The entangling side of the Unruh-Hawking effect

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We show that the Unruh effect can create net quantum entanglement between inertial and accelerated observers depending on the choice of the inertial state. This striking result banishes the extended belief that the Unruh effect can only destroy entanglement and furthermore provides a new and unexpected source for finding experimental evidence of the Unruh and Hawking effects.

The influence of the so-called Unruh and Hawking effects [1] on quantum entanglement has been subject of many studies in the field of relativistic quantum information [2–6]. In all these previous studies it was shown how starting with entangled states from an inertial perspective we end up with a less entangled state when one of the observers is non-inertial. In this letter we show an unexpected outcome of these Unruh and Hawking effects: the appearance of entanglement when one of the observers of a bipartite system undergoes a constant acceleration.

For scalar fields [2, 4] it has been shown that an inertial maximally entangled state loses entanglement from the perspective of a non-inertial observer as he accelerates. In this case the entanglement vanishes as the acceleration is increased. In the fermionic case the situation is more complex: the entanglement is degraded down to a finite limit [3, 4]. In any case no net entanglement generation has ever been reported to happen due to the mere action of the Unruh effect. As a matter of fact, since the Unruh effect consists on the observation of a thermal distribution when an accelerated observer looks at the field vacuum, many times in relativistic quantum information this effect has been regarded as a source of decoherence comparable to a non-zero temperature environment.

We prove in this letter that there are some states whose degree of entanglement increases as seen from the perspective of a non-inertial observer. The phenomenon is thoroughly studied here for Grassman scalar (a scalar field on which anticommutation relations are imposed) and bosonic scalar fields. We will also put it in context of previous studies on the universality of the behaviour of fermionic quantum correlations in non-inertial frames.

This entanglement amplification is very promising in order to detect quantum effects due to acceleration (and therefore gravity). Entanglement, unlike other phenomena (such as thermal noise) does not admit a classical description. Thus, its observation would account for a pure quantum origin of the aforementioned effects.

On the other hand entanglement is very sensitive to any interaction with the environment, which tends to degrade it. This made it very difficult for any experiment relying on entanglement degradation [2] to find evidence for these effects. By the same token, experiments studying entanglement creation are free from these flaws: If a small amount of entanglement is created, no matter how damped by decoherence it may be, the only possible origin is an acceleration-induced quantum effect. The entanglement amplification phenomenon provides a novel way to distinguish genuine quantum effects of gravity from classically induced ones, something worth considering when trying to detect the Unruh and Hawking effects in analog gravity set-ups [12].

The main reason why this phenomenon has gone unnoticed so far is the reliance in the so-called single mode approximation (SMA) [7] that many previous works assumed ([2–5] and many others). Such approximation consists on assumptions that are not generally true about the form of the change of basis between two Fock bases, one built from monochromatic solutions of the Klein-Gordon or Dirac equation in Minkowski coordinates and other built from monochromatic solutions of these equations in Rindler coordinates. The proper way to work with monochromatic Rindler modes was shown in [8] and more detailedly in [6], work that we will follow through all the letter.

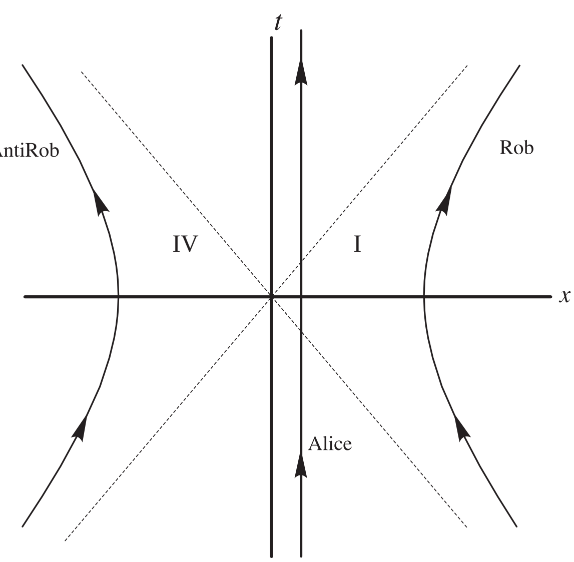

Figure 1: Minkowski spacetime diagram showing the world lines of an inertial observer Alice, and two uniformly accelerated observers moving in the Rindler wedges I and IV which are causally disconnected from each other.

