the entanglement of quantum fields between inside and outside of the horizon [18]. Since the modes of gravitational fluctuations in a black hole background behave as scalar fields, one is able to compute the entanglement entropy of such a field, by tracing over its degrees of freedom inside a sphere. In this way the authors of [18] showed that the black hole entropy is proportional to the area of the sphere when the field is in its ground state, but a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. For large horizon areas, these corrections are relatively small and the area law is recovered. Applying this power-law corrected entropy, we obtain the corrections to Newton's law as well as modified Friedmann equation by adopting the viewpoint that gravity emerges as an entropic force. In a quite different approach, we are able to derive power-law corrected Friedmann equation by starting from the first law of thermodynamics at apparent horizon of a FRW universe, and assuming that the associated entropy with apparent horizon has a power-law corrected relation. We find that both modified Friedmann equations are consistent to each other.

The outline of our paper is as follows. In the next section, we use Verlinde approach to derive Newton's law of gravitation with a correction term. In section III, we derive the power-law entropy-corrected Friedmann equation of FRW universe by considering gravity as an entropic force. Then, in section IV, we obtain modified Friedmann equation by applying the first law of thermodynamics at apparent horizon of a FRW universe. We finish our paper with some closing remarks.

II. ENTROPIC CORRECTION TO NEWTON'S LAW

According to Verlinde's argument, when a test particle moves apart from the holographic screen, the magnitude of the entropic force on this body has the form

$$F\triangle x = T\triangle S, \tag{1}$$

where $\triangle x$ is the displacement of the particle from the holographic screen, while T and $\triangle S$ are the temperature and the entropy change on the screen, respectively.

In Verlinde's discussion, the black hole entropy S plays a crucial role. Indeed, the derivation of Newton's law of gravity depends on the entropy-area relationship $S = k_B A/4\ell_p^2$ of black holes in Einsteins gravity, where $A = 4\pi R^2$ represents the area of the horizon and $\ell_p = \sqrt{G\hbar/c^3}$ is the Planck length. However, the power-law corrections to entropy appear in dealing with the entanglement of quantum fields in and out the horizon [18]. The power-law corrected entropy has the form [19]

$$S = \frac{k_B A}{4\ell_p^2}\left[1 - K_\alpha A^{1-\alpha/2}\right], \tag{2}$$

where α is a dimensionless constant whose value is currently under debate, k_B stands for the Boltzmann constant and

$$K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{2-\alpha}}, \tag{3}$$

where r_c is the crossover scale. Taking the power-law correction to entropy into account, Newton's law of gravitation as well as Friedman equations will be modified accordingly. First of all, we rewrite Eq. (2) in the following form

$$S = k_B\left[\frac{A}{4\ell_p^2} + s(A)\right], \tag{4}$$

where $s(A)$ stands for the correction term in the entropy expression. Suppose we have two masses one a test mass and the other considered as the source with respective masses m and M. Centered around the source mass M, is a spherically symmetric surface $\mathcal{S}$ which will be defined with certain properties that will be made explicit later. To derive the entropic law, the surface $\mathcal{S}$ is between the test mass and the source mass, but the test mass is assumed to be very close to the surface as compared to its reduced Compton wavelength $\lambda_m = \frac{\hbar}{mc}$. When a test mass m is a distance $\triangle x = \eta\lambda_m$ away from the surface $\mathcal{S}$, the entropy of the surface changes by one fundamental unit $\triangle S$ fixed by the discrete spectrum of the area of the surface via the relation

$$\triangle S = \frac{\partial S}{\partial A}\triangle A = k_B\left(\frac{1}{4\ell_p^2} + \frac{\partial s(A)}{\partial A}\right)\triangle A. \tag{5}$$

