

November 8, 2010
V2. December 15, 2010

Superfield equations for the interacting system of $D = 4$ $\mathcal{N} = 1$ supermembrane and scalar multiplet

Igor A. Bandos ^{†*} and Carlos Meliveo ^{*}

[†] *IKERBASQUE, the Basque Foundation for Science, Bilbao, Spain*

^{*} *Department of Theoretical Physics, University of the Basque Country,
P.O. Box 644, 48080 Bilbao, Spain*

We present the superfield action for the dynamical $\mathcal{N} = 1$ $D = 4$ supermembrane in interaction with a dynamical scalar multiplet and use it to derive the superfield equations of motion. These include the supermembrane equations, which formally coincide with equations of supermembrane in a background of the (off-shell) scalar multiplet, and the special chiral superfield equations with supermembrane source. In the case when the scalar supermultiplet part of the action contains only the simplest kinetic term we have also extracted the spacetime component field equations from the superfield equations and solve these in the leading order on supermembrane tension.

1 Introduction

The development of String and M-theory demonstrated the important rôle of supersymmetric extended objects, including eleven dimensional supermembrane [2], called now M2-brane. The consistency of this model in curved 11D superspace requires the torsion and curvature of that to obey the supergravity constraints. These, in its turn, result in the supergravity equations of motion so that the supergravity dynamics is in a way governed by the M2-brane.

Soon after the pioneer papers [2] the simplest nontrivial counterpart of M2-brane, the $D = 4$, $\mathcal{N} = 1$ supermembrane, was studied in [3]. Its consistency in curved superspace also requires a set of superspace constraints. However, in distinction to the 11D case, these $D = 4$, $\mathcal{N} = 1$ constraints are *off-shell* in the sense that they do not produce equations of motion as their consequences. This implies that it is possible to construct the manifestly supersymmetric superfield Lagrangian description of the interacting system of $D = 4$, $\mathcal{N} = 1$ supergravity and supermembrane. Curiously enough, such a system has not been constructed yet. We intend to turn to this problem in the future paper. Here, as a first stage in this direction, we will construct the superfield Lagrangian description and obtain the superfield equations of motion for a simpler interacting system of supermembrane and scalar supermultiplet.

The superfield Lagrangian approach to $D = 4$, $\mathcal{N} = 1$ supergravity—superparticle dynamical system was developed in [6]. The superfield equations for the dynamical system of supergravity, superstring and tensorial supermultiplet were obtained in [7]. Both models were used to study the origin and properties of the complete but gauge fixed Lagrangian description of the supergravity–super- p -brane system by the sum of the spacetime, component action for supergravity (without auxiliary fields) and the purely bosonic limit of the super- p -brane action. These description proposed and developed in [8, 6, 7, 9] can be used also for higher dimensional supergravity plus brane(s) interacting systems. However, the study of lower dimensional superfield equations for the interacting system of supergravity and superbrane, when it is possible, is also of interest as it might provide new insights in the properties of more complicated M-theoretical systems, and also on its own, as a sector of possible phenomenologically interesting models constructed on the basis of four dimensional supergravity.

The system of superfield equation for the interacting system of supergravity, superstring and tensor supermultiplet, which was obtained in [7], happened to be too complicated to be practical. Probably, to obtain a less complex set of superfield equations one have to use the superfield formulation of the so-called new minimal supergravity [10, 11] instead of the 'old' minimal formulation [12, 13] used in [7]. This hope is related to the fact that the new minimal formulation includes an auxiliary antisymmetric tensor which has natural coupling to string model so that one would not need to introduce, as in [7], tensorial multiplet in addition to supergravity one.

On the other hand, one can use the results of [7] to extract the superfield equations for the superstring interacting with tensorial multiplet in flat $D = 4$, $\mathcal{N} = 1$ superspace. The existence of such a nontrivial interaction is related to the fact that, according to [14], tensorial multiplet can be used to construct a supersymmetric closed 3-form in the flat $D = 4$, $\mathcal{N} = 1$ superspace. It is natural to begin with the study of these tensorial multiplet–superstring interacting equations before passing to the superstring interacting with supergravity.

The system of interacting superfield equations could be expected to be even simpler if we were writing it for a supersymmetric extended object interacting with a scalar supermultiplet. This is not possible for superstring but is possible for supermembrane, because the three form field strength cannot be constructed from the scalar multiplet, but the four form field strength can. Thus, as we have already mentioned, the aim of our present study is to construct the Lagrangian description and to obtain superfield equations of motion for this $D = 4$, $\mathcal{N} = 1$ interacting system of supermembrane and scalar multiplet.

This paper is organized as follows. In section 2 we introduce our basic superspace notation and review the generic scalar multiplet action, superfield equations of motion and dual description by a three form potential. To be more precise about the last item, in sec. 2.3 we present the closed four form in $D = 4$, $\mathcal{N} = 1$ superspace which is constructed from generic scalar supermultiplet. The explicit form of the three form potential is given later, in Sec. 4.1, and only for the case of special scalar supermultiplet. In Sec. 3 we present the $D = 4$, $\mathcal{N} = 1$ supermembrane action and obtain the supermembrane equations of motion in the background of (an off-shell) scalar multiplet. In Sec. 4 we derive superfield equations of motion for the special scalar supermultiplet interacting with supermembrane. The supermembrane current superfield determining the *r.h.s.*'s of these equations is presented in Sec. 4.3. Sec. 5 is devoted to extracting the spacetime component field equations from the superfield equations with supermembrane current. For the sake of simplicity the discussion in Sec.5 is restricted to the case where the field theoretical part of the action is given by a free kinetic term only. In sec. 5.3 we present a solution of these dynamical field equations at the leading order in supermembrane tension. In Sec. 6 we present our conclusions and also compare the coupling of the membrane to background scalar field which follows from our superfield supermembrane action with one studied in [15]. Appendix A collects some useful technical details. The complete expressions for the components of the supermembrane current superfields are collected in Appendix B.

2 Scalar multiplet description by chiral superfield.

2.1 Basic notation.

We denote the supervielbein of $\mathcal{N} = 1$, $D=4$ superspace $\Sigma^{(4|4)}$ by

$$E^A := dz^M E_M^A(z) = (E^a, E^\alpha, \bar{E}^{\dot{\alpha}}), \quad \begin{cases} a = 0, 1, 2, 3, \\ \alpha = 1, 2, \\ \dot{\alpha} = 1, 2. \end{cases} \quad (2.1)$$

When superspace is flat, these obeys the constraints

$$dE^a = -2iE \wedge \sigma^a \bar{E}, \quad dE^\alpha = 0, \quad d\bar{E}^{\dot{\alpha}} = 0, \quad (2.2)$$

where $\sigma_{\alpha\dot{\alpha}}^a$ are relativistic Pauli matrices, d is exterior derivative of differential forms ($dd = 0$, $dE^A = d(dZ^M E_M^A(Z)) = dZ^M \wedge dE_M^A(Z) = dZ^M \wedge dZ^N \partial_N E_M^A(Z) = dZ^M \wedge dZ^N \frac{\partial E_M^A(Z)}{\partial Z^N}$ etc.) and \wedge denotes the exterior product of the differential form. This is antisymmetric for bosonic one forms, $E^a \wedge E^b = -E^b \wedge E^a$, symmetric for two fermionic one forms,

$E^\alpha \wedge E^\beta = E^\beta \wedge E^\alpha$, and again antisymmetric for the product of bosonic and fermionic one forms, $E^a \wedge E^b = -E^b \wedge E^a$. The constraints (2.2) can be solved by

$$E^a = dx^a - id\theta^\alpha \sigma_{\alpha\dot{\alpha}}^a \bar{\theta}^{\dot{\alpha}} + i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^a d\bar{\theta}^{\dot{\alpha}}, \quad E^\alpha = d\theta^\alpha, \quad \bar{E}^{\dot{\alpha}} = d\bar{\theta}^{\dot{\alpha}} \quad (2.3)$$

expressing the supervielbein in terms of superspace coordinates

$$z^M = (x^a, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}), \quad \alpha = 1, 2, \quad \dot{\alpha} = 1, 2, \quad a = 0, 1, 2, 3. \quad (2.4)$$

Their set includes the commuting bosonic coordinates x^a ($x^a x^b = x^b x^a$) and anticommuting fermionic coordinates θ^α and $\bar{\theta}^{\dot{\alpha}} = (\theta^\alpha)^*$ ($\theta^\alpha \theta^\beta = -\theta^\beta \theta^\alpha$, $\theta^\alpha x^a = x^a \theta^\alpha$ etc.).