Let us consider a system composed of an inertial observer, Alice, who watches an inertial mode of a quantum field (either a Grassman or bosonic scalar field) and two uniformly accelerated Rindler observers in regions I and IV of Rindler spacetime, called Rob and AntiRob as illustrated in fig.1. Rob (or AntiRob) watches an Unruh mode [1] of the quantum field, which is entangled with Alice's. Unruh modes posess the peculiarity of having a sharp Rindler frequency. In other words, they correspond to monochromatic solutions of the field equation in Rindler coordinates, while an analytic continuation argument shows that they also are purely positive frequency linear combinations of Minkowski modes. This trivially implies that the vacuum state is the same both for Minkowski and Unruh modes. Furthermore there are two kinds of Unruh modes, namely those of left-movers' and 'right movers' [6]. Consideration of both right and left mover modes is necessary for a complete description of an arbitrary solution to the field equation [1, 6].

We will only consider Unruh modes of a given Rindler frequency ω as seen by Rob and AntiRob (who move with proper acceleration a) but which are arbitrary superpositions of left and right mover Unruh modes,

$$C_\omega = q_{\rm L} C_{\omega,\rm L} + q_{\rm R} C_{\omega,\rm R}, \tag{1}$$

where $|q_{\rm L}|^2 + |q_{\rm R}|^2 = 1$, $q_{\rm R} \geq q_{\rm L}$ and $C_{\omega,X}$ for $X = \rm L, R$ are the left and right-mover annihilation operators, whose explicit form can be seen in [6]. The family of states under study is

$$\begin{aligned} |\Psi\rangle = P\,|0\rangle_{\rm A}\left[\alpha\,|1_\omega\rangle + \sqrt{1-\alpha^2}\,|0_\omega\rangle\right] \\ + \sqrt{1-P^2}\,|1\rangle_{\rm A}\left[\beta\,|1_\omega\rangle + \sqrt{1-\beta^2}\,|0_\omega\rangle\right]. \end{aligned} \tag{2}$$

Here, the subscript 'A' refers to Alice's inertial mode, and $|1_\omega\rangle = C_\omega^\dagger\,|0\rangle$ is the Unruh particle excitation. All these states have an implicit dependence on Rob's acceleration a when expressed in the Rindler basis through a parameter r_ω defined by $\tan r_\omega = e^{-\pi c\,\omega/a}$ in the fermionic case, and $\tanh r_\omega = e^{-\pi c\,\omega/a}$ in bosonic case. The difference between the two expressions can be ultimately traced to the presence of anticommutation/commutation relations in the field operators.

As usual [2, 3], we transform the state (2) into the Rindler basis following the same conventions as in [6]. We are interested in quantifying the degree of entanglement between Alice and Rob, and between Alice and AntiRob. To do so, we trace out AntiRob's or Rob's degrees of freedom respectively, and compute the negativities [9] $\mathcal{N}_{AR}$ and $\mathcal{N}_{A\bar{R}}$ of the resulting reduced states. The Alice part of the states does not change with acceleration, so entanglement measures are invariant under relabelling of Alice's orthogonal basis elements.

Most works on this subject also considered the Dirac field, but focused on maximally entangled states [2–6]. Besides, for Dirac fields the study was carried out only for $q_{\rm R} = 1$. In this specific case a remarkable *universality* phenomenon was found [4]: No matter which maximally entangled state is under consideration or if the field is Grassman or Dirac, the negativities of each bipartition as functions of r_ω are exactly the same and follow the simple analytic rules $\mathcal{N}_{AR} = (1/2)\cos^2 r_\omega$, $\mathcal{N}_{A\bar{R}} = (1/2)\sin^2 r_\omega$.