The energy of the surface $\mathcal{S}$ is identified with the relativistic rest mass of the source mass:

$$E = Mc^2. \tag{6}$$

On the surface $\mathcal{S}$, there live a set of "bytes" of information that scale proportional to the area of the surface so that

$$A = QN, \tag{7}$$

where N represents the number of bytes and Q is a fundamental constant which should be specified later. Assuming the temperature on the surface is T, and then according to the equipartition law of energy [20], the total energy on the surface is

$$E = \frac{1}{2}Nk_BT. \tag{8}$$

Finally, we assume that the force on the particle follows from the generic form of the entropic force governed by the thermodynamic equation

$$F = T\frac{\triangle S}{\triangle x}, \tag{9}$$

where $\triangle S$ is one fundamental unit of entropy when $|\triangle x| = \eta\lambda_m$, and the entropy gradient points radially from the outside of the surface to inside. Note that N is the number of bytes and thus $\triangle N = 1$, hence from (7) we have $\triangle A = Q$. Combining Eqs. (5)- (9), we find

$$F = -\frac{Mm}{R^2}\left(\frac{Q^2c^3}{8\pi\hbar\eta\ell_p^2}\right)\left[1 + 4\ell_p^2\frac{\partial s(A)}{\partial A}\right]_{A=4\pi R^2}, \tag{10}$$

This is nothing but the Newton's law of gravitation to the first order provided we define $Q^2 = 8\pi\eta\ell_p^4$. Thus we reach

$$F = -\frac{GMm}{R^2}\left[1 + 4\ell_p^2\frac{\partial s}{\partial A}\right]_{A=4\pi R^2}, \tag{11}$$

Using Eq. (2) we obtain

$$\left(\frac{\partial s}{\partial A}\right)_{A=4\pi R^2} = -\frac{K_\alpha(4-\alpha)}{8\ell_p^2}\left(4\pi R^2\right)^{1-\alpha/2} \tag{12}$$

Substituting Eq. (12) in Eq. (11) we obtain

$$F = -\frac{GMm}{R^2}\left[1 - \frac{K_\alpha}{2}(4-\alpha)\left(4\pi R^2\right)^{1-\alpha/2}\right], \tag{13}$$

Using Eq. (3) the above relation can be rewritten as

$$F = -\frac{GMm}{R^2}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{14}$$

This is the power-law correction to the Newton's law of gravitation. When $\alpha = 0$, one recovers the usual Newton's law. A close look at Eq. (14) shows that the significant of the corrected term in various regions depends on the value of α. Indeed, it was argued that α should be ranges as $2 < \alpha < 4$ [18]. Besides, the satisfaction of the generalized second law of thermodynamics for the universe with the power-law corrected entropy (1) implies that $\alpha > 2$ [19]. In this case ($\alpha > 2$), the corrected term can be comparable to the first term when $r_c \sim R$. On the other hand, for small distance, $R \ll r_c$, the correction becomes significantly large, while for large distance, $R \gg r_c$, the correction term is relatively small and one obtains the usual Newton's law of gravitation. This is consistent with the argument that for large horizon areas, the power-law corrections are relatively small and the area law is recovered [18].

III. MODIFIED FRIEDMANN EQUATIONS FROM ENTROPIC FORCE

Next, we extend our discussion to the cosmological setup. Assuming the background spacetime to be spatially homogeneous and isotropic which is given by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu + R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{15}$$

where $R = a(t)r$, $x^0 = t, x^1 = r$, the two dimensional metric $h_{\mu\nu}$=diag $(-1, a^2/(1-kr^2))$. Here k denotes the curvature of space with $k = 0, 1, -1$ corresponding to open, flat, and closed universes, respectively. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation $h^{\mu\nu}\partial_\mu R\partial_\nu R = 0$. A simple calculation gives the apparent horizon radius for the FRW universe

$$R = ar = \frac{1}{\sqrt{H^2 + k/a^2}} \tag{16}$$

where $H = \dot{a}/a$ is the Hubble parameter. We also assume the matter source in the FRW universe is a perfect fluid of mass density ρ and pressure p with stress-energy tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}. \tag{17}$$