Decomposing the exterior derivative on the supervielbein basis

$$d = E^\alpha D_\alpha + \bar{E}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} + E^a D_a, \quad (2.5)$$

we obtain the expressions for supersymmetric covariant derivatives,

$$D_a = \partial_a, \quad D_\alpha = \partial_\alpha + i(\sigma^a \bar{\theta})_\alpha \partial_a, \quad \bar{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} + i(\theta \sigma^a)_{\dot{\alpha}} \partial_a = -(D_\alpha)^*. \quad (2.6)$$

These obey the superalgebra with the only one nontrivial (anti-)commutation relation

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2i\sigma_{\alpha\dot{\alpha}}^a \partial_a. \quad (2.7)$$

The possibility to construct the $D = 4$, $\mathcal{N} = 1$ supermembrane action is related to that in $D = 4$, $\mathcal{N} = 1$ superspace there exists the following supersymmetric invariant closed 4-form¹

$$h_4 = dc_3 := -\frac{i}{4} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\alpha\beta} + \frac{i}{4} E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \wedge \bar{E}^{\dot{\beta}} \tilde{\sigma}_{ab\dot{\alpha}\dot{\beta}}. \quad (2.8)$$

This describes a 3-cocycle which is nontrivial in Chevalley-Eilenberg (CE) cohomology [4], which implies that h_4 is a supersymmetric invariant closed four form, $dh_4 = 0$ and, despite it can be expressed as an exterior derivative of a 3-form, $h_4 = dc_3$ (and, hence, is trivial cocycle of de Rahm cohomology), the corresponding 3-form c_3 is not invariant under supersymmetry.

2.2 Scalar supermultiplet as described by chiral superfield

In this section we review the well known description of scalar supermultiplet by chiral superfield in superspace [18, 19, 20].

The simplest irreducible representation of the $D = 4$, $\mathcal{N} = 1$ supersymmetry, the scalar supermultiplet, is described by the chiral superfield, this is to say by complex superfield obeying the so-called chirality equation

$$\bar{D}_{\dot{\alpha}} \Phi = 0. \quad (2.9)$$

The complex conjugate (*c.c.*), $\bar{\Phi} = (\Phi)^*$, obeys

$$D_\alpha \bar{\Phi} = 0 \quad (2.10)$$

¹Notice that in our notation the exterior derivative acts from the right, so that for any p -form Ω_p and q -form Ω_q , $d(\Omega_p \wedge \Omega_q) = \Omega_p \wedge d\Omega_q + (-)^q d\Omega_p \wedge \Omega_q$.

and is called anti-chiral superfield. The free equations of motion for the physical fields of a massless scalar supermultiplet ($\phi(x) = \Phi|_{\theta=0}$ and $i\psi_\alpha(x) = D_\alpha\Phi|_{\theta=0}$) are collected in the superfield equation

$$DD\Phi := D^\alpha D_\alpha \Phi = 0 . \quad (2.11)$$

These equations can be derived from the action

$$S_{kin} = \int d^8z \Phi \bar{\Phi} , \quad (2.12)$$

where the superspace integration measure $d^8z = d^4x d^2\theta d^2\bar{\theta}$ is normalized as²

$$d^8z = d^4x \bar{D}\bar{D} DD := d^4x \bar{D}_\alpha \bar{D}^{\dot{\alpha}} D^\alpha D_\alpha . \quad (2.13)$$

Indeed, the variation of this functional reads $\delta S_{kin} = \int d^8z (\Phi \delta \bar{\Phi} + \delta \Phi \bar{\Phi})$. As far as the variation of chiral superfield should be chiral, $\bar{D}_\alpha \delta \Phi = 0$, and $D_\alpha \delta \bar{\Phi} = 0$, we can equivalently write the action variation as $\delta S_{kin} = \int d^4x \bar{D}_\alpha \bar{D}^{\dot{\alpha}} ((DD\Phi)\delta\bar{\Phi}) + c.c.$, which results in the equations of motion (2.11).

The most general selfinteraction of the scalar supermultiplet is described by the superfield action

$$\begin{aligned} S_{s-int}[\Phi; \bar{\Phi}] &= \int d^8z \mathcal{K}(\Phi, \bar{\Phi}) + \int d^6\zeta_L W(\Phi) + \int d^6\zeta_R \bar{W}(\bar{\Phi}) = \\ &= \int d^4x \bar{D}\bar{D} DD \mathcal{K}(\Phi, \bar{\Phi}) + \int d^4x DD W(\Phi) + \int d^4x \bar{D}\bar{D} \bar{W}(\bar{\Phi}) , \end{aligned} \quad (2.14)$$

where $\mathcal{K}(\Phi, \bar{\Phi})$ is an arbitrary function of chiral superfield and its complex conjugate called *Kähler potential* and $W(\Phi)$ ($= \bar{W}(\bar{\Phi})^*$) is an arbitrary holomorphic function of the complex scalar superfield Φ called *superpotential*. This latter is chiral, $\bar{D}_\alpha W(\Phi) = W'(\Phi)\bar{D}_\alpha\Phi = 0$, and hence is integrated with chiral measure defined by $d^6\zeta_L = d^4x DD$ (and $d^6\zeta_R = d^4x \bar{D}\bar{D}$). To have the standard kinetic term for the scalar field of the supermultiplet, the Kähler potential is usually chosen to obey

$$\mathcal{K}''_{\varphi\bar{\varphi}}(\varphi, \bar{\varphi}) := \frac{\partial}{\partial\varphi} \frac{\partial}{\partial\bar{\varphi}} \mathcal{K}(\varphi, \bar{\varphi}) \neq 0 . \quad (2.15)$$

The superfield equations of motion following from the action $S_{s-int}[\Phi; \bar{\Phi}]$ (2.14) are

$$\begin{aligned} \bar{\mathcal{E}} &:= DD\mathcal{K}'_{\bar{\Phi}} + \bar{W}'_{\bar{\Phi}} = \\ &= DD\Phi \mathcal{K}''_{\Phi\bar{\Phi}}(\Phi, \bar{\Phi}) + D^\alpha\Phi D_\alpha\Phi \mathcal{K}'''_{\Phi\Phi\bar{\Phi}}(\Phi, \bar{\Phi}) + \bar{W}'_{\bar{\Phi}}(\bar{\Phi}) = 0 , \end{aligned} \quad (2.16)$$

$$\begin{aligned} \mathcal{E} &:= \bar{D}\bar{D}\mathcal{K}'_{\Phi} + W'_{\Phi} = \\ &= \bar{D}\bar{D}\bar{\Phi} \mathcal{K}''_{\Phi\bar{\Phi}}(\Phi, \bar{\Phi}) + \bar{D}_\alpha\bar{\Phi} \bar{D}^{\dot{\alpha}}\bar{\Phi} \mathcal{K}'''_{\Phi\bar{\Phi}\bar{\Phi}}(\Phi, \bar{\Phi}) + W'_{\Phi}(\Phi) = 0 , \end{aligned} \quad (2.17)$$

where prime denotes the derivative with respect to argument, $\mathcal{K}'_{\bar{\Phi}} := \frac{\partial\mathcal{K}(\Phi, \bar{\Phi})}{\partial\bar{\Phi}}$, $\mathcal{K}''_{\Phi\bar{\Phi}} := \frac{\partial^2}{\partial\Phi\partial\bar{\Phi}}\mathcal{K}(\Phi, \bar{\Phi})$, etc. These equations can be obtained by solving the chirality conditions (2.9) and (2.10) in terms of prepotential, generic complex superfield P ($= (\bar{P})^*$),

$$\Phi = \bar{D}\bar{D}P , \quad \bar{\Phi} = DD\bar{P} , \quad (2.18)$$

²Notice that, although the *r.h.s.* of this equation is not manifestly hermitian, its imaginary part is integral of complete derivative (as far as $\bar{D}\bar{D}DD = DD\bar{D}\bar{D} - 4i\bar{\sigma}^a{}^{\dot{\alpha}\alpha}\partial_a[D_\alpha, \bar{D}_{\dot{\alpha}}]$) and, as such, can be ignored in our discussion.