Negativity in the AR bipartition is transferred to $A\bar{R}$ and, quite remarkably, total negativity is conserved. It is natural then to ask if this universality still holds when $q_{\rm R} \neq 1$, however, only a study for the Grassman field has been carried out [6], but since all maximally entangled states of this field are related to each other by a relabelling of Alice's states the question of universality could not be addressed. Nevertheless, due to the fact that for fixed spin z-component the Dirac field has the same algebraic structure as the Grassman field, universality still holds provided that we focus only on those Dirac states with Grassman analog. By this we mean a state which can be formally converted into a Grassman state by the replacement $|\uparrow\rangle \to |1\rangle$, or $|\downarrow\rangle \to |1\rangle$. The states need no longer be maximally entangled to show this form of universality. The main point of this discussion is to increase the range of validity of our results, since any result for the Grassman field can be automatically carried over to the Dirac field. The converse holds as well, as long as we consider only states with Grassman analog.

When one studies entanglement of fermionic fields in non-inertial frames a somewhat surprising result appears: some entanglement survives in the infinite acceleration limit [3] whereas a naive reasoning might conclude that since the Unruh effect acts as a thermal bath with $T \propto a$, all the entanglement should be washed out by an infinite temperature thermal decoherence. This argument is flawed because the Unruh thermal state is derived for the Minkowski vaccuum state, not for states containing excitations. Yet, previous works under the SMA found that entanglement in the AR bipartition was a monotone decreasing function of r_ω. So it still seemed true that acceleration tends to degrade entanglement. When studying the general states (2) these trends would be expected to continue holding in principle.

Instead, for $q_{\rm R} \neq 1$ and a rather simple choice of parameters the surprise appears: there can be a peaked entanglement amplification due to acceleration, as seen in fig. 2. Furthermore, the peak gets sharper as $q_{\rm R}$ approaches the extremal value $q_{\rm R} = 1/\sqrt{2}$, with AR and $A\bar{R}$ behaving the same way in this case since the symmetry between regions I and IV is not explicitly broken (see eq. (1) and ref. [6]). As $q_{\rm R}$ tends to 1 (limit referred in previous works as the SMA), the effect vanishes.

More importantly, it is also possible to obtain high entanglement amplification considering almost separable states. In some of these cases the negativity is a monotone function of r_ω (as for instance in (2) when $P = 0.1$, $\alpha = 0.8695$, $\beta = 0.909$). Therefore, there are states for which the Unruh effect does exactly the opposite of what was expected. That is to say, entanglement is monotonically created rather than being monotonically destroyed.

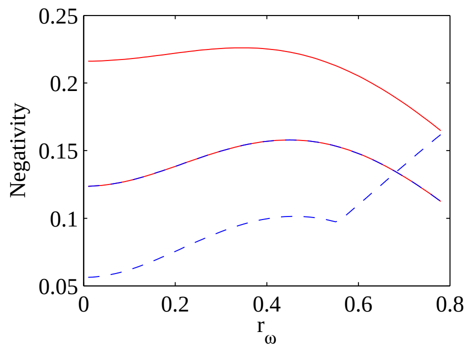

Figure 2: Entanglement creation for the Grassman scalar field. Negativity for AR (red continuous) and $A\bar{R}$ (blue dashed) as a function of r_ω for $q_{\rm R}=0.85$ and $q_{\rm R}=1/\sqrt{2}$ (where both contributions are equal). The state considered is of the general form (2), with $P=0.4$, $\alpha=0$, $\beta=1$.