Due to the pressure, the total mass $M = \rho V$ in the region enclosed by the boundary $\mathcal{S}$ is no longer conserved, the change in the total mass is equal to the work made by the pressure $dM = -pdV$, which leads to the well-known continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{18}$$

It is instructive to first derive the dynamical equation for Newtonian cosmology. Consider a compact spatial region V with a compact boundary $\mathcal{S}$, which is a sphere with physical radius $R = a(t)r$. Note that here r is a dimensionless quantity which remains constant for any cosmological object partaking in free cosmic expansion. Combining the second law of Newton for the test particle m near the surface with gravitational force (14) we get

$$F = m\ddot{R} = m\ddot{a}r = -\frac{GMm}{R^2}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{19}$$

We also assume $\rho = M/V$ is the energy density of the matter inside the the volume $V = \frac{4}{3}\pi a^3 r^3$. Thus, Eq. (19) can be rewritten as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right], \tag{20}$$

This is nothing but the power-law entropy-corrected dynamical equation for Newtonian cosmology. The main difference between this equation and the standard dynamical equation for Newtonian cosmology is that the correction terms now depends explicitly on the radius R. However, we can remove this confusion. Assuming that for Newtonian cosmology the spacetime is Minkowskian with $k = 0$, then we get $R = 1/H$, and we can rewrite Eq. (20) in the form

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left[1 - \frac{\alpha}{2}{r_c}^{\alpha-2}\left(\frac{\dot{a}}{a}\right)^{\alpha-2}\right]. \tag{21}$$

It was argued in [21] that for deriving the Friedmann equations of FRW universe in general relativity, the quantity that produces the acceleration is the active gravitational mass $\mathcal{M}$ [22], rather than the total mass M in the spatial region V. With the entropic correction term, the active gravitational mass $\mathcal{M}$ will also modified as well. On one side, from Eq. (20) with replacing M with $\mathcal{M}$ we have

$$\mathcal{M} = -\frac{\ddot{a}a^2}{G}r^3\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]^{-1}. \tag{22}$$

On the other side, the active gravitational mass is defined as [21]

$$\mathcal{M} = 2\int_V dV\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)u^\mu u^\nu. \tag{23}$$

A simple calculation leads

$$\mathcal{M} = (\rho + 3p)\frac{4\pi}{3}a^3 r^3. \tag{24}$$

Equating Eqs. (22) and (24), we find

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \tag{25}$$

Multiplying $\dot{a}a$ on both sides of Eq. (25), and using the continuity equation (18) we reach

$$\frac{d}{dt}(\dot{a}^2) = \frac{8\pi G}{3}\frac{d}{dt}(\rho a^2)\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \tag{26}$$

Integrating of Eq. (26), we find

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho\left[1 - \frac{\alpha}{2\rho a^2}\left(\frac{r_c}{r}\right)^{\alpha-2}\int\frac{d(\rho a^2)}{a^{\alpha-2}}\right], \tag{27}$$

where k is a constant of integration. Now, in order to calculate the integral we need to find $\rho = \rho(a)$. Assume the equation of state parameter $w = p/\rho$ is a constant, the continuity equation (18) can be integrated immediately to give

$$\rho = \rho_0 a^{-3(1+w)}, \tag{28}$$

where ρ_0, an integration constant, is the present value of the energy density. Inserting relation (28) in Eq. (27), after integration, we obtain

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \left[1 - \frac{\alpha}{2}\frac{(3w+1)}{(3w+\alpha-1)}\left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \tag{29}$$

Using Eq. (16), we can rewrite the above equation as

$$\left(H^2 + \frac{k}{a^2}\right)\left[1 - \frac{\alpha}{2}{r_c}^{\alpha-2}\frac{(3w+1)}{(3w+\alpha-1)}\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2-1}\right]^{-1} = \frac{8\pi G}{3}\rho. \tag{30}$$