and vary with respect to this prepotential and its complex conjugate,

$$\mathcal{E} = \frac{\delta S_{s-int}[\Phi; \bar{\Phi}]}{\delta P}, \quad \bar{\mathcal{E}} = \frac{\delta S_{s-int}[\Phi; \bar{\Phi}]}{\delta \bar{P}}. \quad (2.19)$$

2.3 Four form field strength constructed from the scalar supermultiplet

Having a chiral superfield K ,

$$\bar{D}_{\dot{\alpha}} K = 0, \quad D_{\alpha} \bar{K} = 0, \quad (2.20)$$

one can construct the following supersymmetric invariant closed four form (CE cocycle) in flat $D = 4$, $\mathcal{N} = 1$ superspace [14]

$$\begin{aligned} F_4 = dC_3 := & \frac{1}{4} E^b \wedge E^a \wedge E^{\alpha} \wedge E^{\beta} \sigma_{ab\alpha\beta} \bar{K} + \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \wedge \bar{E}^{\dot{\beta}} \tilde{\sigma}_{ab\dot{\alpha}\dot{\beta}} K + \\ & + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge E^{\alpha} \epsilon_{abcd} \sigma_{\alpha\dot{\beta}}^d \bar{D}^{\dot{\beta}} \bar{K} + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge \bar{E}^{\dot{\beta}} \epsilon_{abcd} \sigma_{\alpha\dot{\beta}}^d D^{\alpha} K - \\ & + \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a \frac{i}{4} \epsilon_{abcd} (\bar{D} \bar{D} \bar{K} - D D K). \end{aligned} \quad (2.21)$$

Notice that we intentionally have not used the notation Φ for chiral superfield to stress that, *e.g.* having a free chiral superfield of Eqs. (2.9) satisfying equations of motion (2.11), one can construct the three form using some holomorphic functions $K = K(\Phi)$, $\bar{K} = \bar{K}(\bar{\Phi})$ which obey $DDK = K''(\Phi) D^{\alpha} \Phi D_{\alpha} \Phi$ instead of (2.11).

Interestingly enough, F_4 in (2.21) can be considered as real part of the complex closed form \mathcal{F}_4^L ,

$$F_4 = \Re e(\mathcal{F}_4^L) := \frac{1}{2} (\mathcal{F}_4^L + \mathcal{F}_4^R), \quad (2.22)$$

$$\begin{aligned} \mathcal{F}_4^L = & \frac{1}{4} E^b \wedge E^a \wedge E^{\alpha} \wedge E^{\beta} \sigma_{ab\alpha\beta} \bar{K} + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge E^{\alpha} \epsilon_{abcd} \sigma_{\alpha\dot{\beta}}^d \bar{D}^{\dot{\beta}} \bar{K} + \\ & + \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a \frac{i}{16} \epsilon_{abcd} \bar{D} \bar{D} \bar{K}, \quad D_{\alpha} \bar{K} = 0, \end{aligned} \quad (2.23)$$

$$\begin{aligned} \mathcal{F}_4^R = & \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \wedge \bar{E}^{\dot{\beta}} \tilde{\sigma}_{ab\dot{\alpha}\dot{\beta}} K + \frac{1}{4!} E^c \wedge E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \epsilon_{abcd} \sigma_{\beta\dot{\alpha}}^d D^{\beta} K + \\ & + \frac{1}{4!} E^d \wedge E^c \wedge E^b \wedge E^a \frac{i}{16} \epsilon_{abcd} D D K, \quad \bar{D}_{\dot{\alpha}} K = 0. \end{aligned} \quad (2.24)$$

The fact that these forms are closed as a consequences of (2.20),

$$d\mathcal{F}_4^L = 0 \quad \Leftrightarrow \quad D_A \bar{K} = 0, \quad (2.25)$$

$$d\mathcal{F}_4^R = 0 \quad \Leftrightarrow \quad \bar{D}_{\dot{A}} K = 0, \quad (2.26)$$

suggests the existence of the complex 3-form potentials C_3^L and $C_3^R = (C_3^L)^*$ such that $\mathcal{F}_4^L = dC_3^L$ and $\mathcal{F}_4^R = dC_3^R$.

To study a supermembrane in the background of scalar multiplet, which will be the subject of the next section, we do not need an explicit expression for C_3 . However, we do need it to obtain the equations for the scalar multiplet fields with a source from supermembrane, so that we will come back to discussing the problem of constructing potentials in Sec. 4.

3 $D = 4, \mathcal{N} = 1$ supermembrane and its interaction with scalar multiplet background

3.1 Free supermembrane in flat $D = 4 \mathcal{N} = 1$ superspace

The action for a free supermembrane in $D = 4, \mathcal{N} = 1$ superspace reads [3]

$$\begin{aligned} S_{p=2} &= \frac{1}{2} \int d^3\xi \sqrt{g} - \int_{W^3} \hat{c}_3 = \\ &= -\frac{1}{6} \int_{W^3} * \hat{E}_a \wedge \hat{E}^a - \int_{W^3} \hat{c}_3, \end{aligned} \quad (3.1)$$

where, in the first line $g = \det(g_{mn})$ is the determinant of the induced metric,

$$g_{mn} = \hat{E}_m^a \eta_{ab} \hat{E}_n^b, \quad \hat{E}_m^a := \partial_m \hat{x}^a - i \partial_m \hat{\theta}^\alpha \sigma_{\alpha\dot{\alpha}}^a \hat{\theta}^{\dot{\alpha}} + i \hat{\theta}^\alpha \sigma_{\alpha\dot{\alpha}}^a \partial_m \hat{\theta}^{\dot{\alpha}}, \quad (3.2)$$

W^3 is the supermembrane worldvolume the embedding of which into the target superspace $\Sigma^{(4|4)}$ is defined parametrically by the coordinate functions $\hat{z}^M(\xi) = (\hat{x}^a(\xi), \hat{\theta}^\alpha(\xi), \hat{\theta}^{\dot{\alpha}}(\xi))$; $\xi^m = (\xi^0, \xi^1, \xi^2)$ are local coordinates on W^3 ,

$$W^3 \subset \Sigma^{(4|4)} : \quad z^M = \hat{z}^M(\xi) = (\hat{x}^a(\xi), \hat{\theta}^\alpha(\xi), \hat{\theta}^{\dot{\alpha}}(\xi)). \quad (3.3)$$

Finally,

$$\begin{aligned} \hat{c}_3 &:= \frac{1}{3!} \hat{E}^{a_3} \wedge \hat{E}^{a_2} \wedge \hat{E}^{a_1} c_{a_1 a_2 a_3}(\hat{Z}) = \frac{1}{3!} d\xi^{m_3} \wedge d\xi^{m_2} \wedge d\xi^{m_1} \hat{c}_{m_1 m_2 m_3} = \\ &= -\frac{1}{6} d^3\xi \epsilon^{m_1 m_2 m_3} \hat{c}_{m_1 m_2 m_3} \end{aligned} \quad (3.4)$$

is the pull-back of the 3-form defined in Eq. (2.8) to W^3 , so that the second, Wess–Zumino part of the action can be written in the form of (see [3]) $\int_{W^3} \hat{c}_3 = -\frac{1}{6} \int d^3\xi \epsilon^{m_1 m_2 m_3} \hat{c}_{m_1 m_2 m_3}$.

In this paper we consider the case of closed supermembrane so that the worldvolume W^3 has no boundary, $\partial W^3 = \emptyset$, and $\int_{W^3} d(\dots) = 0$. Then we do not need in the explicit form of $\hat{c}_{m_1 m_2 m_3}$ in (3.4) as far as variation of its integral in (3.1) can be calculated (using the Lie derivative formula, $\delta c_3 = i_\delta c_3 + d i_\delta c_3$) through its exterior derivative, the pull-back $\hat{h}_4 := h_4(\hat{Z})$ of the CE cocycle (2.8), $h_4 = dc_3$.