The phenomenon of entanglement creation also shows up for the bosonic scalar field, much in the same way as it did in the Grassman case. The main difference is that in the bosonic case entanglement is bound to vanish in the infinite acceleration limit, in concordance with previous results [6]. Therefore, entanglement can be created only for a finite range of accelerations, as shown in fig.3. For $q_{\rm R}=1/\sqrt{2}$, Alice-Rob negativity attains a maximum of 0.127 at $r_\omega=0.191$. This is 3.1 % above inertial level. The definition of r_ω in bosonic fields differs from that of fermionic fields in the replacement tan $\to$ tanh, and therefore the allowed range of r_ω is $[0,\infty)$ rather than $[0,\frac{\pi}{4})$. Considering frequencies of order ≈ 1 GHz, which correspond to reasonable experimental possibilities [10] the acceleration corresponding to this value of r_ω is $a\approx 10^{17}g$, much closer to experimental feasibility than previous proposals [11] which suggested accelerations of $\approx 10^{26}g$. The appearance of entanglement creation in the bosonic case shows that this is a truly universal phenomenon, independent of field statistics.

In order to study the experimental implications of this phenomenon, let us introduce a specific scenario. The state (2) for fixed P and β over certain threshold shows an unbounded relative increase of negativity as one approaches the limit $\alpha\to\beta$ (separable limit). This means that there are states for which an arbitrary small acceleration produces an arbitrary large relative increase in negativity. The same happens as we approach a separable state taking $P\to 0$ for certain values of α and β. This behaviour is quite general and appears for both fermionic and bosonic fields. However the more relative entanglement increase (signal-to-background ratio) we want to achieve, the more separable the inertial states should be. Dealing with quasi-separable states would be the experimental challenge to detect the Unruh effect by means of these techniques.

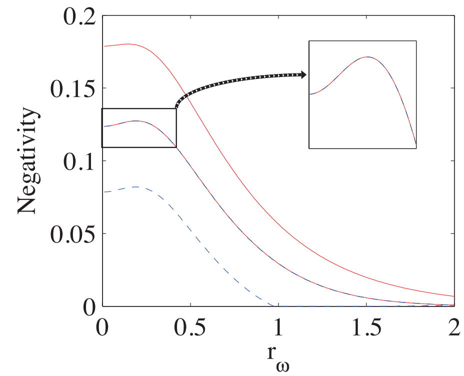

Figure 3: Entanglement creation for the bosonic scalar field. Negativity for AR (red continuous) and $A\bar{R}$ (blue dashed) as a function of r_ω for $q_{\rm R}=0.85$ and $q_{\rm R}=1/\sqrt{2}$ (where both contributions are equal). The state considered is of the general form (2), with $P=0.4$, $\alpha=0$, $\beta=1$.

Any such experiment would be naturally interested in negativity behaviour in the vicinity of $r_\omega=0$, easier to obtain in laboratory conditions. This means that in order to maximise experimental feasibility we are interested in states whose negativity shows a quick growth for small r_ω. We study the relative increase of AR negativity with respect to its inertial value for the family of states (2) at fixed r_ω. As an example we choose $r_\omega=0.15$ which corresponds to accelerations from $a\approx 5\cdot 10^{13}g$ to $5\cdot 10^{16}g$ for frequencies from 1 MHz to 1 GHz. This unbounded entanglement creation can be seen in fig. 4.

We obtain huge 'signal-to-background' ratios and the better the negativity can be experimentally determined, the bigger this ratio can become. If we relied on entanglement degradation to detect the Unruh effect [2], the percental change in negativity would be bounded by 100 %. With the plethora of new states presented in this work, this relative change can be made arbitrarily high.

The same analysis carried out for Unruh modes can be repeated if we consider that the excitations $|1_\omega\rangle$ are Gaussian wavepackets from the inertial perspective. As detailedly shown in section IV of [6], Gaussian wavepackets of Minkowski modes transform into Gaussian wavepackets of Rindler modes. We can consider at the same time peaked wavepackets in the Minkowskian basis and in the Rindler basis such that the analysis would be completely analogous to the monochromatic case. In this case different choices of $q_{\rm R}$ and $q_{\rm L}$ represent different spatial localisation of the Gaussian wavepackets.