If α is taken as small quantity, then the above equation can be expanded up to the linear order of α. The result is

$$\left(H^2 + \frac{k}{a^2}\right) + \beta {r_c}^{\alpha-2}\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2} = \frac{8\pi G}{3}\rho, \tag{31}$$

where

$$\beta = \frac{\alpha(3w+1)}{2(3w+\alpha-1)}. \tag{32}$$

Thus we have derived the power-law entropy-corrected Friedmann equation of FRW universe by considering gravity as an entropic force caused by changes in the information associated with the positions of material bodies. In the absence of the correction terms ($\alpha = 0 = \beta$), one recovers the well-known Friedmann equation in standard cosmology. Since $\alpha > 2$ the correction term in (31) can be comparable to the first term only when a is very small, thus the corrections make sense only at early stage of the universe where $a \to 0$. When the universe becomes large, the power-law entropy-corrected Friedmann equation reduces to the standard Friedman equation.

IV. MODIFIED FRIEDMANN EQUATIONS FROM THE FIRST LAW

To show the correctness of our final result (31) in the previous section, here we adopt another approach. Indeed, we are able to derive modified Friedmann equation by applying the first law of thermodynamics at apparent horizon of a FRW universe, with the assumption that the associated entropy with apparent horizon has the power-law corrected form (2). It was already shown that the differential form of the Friedmann equation in the FRW universe can be written in the form of the first law of thermodynamics on the apparent horizon [23]. We follow the method developed in [24]. Throughout this section we set $\hbar = c = k_B = 1$ for simplicity. The associated temperature with the apparent horizon can be defined as [25]

$$T = \frac{\kappa}{2\pi} = -\frac{1}{2\pi R}\left(1 - \frac{\dot{R}}{2HR}\right). \tag{33}$$

where κ is the surface gravity. When $\dot{R} \leq 2HR$, the temperature becomes negative $T \leq 0$. Physically it is not easy to accept the negative temperature. In this case the temperature on the apparent horizon should be defined as $T = |\kappa|/2\pi$. The work density is obtained as [26]

$$W = \frac{1}{2}(\rho - p). \tag{34}$$

The work density term is regarded as the work done by the change of the apparent horizon. We also assume the first law of thermodynamics on the apparent horizon is satisfied and has the form

$$dE = T_h dS_h + W dV, \tag{35}$$

where S_h is the power-law corrected entropy associated with the apparent horizon which has the form (2). We also assume $E = \rho V$ is the total energy content of the universe inside a 3-sphere of radius R, where $V = \frac{4\pi}{3}R^3$ is the volume enveloped by 3-dimensional sphere with the area of apparent horizon $A = 4\pi R^2$. Taking differential form of the relation $E = \frac{4\pi}{3}\rho R^3$ for the total matter and energy inside the apparent horizon, and using the continuity equation (18), we get

$$dE = 4\pi\rho R^2 dR - 4\pi H R^3(\rho + p)dt. \tag{36}$$

Taking differential form of the corrected entropy (2), we have

$$dS_h = \frac{2\pi R}{G}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right] dR. \tag{37}$$

Inserting Eqs. (33), (34), (36) and (37) in the first law (35), we can get the differential form of the modified Friedmann equation

$$\frac{1}{4\pi G}\frac{dR}{R^3}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right] = H(\rho + p)dt. \tag{38}$$

Using the continuity equation (18), we can rewrite it as

$$-\frac{2}{R^3}\left[1 - \frac{\alpha}{2}\left(\frac{r_c}{R}\right)^{\alpha-2}\right] dR = \frac{8\pi G}{3} d\rho. \tag{39}$$