In the second line of Eq. (3.1) we have written the first, Nambu–Goto term of the action as an integral of a differential three form. This is constructed from the pull-back of the bosonic vielbein form

$$\hat{E}^a = d\xi^m \hat{E}_m^a, \quad \hat{E}_m^a := \partial_m \hat{x}^a - i \partial_m \hat{\theta}^\alpha \sigma_{\alpha\dot{\alpha}}^a \hat{\theta}^{\dot{\alpha}} + i \hat{\theta}^\alpha \sigma_{\alpha\dot{\alpha}}^a \partial_m \hat{\theta}^{\dot{\alpha}}, \quad (3.5)$$

using the worldvolume Hodge star operation,

$$* \hat{E}^a := \frac{1}{2} d\xi^m \wedge d\xi^n \sqrt{g} \epsilon_{mnk} g^{kl} \hat{E}_l^a. \quad (3.6)$$

The action (3.1) is invariant under the local fermionic κ -symmetry transformations. These have the form of

$$\delta_\kappa x^\mu = i \kappa^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} - i \theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\kappa}^{\dot{\alpha}}, \quad \delta_\kappa \theta^\alpha = \kappa^\alpha, \quad \delta_\kappa \bar{\theta}^{\dot{\alpha}} = \bar{\kappa}^{\dot{\alpha}}, \quad (3.7)$$

where the spinorial fermionic parameter $\kappa^\alpha = \kappa^\alpha(\xi) = (\bar{\kappa}^{\dot{\alpha}})^*$ has actually only two independent components because it obeys the equations

$$\bar{\kappa}_{\dot{\alpha}} = \kappa^\beta \bar{\gamma}_{\beta\dot{\alpha}} \quad \Leftrightarrow \quad \kappa^\alpha = \bar{\kappa}_{\dot{\alpha}} \tilde{\gamma}^{\dot{\beta}\alpha} \quad (3.8)$$

with

$$\bar{\gamma}_{\beta\dot{\alpha}} = \epsilon_{\beta\alpha} \epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\gamma}^{\dot{\beta}\alpha} = \frac{i}{3! \sqrt{g}} \sigma_{\beta\dot{\alpha}}^a \epsilon_{abcd} \epsilon^{mnk} \hat{E}_m^b \hat{E}_n^c \hat{E}_k^d . \quad (3.9)$$

By construction, the matrix $\bar{\gamma}$ obeys

$$\bar{\gamma}_{\beta\dot{\beta}} \tilde{\gamma}^{\dot{\beta}\alpha} = \delta_\beta^\alpha \quad (3.10)$$

which makes two equations in (3.8) equivalent.

To prove the κ -symmetry one have to use the identities

$$\frac{1}{2} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \sigma_{bc\alpha\beta} = * \hat{E}_a \wedge \hat{E}^\alpha (\sigma^a \tilde{\gamma})_{\alpha\beta} \quad (3.11)$$

which allows to present the variation of the kinetic, Nambu-Goto type, and the Wess-Zumino terms in similar form.

It is convenient to write the κ -symmetry transformations in the form of

$$i_\kappa \hat{E}^a := \delta_\kappa \hat{Z}^M E_M^a(\hat{Z}) = 0, \quad \begin{cases} i_\kappa \hat{E}^\alpha := \delta_\kappa \hat{Z}^M E_M^\alpha(\hat{Z}) = \kappa^\alpha = \bar{\kappa}_{\dot{\alpha}} \tilde{\gamma}^{\dot{\beta}\alpha}, \\ i_\kappa \hat{E}^{\dot{\alpha}} := \delta_\kappa \hat{Z}^M E_M^{\dot{\alpha}}(\hat{Z}) = \bar{\kappa}_{\dot{\alpha}} = \kappa^\beta \bar{\gamma}_{\beta\dot{\alpha}}. \end{cases} \quad (3.12)$$

3.2 Supermembrane action in the scalar multiplet background

The action of supermembrane in the background of a scalar multiplet can be written in the form

$$\begin{aligned} S_{p=2} &= \frac{1}{2} \int d^3\xi \sqrt{K \bar{K}} \sqrt{g} - \int_{W^3} \hat{C}_3, \\ &= -\frac{1}{6} \int_{W^3} * \hat{E}_a \wedge \hat{E}^a \sqrt{K \bar{K}} - \int_{W^3} \hat{C}_3, \end{aligned} \quad (3.13)$$

where \hat{C}_3 is the pull-back of the C_3 potential defined by Eq. (2.21) involving the chiral superfields K and \bar{K} (2.20). For simplify we omit the hat symbol form the pull-backs of superfields here and below in the places where this cannot produce a confusion.

The action (3.13) is invariant under the κ -symmetry transformations

$$i_\kappa \hat{E}^a = 0, \quad i_\kappa \hat{E}^\alpha = \kappa^\alpha, \quad i_\kappa \hat{E}^{\dot{\alpha}} = \bar{\kappa}_{\dot{\alpha}}, \quad (3.14)$$

with $i_\kappa d\hat{Z}^M := \delta_\kappa \hat{Z}^M$, similar to ones in (3.12) and (3.7) but with the spinorial parameter obeying the reducibility conditions

$$\bar{\kappa}_{\dot{\alpha}} = -i \kappa^\beta \bar{\gamma}_{\beta\dot{\alpha}} \sqrt{\bar{K}/K} \quad \Leftrightarrow \quad \kappa^\alpha = i \bar{\kappa}_{\dot{\alpha}} \tilde{\gamma}^{\dot{\beta}\alpha} \sqrt{K/\bar{K}} \quad (3.15)$$

defined by a projector which differs from the one in (3.8) by a (super)field dependent phase factor $i\sqrt{K/\bar{K}}$.

Notice that, if we write the counterpart of the action (3.13) with an arbitrary function $\mathcal{S}(K, \bar{K})$ instead of $\sqrt{K\bar{K}}$ and perform the fermionic variation (3.14) of such an action, we find that the local fermionic κ -symmetry parameter should obey the equations $\kappa^\alpha \partial \mathcal{S} / \partial K = i/2 \bar{\kappa}_{\dot{\beta}} \tilde{\gamma}_{\dot{\beta}\alpha}$ and $\bar{\kappa}_{\dot{\beta}} \partial \mathcal{S} / \partial \bar{K} = \kappa^\alpha \gamma_{\alpha\dot{\beta}}$. This system of equations has a nontrivial solution when $\partial \mathcal{S} / \partial \bar{K} = \frac{1}{4\partial \mathcal{S} / \partial K}$. This latter equation is solved by $\mathcal{S}(K, \bar{K}) = \sqrt{K\bar{K}}$ so that the action (3.13) for scalar multiplet in supergravity background can be constructed from the requirement of the κ -symmetry.

3.3 Equations of motion for supermembrane in a background of an off-shell scalar supermultiplet

The supermembrane equations of motion can be obtained by varying the action (3.13) with respect to coordinate functions $\hat{Z}^M(\xi)$, so that we can write them in the form of $\frac{\delta S_{p=2}}{\delta \hat{Z}^M(\xi)} = 0$. The convenient form of the bosonic and fermionic equations can be extracted by multiplying this on the inverse supervielbein, $E_\alpha^M(\hat{Z}) \frac{\delta S_{p=2}}{\delta \hat{Z}^M(\xi)} = 0$ and $E_a^M(\hat{Z}) \frac{\delta S_{p=2}}{\delta \hat{Z}^M(\xi)} = 0$. These combinations appear as the coefficients for $i_\delta \hat{E}^\alpha := \delta \hat{Z}^M(\xi) E_M^\alpha(\hat{Z})$, $i_\delta \hat{E}^{\dot{\alpha}} := \delta \hat{Z}^M(\xi) \bar{E}_M^{\dot{\alpha}}(\hat{Z})$ and $i_\delta \hat{E}^a := \delta \hat{Z}^M(\xi) E_M^a(\hat{Z})$ in the integrand of the action variation. This implies the possibility to write the formal expression for supermembrane equations of motion in the form

$$\delta S_{p=2} / i_\delta \hat{E}^\alpha := E_\alpha^M(\hat{Z}) \frac{\delta S_{p=2}}{\delta \hat{Z}^M(\xi)} = 0 \quad , \quad \delta S_{p=2} / i_\delta \hat{E}^{\dot{\alpha}} := E_{\dot{\alpha}}^M(\hat{Z}) \frac{\delta S_{p=2}}{\delta \hat{Z}^M(\xi)} = 0 \quad , \quad (3.16)$$

$$\delta S_{p=2} / i_\delta \hat{E}^a := E_a^M(\hat{Z}) \frac{\delta S_{p=2}}{\delta \hat{Z}^M(\xi)} = 0 \quad . \quad (3.17)$$