All these results can be readily exported to a setting consisting in an observer hovering at certain distance close to the event horizon of an Schwarzschild black hole, following [8], and therefore the same conclusions drawn for the Unruh effect are also valid for the Hawking effect.

We have shown that the Unruh effect can not only de-

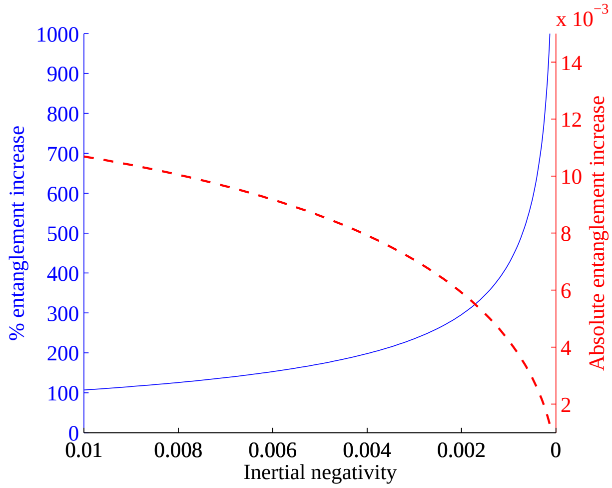

Figure 4: Relative (blue continuous) and absolute (red dashed) entanglement creation as a function of the inertial negativity for a Grassman field state of the family (2) with $P=0.4$, $\beta=0.8$ and different values of α for $q_{\text{R}}=1/\sqrt{2}$ and $r_\omega=0.15$ (accelerations from $a\approx 5\cdot 10^{13}g$ to $5\cdot 10^{16}g$ for frequencies from 1 MHz to 1 GHz). Notice an unbounded growth of the 'signal-to-background' ratio.

grade quantum entanglement but also amplificate it, banishing previous fundamental misconceptions such as the belief that the Unruh and Hawking effects are sources of entanglement degradation. We have demonstrated that there are families of states whose entanglement can be increased by an arbitrarily high relative factor.

Furthermore, with these results we have moved the experimental difficulties from generating and sustaining high accelerations to the problem of being able to prepare and measure quasi-separable entangled states. Measuring increases of entanglement as the ones showed in fig. 4 can be attained by means of entanglement distillation protocols [13], hence, we are presenting a new way of detecting the Unruh effect with experimental techniques already accessible. As a matter of fact, our results are independent of the specific implementation to detect the entanglement magnification, hence they can be exported to a huge variety of experimental set-ups as for instance analog gravity settings.

As a final remark, one could ask why entanglement seems to be created if, after all, to change to the accelerated observer basis we are forced to trace out part of the density matrix. The answer seems evident when we consider, for instance, the Minkowski vacuum which is itself separable. When we move to the Rindler basis it transforms into a two mode squeezed state between region I and IV modes, which is more and more entangled as the acceleration is increased [2]. It is when we trace out either region I or IV that a thermal state emerges. Therefore, observing the field from the perspective of an accelerated observer implies two processes: 1) a generation of entanglement due to the Bogoliubov relationships implied in the change of basis and 2) an erasure of correlations due to the tracing over one of the Rindler regions. Going beyond the single mode approximation the correlations created due to the change of basis are not completely erased when we trace out one of the Rindler regions. These competing trends explain why for a certain acceleration the amplification of entanglement is maximal.

Although only scalar and Grassman scalar fields have been analysed here, a qualitatively similar phenomenon also happens in higher spins such as Dirac and electromagnetic fields whose study constitutes itself an interesting piece of work that will be reported elsewhere.

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