Integrating (39) yields

$$\frac{1}{R^2} - \frac{r_c^{\alpha-2}}{R^\alpha} = \frac{8\pi G}{3}\rho + C, \tag{40}$$

where C is the integration constant to be determined later. Substituting R from Eq.(16) we obtain entropy-corrected Friedmann equation

$$H^2 + \frac{k}{a^2} - r_c^{\alpha-2}\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2} = \frac{8\pi G}{3}\rho + C. \tag{41}$$

The constant C can be determined by taking the $\alpha \longrightarrow 0$ limit of the above expression. In this limit Eq. (41) reduces to the usual Friedmann equation provided $C = -r_c^{-2}$. Thus we reach

$$H^2 + \frac{k}{a^2} - r_c^{-2}\left[r_c^\alpha\left(H^2 + \frac{k}{a^2}\right)^{\alpha/2} - 1\right] = \frac{8\pi G}{3}\rho. \tag{42}$$

On the other hand the constant C can be absorbed in ρ. In this case the Friedmann equation (41) is consistent with Eq. (31) derived in the previous section provided

$$\alpha = \frac{2(1-3w)}{3(1+w)}. \tag{43}$$

This shows a strong consistency check on Verlinde's model. It is also notable to mention that Eq. (42) is consistent with the result obtained in [27]. However, our derivation is quite different from [27]. Let us stress the difference between our derivation in this section and [27]. First of all, the authors of [27] have derived modified Friedmann equations by applying the first law of thermodynamics, $TdS = -dE$, to the apparent horizon of a FRW universe with the assumption that the apparent horizon has corrected-entropy like (2). It is worthy to note that the notation dE in [27] is quite different from the same we used in this section. In [27], $-dE$ is actually just the heat flux crossing the apparent horizon within an infinitesimal internal of time dt. But, here dE is change in the the matter energy inside the apparent horizon. Besides, in [27] the apparent horizon radius R has been assumed to be fixed. But, here, the apparent horizon radius changes with time. This is the reason why we have included the term WdV in the first law (35). Indeed, the term $4\pi R^2 \rho dR$ in Eq. (36) contributes to the work term, while this term is absent in dE of [27]. This is consistent with the fact that in thermodynamics the work is done when the volume of the system is changed.

V. CLOSING REMARKS

It was argued that a possible source of black hole entropy could be the entanglement of quantum fields in and out the horizon [18]. The entanglement entropy of the ground state of field obeys the well-known area law. However, the power-law correction to the area law appears when the wave-function of the quantum field is chosen to be a superposition of ground state and exited state [18]. Indeed, the excited states contribute to the correction, and more excitations produce more deviation from the area law [28, 29]. Therefore, the correction terms are more significant for higher excitations.

Motivated by the power-law corrected entropy and adopting the viewpoint that gravity emerges as an entropic force, we derived modified Newton's law of gravitation as well as power-law correction to Friedmann equations. We found that the correction term for Friedmann equation falls off rapidly with apparent horizon radius and can be comparable to the first term only when the scale factor a is very small. Thus the corrections make sense only at early stage of the universe. When the universe becomes large, the power-law entropy-corrected Friedmann equation reduces to the standard Friedman equation. This can be understood easily. At late time where a is large, i.e., at low energies, it is difficult to excite the modes and hence, the ground state modes contribute to most of the entanglement entropy. However, at the early stage, i.e., at high energies, a large number of field modes can be excited and contribute significantly to the correction causing deviation from the area law and hence deviation from the standard Friedmann equation.

To show the correctness of our result, we also derived modified Friedmann equation from different approach. We started from the first law of thermodynamics at apparent horizon of a FRW universe, and assuming that the associated entropy with apparent horizon has power-law corrected form (2), we obtained modified Friedmann equation which is consistent with the result we obtained by Verlinde's technique. This provides a strong consistency check on Verlinde's model.

It is worth noting that although we derived modified Friedmann equations corresponding to the corrected entropy-area relation (2), it would be of great interest to see whether one is able to get modified Einstein field equation as well as Poisson equation by following Verlinde's argument [1]. We leave more investigations on this issue for future works.

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