The straightforward calculation gives the following explicit form of these equations of motion

$$*\hat{E}_a \wedge \left(i \hat{E}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^a \sqrt{K\bar{K}} - \hat{E}_\beta (\bar{\gamma} \sigma^a)_\alpha{}^\beta \bar{K} \right) + \frac{1}{12} * \hat{E}_a \wedge \hat{E}^a \left(\sqrt{\bar{K}/K} D_\alpha K - i \bar{\gamma}_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \bar{K} \right) = 0 \quad , \quad (3.18)$$

$$*\hat{E}_a \wedge \left(i \hat{E}^\alpha \sigma_{\alpha\dot{\alpha}}^a \sqrt{K\bar{K}} - \hat{E}_{\dot{\beta}} (\sigma^a \bar{\gamma})^{\dot{\beta}}{}_{\dot{\alpha}} K \right) + \frac{1}{12} * \hat{E}_a \wedge \hat{E}^a \left(\sqrt{K/\bar{K}} \bar{D}_{\dot{\alpha}} \bar{K} - i D^\alpha K \bar{\gamma}_{\alpha\dot{\alpha}} \right) = 0 \quad , \quad (3.19)$$

$$\begin{aligned} D(*\hat{E}_a) &- \frac{1}{6} * \hat{E}_b \wedge \hat{E}^b (D_a \ln \bar{K} + D_a \ln K) + \frac{1}{2} * \hat{E}_a \wedge (d \ln \hat{K} + d \ln \hat{K}) - \\ &- \frac{i}{12\sqrt{K\bar{K}}} \hat{E}^d \wedge \hat{E}^c \wedge \hat{E}^b \epsilon_{abcd} (\bar{D} \bar{D} \bar{K} - D D K) - \\ &- \frac{1}{4\sqrt{K\bar{K}}} \hat{E}^c \wedge \hat{E}^b \wedge \epsilon_{abcd} \sigma_{\alpha\dot{\alpha}}^d (\hat{E}^\alpha \bar{D}^{\dot{\alpha}} \bar{K} + \hat{E}^{\dot{\alpha}} D^\alpha K) - \\ &- \hat{E}^b \wedge \hat{E}^\alpha \wedge \hat{E}^\beta \sigma_{ab\alpha\beta} \sqrt{\bar{K}/K} - \hat{E}^b \wedge \hat{E}^{\dot{\alpha}} \wedge \hat{E}^{\dot{\beta}} \tilde{\sigma}_{ab\dot{\alpha}\dot{\beta}} \sqrt{K/\bar{K}} = 0 \quad . \quad (3.20) \end{aligned}$$

Notice that the above equations of motion are not independent. According to the second Noether theorem, the gauge symmetries of a dynamical system result in the so-called Noether identities relating the left-hand sides of equations of motion of this system. The supermembrane possesses a number of gauge symmetries, including the local fermionic κ -symmetry (3.14), (3.15). This is reflected by the fact that contracting our fermionic equation (3.18) with $i\sqrt{\bar{K}/K}\tilde{\gamma}^{\beta\alpha}$ we arrive at Eq. (3.19). Denoting the *l.h.s.*'s of Eqs. (3.18) and (3.19) by Ψ_α and $\bar{\Psi}_{\dot{\alpha}}$, respectively, we can write the above described Noether identity for the κ -symmetry in the form of

$$\tilde{\gamma}^{\beta\alpha}\Psi_\alpha \equiv -i\sqrt{\bar{K}/K}\epsilon^{\beta\dot{\alpha}}\bar{\Psi}_{\dot{\alpha}}. \quad (3.21)$$

4 Superfield equations for the dynamical system of special scalar supermultiplet interacting with supermembrane

4.1 Special scalar multiplet and its dual three form potential

In our discussion below we will be considering not generic but *special scalar multiplet* described by the chiral superfield constructed from the *real prepotential* $V = (V)^*$,

$$\Phi = \bar{D}\bar{D}V, \quad \bar{\Phi} = DDV. \quad (4.22)$$

On the level of auxiliary fields the distinction of this special case is that one of the real auxiliary scalars of the generic scalar multiplet is replaced in it by a divergence of a real vector, $\partial_\mu k^\mu$ or, equivalently, by the field strength of a three form potential $k_{\nu\rho\sigma} = k^\mu\epsilon_{\mu\nu\rho\sigma}$ (in this latter form it was described in [14] and, as one of 'variant superfield representations', in [16]).

Indeed, the complex prepotential P of the generic chiral multiplet, $\Phi = \bar{D}\bar{D}P$, $\bar{\Phi} = DD\bar{P}$, is defined up to the gauge transformations, $\bar{P} \mapsto \bar{P} + D_\alpha\Xi^\alpha$. These imply that the imaginary part of the generic prepotential is transformed by $\Im m P := (P - \bar{P})/2i \mapsto \Im m P + (D_\alpha\Xi^\alpha - D_{\dot{\alpha}}\bar{\Xi}^{\dot{\alpha}})/2i$. Hence not-pure gauge parts of the superfield parameter $\Im m P$ are the ones which do not have their exact counterparts in the composed superfield $(D_\alpha\Xi^\alpha - D_{\dot{\alpha}}\bar{\Xi}^{\dot{\alpha}})/2i$. One can check that the superfield parameter $D_\alpha\Xi^\alpha - D_{\dot{\alpha}}\bar{\Xi}^{\dot{\alpha}}$ has all the components but one having contributions of different independent functions without derivatives. The only exception is the highest component in its decomposition which reads $-4i\theta\bar{\theta}\bar{\theta}\partial_a(k^a + \bar{k}^a)$ and includes the divergence of the real part of the complex vector $k_a = \tilde{\sigma}_a^{\dot{\alpha}\alpha}(\bar{D}_{\dot{\alpha}}\Xi_\alpha)|_{\theta=0} = (\bar{k}_a)^*$ versus an arbitrary function in a generic real scalar superfield, like P . Then one can guess that the equations of motion for the special scalar supermultiplet will differ from the set of equations for a generic scalar supermultiplet by that one of the algebraic auxiliary field equations of the latter ($(\mathcal{E} - \mathcal{E})|_0 = 0$) will be replaced by its derivative ($\partial_a(\mathcal{E} - \mathcal{E})|_0 = 0$). In other words, the general solution of the (auxiliary) field equations of the special scalar supermultiplet involves one additional (with respect to the generic case) arbitrary real constant. Furthermore, this indicates that the above mentioned auxiliary field equation of the special scalar supermultiplet ($\partial_a(\mathcal{E} - \mathcal{E})|_0 = 0$) is dependent, *i.e.* can be obtained as a consequence of other equations; this implies that the only effect of the use of the complex prepotential in the generic case

is vanishing of a real constant which is indefinite in the case of special scalar multiplet (where $\partial_a(\mathcal{E} - \bar{\mathcal{E}})|_0 = 0 \Rightarrow (\mathcal{E} - \bar{\mathcal{E}})|_0 = -2ic$). We will see that this is indeed the case.

4.1.1 Equations of motion of special scalar multiplet

The variation of the general action (2.14) for the special chiral superfields (4.22) with respect to real prepotential V apparently produces only the real part of the complex equation (2.17),

$$\frac{\delta S_{s-int}[\bar{D}\bar{D}V; DDV]}{\delta V} = 0 \Rightarrow \quad \mathcal{E} + \bar{\mathcal{E}} := \bar{D}\bar{D}\mathcal{K}'_{\Phi} + DD\mathcal{K}'_{\Phi} + W'_{\Phi} + \bar{W}'_{\Phi} = 0. \quad (4.23)$$

However, as far as the left hand sides of the equations of motion for generic scalar multiplet, Eqs. (2.17) and (2.16), are, respectively, anti-chiral and chiral, $D_{\alpha}\bar{\mathcal{E}} = 0$ and $\bar{D}_{\alpha}\mathcal{E} = 0$, Eq. (4.23) implies that the imaginary part of the complex equation (2.17) is equal to a constant,

$$\mathcal{E} + \bar{\mathcal{E}} = 0 \quad \Rightarrow \quad D_{\alpha}(\mathcal{E} - \bar{\mathcal{E}}) = 0 \quad \Rightarrow \quad \partial_a(\mathcal{E} - \bar{\mathcal{E}}) = 0. \quad (4.24)$$

Hence the only effect of the use of the special chiral superfields (4.22) instead of the generic scalar superfield (2.18) is that the equation $\mathcal{E} = 0$ is replaced by $\mathcal{E} = -ic$ where c is an arbitrary real constant.

4.1.2 On spontaneous supersymmetry breaking

The presence of this arbitrary constant in the right hand side of the superfield equations of motion, $\mathcal{E} = -ic$, actually suggests a possible spontaneous supersymmetry breaking in the theory of special chiral multiplet. To clarify this, let us discuss the simple case of a free massless special scalar multiplet, in which the action reads $\int d^8z \Phi\bar{\Phi} = \int d^8z DDV\bar{D}\bar{D}V$ so that $\bar{\mathcal{E}} = DD\bar{\Phi}$ and the equations of motion (4.23) simplify to

$$DD\Phi + \bar{D}\bar{D}\bar{\Phi} = 0. \quad (4.25)$$

As it has been discussed above (Eq. (4.24)) these equations lead to $D_{\alpha}(DD\Phi - \bar{D}\bar{D}\bar{\Phi}) = 0$ and $\partial_a(DD\Phi - \bar{D}\bar{D}\bar{\Phi}) = 0$. Algebraically all this set of equations is solved by $DD\Phi = -ic$ with the above mentioned arbitrary real constant c ,

$$DD\Phi + \bar{D}\bar{D}\bar{\Phi} = 0 \quad \Rightarrow \quad DD\Phi = -ic, \quad c = const. \quad (4.26)$$

In particular, this constant enters the solution of auxiliary field equations which now reads

$$DD\Phi|_0 = -ic, \quad c = const. \quad (4.27)$$

As the on-shell supersymmetry transformations of the fermionic fields $\psi_{\alpha} = -iD_{\alpha}\Phi|_0$ are obtained from the off-shell ones, $\delta\psi_{\alpha} = \frac{i}{2}\varepsilon_{\beta}DD\Phi|_0 + 2(\sigma^a\bar{\varepsilon})_{\alpha}\partial_a\phi$, by inserting the above solution of the auxiliary field equations, they read

$$\delta\psi_{\alpha} = \frac{c}{2}\varepsilon_{\beta} + 2(\sigma^a\bar{\varepsilon})_{\alpha}\partial_a\phi. \quad (4.28)$$

Hence, for nonvanishing value of c , the on-shell supersymmetry transformations of $\delta\psi_\alpha$ contains the additive contribution of supersymmetry parameter ε_β characteristic of the transformation rules of the Volkov-Akulov Goldstone fermion [17] the presence of which may be considered as an indication of the spontaneous supersymmetry breaking.

However, studying more carefully the case of free special scalar multiplet, one finds that such a spontaneous symmetry breaking actually does not occur if nontrivial boundary conditions are not introduced³. Indeed, the constant in the superfield equations (4.26) can be reproduced from the generic scalar supermultiplet action which includes the superpotential linear in chiral superfield, $W(\Phi) = -ic\Phi$. As it was observed already in [22], such a term can be removed from the action by a field redefinition. However, the boundary term contribution may change the situation; this role can be also played by supermembrane contribution. We leave the discussion on spontaneous supersymmetry breaking in the interacting system of scalar multiplet and supermembrane for future and now turn to the three form potential presentation of the special chiral supermultiplet.

4.1.3 Dual three form potential

The four form field strength constructed with the use of special scalar multiplet (4.22) is obtained from (2.21) by substituting

$$\bar{K} = \bar{\Phi} = DDV, \quad K = \Phi = \bar{D}\bar{D}V. \quad (4.29)$$

It reads

$$\begin{aligned} F_4 = dC'_3 &= \frac{1}{4}E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\alpha\beta} DDV + \frac{1}{4}E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \wedge \bar{E}^{\dot{\beta}} \tilde{\sigma}_{ab\dot{\alpha}\dot{\beta}} \bar{D}\bar{D}V + \\ &+ \frac{1}{4!}E^c \wedge E^b \wedge E^a \wedge \left(E^\alpha \epsilon_{abcd} \sigma_{\alpha\dot{\beta}}^d \bar{D}^{\dot{\beta}} DDV + \bar{E}^{\dot{\alpha}} \epsilon_{abcd} \sigma_{\beta\dot{\alpha}}^d D^\beta \bar{D}\bar{D}V \right) + \\ &+ \frac{1}{4!}E^d \wedge E^c \wedge E^b \wedge E^a \frac{i}{4} \epsilon_{abcd} (\bar{D}\bar{D}DDV - DD\bar{D}\bar{D}V). \end{aligned} \quad (4.30)$$

The corresponding 3-form potential C'_3 can be written in terms of the real prepotential as follows [14]

$$\begin{aligned} C'_3 &= 2iE^c \wedge E^\alpha \wedge \bar{E}^{\dot{\beta}} \sigma_{c\alpha\dot{\beta}} V + \frac{1}{2}E^c \wedge E^b \wedge E^\alpha \sigma_{bc\alpha}{}^\beta D_\beta V - \\ &- \frac{1}{2}E^b \wedge E^a \wedge \bar{E}^{\dot{\beta}} \tilde{\sigma}_{ab}{}^{\dot{\alpha}}{}_{\dot{\beta}} \bar{D}_{\dot{\alpha}} V - \frac{1}{4!}E^c \wedge E^b \wedge E^a \epsilon_{abcd} \tilde{\sigma}^{d\dot{\beta}\alpha} [D_\alpha, \bar{D}_{\dot{\beta}}] V. \end{aligned} \quad (4.31)$$

Of course, this expression can be changed on an equivalent one using gauge transformations $\delta C_3 = d\alpha_2$. These do not change the field strength (4.30) and are responsible for the possibility to do not have the lower dimensional form contributions ($\propto E^\alpha \wedge E^\beta \wedge E^\gamma$ etc.) in the above C'_3 .

The existence of this simple three form C'_3 giving a dual description of the special chiral supermultiplet (4.22) is the main reason to restrict our discussion below by this special case.

4.2 Superfield equations of motion for interacting system

Let us consider the most general interaction of the special scalar supermultiplet with supermembrane as described by the action (3.13) with $K = \Phi = \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} V$ and $\bar{K} = \bar{\Phi} =$

³One of the authors (I.B.) is grateful to Warren Siegel for the discussion on this issue.

$D^\alpha D_\alpha V$ as in (4.29), *i.e.* by

$$\begin{aligned}
S &= \int d^8z \mathcal{K}(\Phi, \bar{\Phi}) + \int d^6\zeta_L W(\Phi) + c.c. + \frac{1}{2} \int d^3\xi \sqrt{g} \sqrt{\hat{\Phi}\hat{\Phi}} - \int_{W^3} \hat{C}'_3 = \\
&= \int d^4x \bar{D}\bar{D}DD\mathcal{K}(\Phi, \bar{\Phi}) + \int d^4x (DDW(\Phi) + c.c.) - \frac{1}{6} \int_{W^3} * \hat{E}_a \wedge \hat{E}^a \sqrt{\hat{\Phi}\hat{\Phi}} - \int_{W^3} \hat{C}'_3
\end{aligned} \tag{4.32}$$

with special chiral superfield (4.22),

$$\Phi = \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} V, \quad \bar{\Phi} = D^\alpha D_\alpha V. \tag{4.33}$$

The variation of the interacting action (4.32) with respect to supermembrane variables gives formally the same equations of motion as for the supermembrane in the background, (3.18)–(3.20), but with $K = \Phi = DDV$,

$$* \hat{E}_a \wedge \left(i \hat{E}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^a \sqrt{\hat{\Phi}\hat{\Phi}} - \hat{E}_\beta (\tilde{\gamma} \tilde{\sigma}^a)_{\alpha}{}^{\beta} \hat{\Phi} \right) + \frac{1}{12} * \hat{E}_a \wedge \hat{E}^a \left(\sqrt{\hat{\Phi}/\hat{\Phi}} \widehat{D}_\alpha \hat{\Phi} - i \tilde{\gamma}_{\alpha\dot{\alpha}} \widehat{D}^{\dot{\alpha}} \hat{\Phi} \right) = 0, \tag{4.34}$$

$$* \hat{E}_a \wedge \left(i \hat{E}^{\alpha} \sigma_{\alpha\dot{\alpha}}^a \sqrt{\Phi\bar{\Phi}} - \hat{E}_{\dot{\beta}} (\tilde{\sigma}^a \tilde{\gamma})^{\dot{\beta}}{}_{\alpha} \Phi \right) + \frac{1}{12} * \hat{E}_a \wedge \hat{E}^a \left(\sqrt{\Phi/\bar{\Phi}} \bar{D}_{\dot{\alpha}} \bar{\Phi} - i D^\alpha \Phi \tilde{\gamma}_{\alpha\dot{\alpha}} \right) = 0, \tag{4.35}$$

$$\begin{aligned}
D(*\hat{E}_a) &- \frac{1}{6} * \hat{E}_b \wedge \hat{E}^b (D_a \ln \bar{\Phi} + D_a \ln \Phi) + \frac{1}{2} * \hat{E}_a \wedge (d \ln \hat{\Phi} + d \ln \hat{\Phi}) - \\
&- \frac{i}{12 \sqrt{\hat{\Phi}\hat{\Phi}}} \hat{E}^d \wedge \hat{E}^c \wedge \hat{E}^b \epsilon_{abcd} (\bar{D}\bar{D}\bar{\Phi} - DD\Phi) - \\
&- \frac{1}{4 \sqrt{\Phi\bar{\Phi}}} \hat{E}^c \wedge \hat{E}^b \wedge \epsilon_{abcd} \sigma_{\alpha\dot{\alpha}}^d (\hat{E}^\alpha \bar{D}^{\dot{\alpha}} \bar{\Phi} + \hat{E}^{\dot{\alpha}} D^\alpha \Phi) - \\
&- \hat{E}^b \wedge \hat{E}^a \wedge \hat{E}^\beta \sigma_{ab\alpha\beta} \sqrt{\Phi/\bar{\Phi}} - \hat{E}^b \wedge \hat{E}^{\dot{\alpha}} \wedge \hat{E}^{\dot{\beta}} \tilde{\sigma}_{ab\dot{\alpha}\dot{\beta}} \sqrt{\Phi/\bar{\Phi}} = 0.
\end{aligned} \tag{4.36}$$

However, the target superspace superfields the pull-backs of which enter these equations have to be the solutions of interacting equations with the source terms from the supermembrane. These superfield interacting equations read

$$\mathcal{E} + \bar{\mathcal{E}} = J, \tag{4.37}$$

where

$$\mathcal{E} = \bar{D}\bar{D}\bar{\Phi} \mathcal{K}''_{\Phi\bar{\Phi}}(\Phi, \bar{\Phi}) + \bar{D}_{\dot{\alpha}} \bar{\Phi} \bar{D}^{\dot{\alpha}} \bar{\Phi} \mathcal{K}'''_{\Phi\bar{\Phi}\bar{\Phi}}(\Phi, \bar{\Phi}) + W'_\Phi(\Phi) \tag{4.38}$$

(see (2.17) and (4.23)) and

$$J(z) = -\frac{\delta S_{p=2}}{\delta V(z)} \tag{4.39}$$

is the current superfield from the supermembrane. The problem of obtaining the complete set of interacting equations for the dynamical system of supermembrane and special chiral supermultiplet is now reduced to the problem of calculating this supermembrane current.

4.3 Supermembrane current

The supermembrane current is split naturally on the contributions from the Nambu–Goto and the Wess–Zumino terms of the action (3.13) with (4.22)

$$J(z) = J^{NG}(z) + J^{WZ}(z) = -\frac{\delta S_{p=2}}{\delta V(z)} \quad (4.40)$$

The Nambu-Goto part of the current

$$J^{NG}(z) := -\frac{\delta}{\delta V(z)} \frac{1}{2} \int d^3\xi \sqrt{g} \sqrt{\widehat{DDV} \widehat{DDV}} \quad (4.41)$$

($\widehat{DDV} := D^\alpha D_\alpha V(z)|_{z^M = \hat{z}^M(\xi)}$) is calculated by first using the properties of the superspace delta function

$$\delta^8(z) := \frac{1}{16} \delta^4(x) \theta\theta \bar{\theta}\bar{\theta}, \quad \int d^8z \delta^8(z - z') f(z) = f(z') \quad (4.42)$$

to present (4.41) in the form

$$J^{NG}(z) = -\frac{\delta}{\delta V(z)} \frac{1}{2} \int d^8z' \sqrt{\widehat{DDV}(z') \widehat{DDV}(z')} \int d^3\xi \sqrt{g} \delta^8(z' - \hat{z}). \quad (4.43)$$

Then the calculation reduces to using the definition of variation $\frac{\delta V(z')}{\delta V(z)} = \delta^8(z' - z)$ and performing the superspace integration. In such a way one arrives at

$$J^{NG}(Z) = -\frac{1}{4} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\Phi}}{\hat{\Phi}}} DD \delta^8(z - \hat{z}) - \frac{1}{4} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\Phi}}{\hat{\Phi}}} \bar{D}\bar{D} \delta^8(z - \hat{z}), \quad (4.44)$$

where $\Phi = \bar{D}\bar{D}V$, $\bar{\Phi} = DDV$ (see Eqs. (2.18)) and $\hat{\Phi} := \Phi(\hat{z}(\xi))$ etc. Similarly one can present the Wess–Zumino current in the form of

$$\begin{aligned} J^{WZ}(Z) = & \left(2i \int_{W^3} \hat{E}^c \wedge \hat{E}^\alpha \wedge \hat{E}^{\dot{\alpha}} \sigma_{c\alpha\dot{\alpha}} + \right. \\ & + \frac{1}{2} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^\alpha \sigma_{bc\alpha}{}^\beta D_\beta - \frac{1}{2} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^{\dot{\alpha}} \tilde{\sigma}_{bc}{}^{\dot{\beta}}{}_{\dot{\alpha}} \bar{D}_{\dot{\beta}} - \\ & \left. - \frac{1}{4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \tilde{\sigma}^{d\dot{\alpha}\alpha} [D_\alpha, \bar{D}_{\dot{\alpha}}] \right) \delta^8(z - \hat{z}). \end{aligned} \quad (4.45)$$

5 Simplest equations of motion for spacetime fields interacting with dynamical supermembrane

Having the superfield equations with supermembrane current contributions, the next stage is to extract the equations of motion for the physical fields of the supermultiplet. In this paper we will do this for the simplest case when the special scalar multiplet part of the interacting action is given by the kinetic term (2.14) only, this is to say for the interacting system described by the action

$$S = S_{kin} + S_{p=2} = \int d^8z \Phi \bar{\Phi} + \frac{1}{2} \int d^3\xi \sqrt{\hat{\Phi} \hat{\Phi}} \sqrt{g} - \int_{W^3} \hat{C}_3' \quad (5.1)$$

where $\Phi = \bar{D}\bar{D}V$, $\bar{\Phi} = DDV$ (4.33) and \hat{C}'_3 is the pull-back to W^3 of the 3-form C'_3 defined in (4.31). The interacting equations of motion for the bulk superfields, Eqs. (4.38), in this case simplifies to

$$DD\Phi + \bar{D}\bar{D}\bar{\Phi} = J(z) \quad (5.2)$$

where the current $J(z)$ is given by (4.40), (4.44) and (4.45).

5.1 General structure of the simplest special scalar multiplet equations with a superfield source

Superfield equation (5.2) encodes the dynamical equations for the physical fields of the scalar multiplet, $\phi(x) = \Phi|_0$ and $\psi_\alpha(x) = -i(D_\alpha\Phi)|_0$, as well as algebraic equations for auxiliary fields $DD\Phi|_0$ and $\bar{D}\bar{D}\bar{\Phi}|_0$. These latter include the leading component of the real superfield equation (5.2)

$$DD\Phi|_0 + \bar{D}\bar{D}\bar{\Phi}|_0 = J(z)|_0 \quad (5.3)$$

as well as the first order equation

$$\partial_a(DD\Phi|_0 - \bar{D}\bar{D}\bar{\Phi}|_0) = -\frac{i}{4}\bar{\sigma}_a^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]J(z)|_0. \quad (5.4)$$

The set of dynamical field equations include the Dirac (actually Weyl) equation with the source from supermembrane,

$$\sigma_{\alpha\dot{\alpha}}^a\partial_a\psi^\alpha := -i\partial_{\alpha\dot{\alpha}}D_\alpha\Phi|_0 = \frac{1}{4}\bar{D}_{\dot{\alpha}}J(z)|_0, \quad (5.5)$$

and the Klein-Gordon equation, also with the source,

$$\square\phi(x) := \square\Phi|_0 = -\frac{1}{16}\bar{D}\bar{D}J(z)|_0. \quad (5.6)$$

Now, to specify supermembrane contributions to the scalar multiplet field equations we have to calculate the derivatives of the supermembrane current.

5.2 Dynamical scalar multiplet equations with supermembrane source contributions

5.2.1 Auxiliary field equations

The leading components $J|_0$ of the current J in (5.3) is the sum of

$$J^{NG}|_0 = \frac{1}{16}\sqrt{\frac{\phi}{\bar{\phi}}}\int d^3\xi\sqrt{g}\hat{\theta}\hat{\theta}\delta^4(x-\hat{x}) + \frac{1}{16}\sqrt{\frac{\bar{\phi}}{\phi}}\int d^3\xi\sqrt{g}\hat{\theta}\hat{\theta}\delta^4(x-\hat{x}) \quad (5.7)$$

and

$$J^{WZ}(Z)|_0 = \frac{1}{48}\int_{W^3}\hat{E}^c\wedge\hat{E}^b\wedge\hat{E}^a\epsilon_{abcd}\hat{\theta}\sigma^{d\hat{\theta}}\delta^4(x-\hat{x}) + \mathcal{O}(f^4), \quad (5.8)$$

where $\mathcal{O}(f^4)$ denotes the terms of the fourth order in fermions (in this case, these are worldvolume fermionic fields $\hat{\theta}, \hat{\bar{\theta}}$ and their worldvolume derivatives, $\partial_m \hat{\theta} := \partial \hat{\theta} / \partial \xi^m$ and *c.c.*); the explicit form of these one can find in the Appendix B (Eq. (B.8)).

Substituting the above expressions into Eq. (5.3), one finds that the real part of the auxiliary fields of the chiral multiplet has quite a complex form in terms of supermembrane variables

$$DD\Phi|_0 + \bar{D}\bar{D}\bar{\Phi}|_0 = \frac{1}{16} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} \hat{\theta} \hat{\bar{\theta}} \delta^4(x - \hat{x}) + \frac{1}{16} \sqrt{\frac{\bar{\phi}}{\phi}} \int d^3\xi \sqrt{g} \hat{\theta} \hat{\bar{\theta}} \delta^4(x - \hat{x}) + \frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \hat{\theta} \sigma^d \hat{\bar{\theta}} \delta^4(x - \hat{x}) + \mathcal{O}(f^4), \quad (5.9)$$

where $\mathcal{O}(f^4)$ are the same as in Eq. (5.8) (and thus can be read off Eq. (B.8)).

The second auxiliary field equation, Eq. (5.4), reads

$$\partial_a (DD\Phi|_0 - \bar{D}\bar{D}\bar{\Phi}|_0) = -\frac{i}{8 \cdot 4!} \int_{W^3} \hat{E}^d \wedge \hat{E}^c \wedge \hat{E}^b \epsilon_{abcd} \delta^4(x - \hat{x}) + \mathcal{O}(f^2), \quad (5.10)$$

where the terms of higher order in fermions, $\mathcal{O}(f^2)$ can be found in Eqs. (B.16) and (B.17) of Appendix B (multiplying the expressions presented there by $-\frac{i}{4} \tilde{\sigma}_a^{\dot{\alpha}\alpha}$). On the first look it might seem that Eq. (5.10) imposes additional restrictions on the supermembrane motion. Such possible restrictions might come from the selfconsistency condition of Eq. (5.10); at zero order in fermions that reads ⁴

$$\partial_{[a} \epsilon_{b]c_1 c_2 c_3} \int_{W^3} d\hat{x}^{c_3} \wedge d\hat{x}^{c_2} \wedge d\hat{x}^{c_1} \delta^4(x - \hat{x}) = 0. \quad (5.11)$$

However, one can check that this equation is satisfied identically. Indeed, using the identity $\epsilon_{c_1 c_2 c_3 [a} \partial_{b]} \equiv -\frac{3}{2} \epsilon_{ab[c_1 c_2} \partial_{c_3]}$ one can write the *l.h.s.* of Eq. (5.11) in the form of $-\frac{3}{2} \epsilon_{abc_1 c_2} \int_{W^3} d\hat{x}^{c_2} \wedge d\hat{x}^{c_1} \wedge d\delta^4(x - \hat{x}) = -\frac{3}{2} \epsilon_{abc_1 c_2} \int_{W^3} d(d\hat{x}^{c_2} \wedge d\hat{x}^{c_1} \delta^4(x - \hat{x}))$ which vanishes as an integral of total derivative in the case of closed supermembrane which we are studying in this paper ($\partial W^3 = \emptyset \Rightarrow \int_{W^3} d(\dots) = 0$).

As we have discussed in Sec. 4.1, the on-shell transformations are obtained from the off-shell ones, $\delta\psi_\alpha = 2(\sigma^a \bar{\epsilon})_\alpha \partial_a \phi + \frac{i}{2} \varepsilon_\beta DD\Phi|_0$ for the case of fermions, by substituting the solution of the equations for the auxiliary fields. This implies that the on-shell supersymmetry transformation of fermions will be quite complicated due to the complicated structure of the auxiliary field equations (5.9) and (5.4). As the on-shell fermionic supersymmetry transformations can be used to extract BPS conditions for the supersymmetric solutions, their further study in our simple system might lead to useful suggestions for the investigation of the backreaction of D=10,11 super-*p*-branes on the BPS solutions of supergravity equations.

5.2.2 Dynamical field equations

Fortunately, the dynamical equations for the physical fields of the special scalar multiplet following from the simplest interacting action Eq. (5.1) do not obtain contributions from

⁴ $\int_{W^3} \hat{E}^d \wedge \hat{E}^c \wedge \hat{E}^b \epsilon_{abcd} \delta^4(x - \hat{x}) = \int_{W^3} d\hat{x}^d \wedge d\hat{x}^c \wedge d\hat{x}^b \epsilon_{abcd} \delta^4(x - \hat{x}) + \text{fermionic contributions.}$

the auxiliary fields of the scalar multiplet, which on the mass shell are expressed by quite complicated Eqs. (5.9) and (5.4).

Specifying the current contributions to (5.5) and (5.6), we find the massless Dirac equation with the supermembrane contributions,

$$\begin{aligned} \sigma_{\alpha\dot{\alpha}}^a \partial_a \psi^\alpha &= \frac{1}{32} \sqrt{\frac{\hat{\phi}}{\phi}} \int d^3\xi \sqrt{g} \hat{\theta}_{\dot{\alpha}} \delta^4(x - \hat{x}) - \\ &\quad - \frac{1}{8 \cdot 4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta} \sigma^d)_{\dot{\alpha}} \delta^4(x - \hat{x}) + \mathcal{O}(f^3), \end{aligned} \quad (5.12)$$

and the Klein-Gordon equation, also with the source from supermembrane,

$$\begin{aligned} \square \phi(x) &= \frac{1}{64} \sqrt{\frac{\hat{\phi}}{\phi}} \int d^3\xi \sqrt{g} \delta^4(x - \hat{x}) - \\ &\quad - \frac{1}{64} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \sigma_{bc} \hat{\theta} \delta^4(x - \hat{x}) - \frac{i}{64} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d \delta^4(x - \hat{x}) + \\ &\quad + \frac{i}{64} \int d^3\xi \sqrt{g} (\hat{\theta} \sigma^a \hat{\theta}) \sqrt{\frac{\hat{\phi}}{\phi}} \partial_a \delta^4(x - \hat{x}) + \mathcal{O}(f^4). \end{aligned} \quad (5.13)$$

The explicit form of the terms of higher order in fermions, $\mathcal{O}(f^4)$ in (5.13) and $\mathcal{O}(f^3)$ in (5.12), can be extracted from the Eqs. (B.6) and (B.14) in Appendix B.

5.3 Simplest solution of the dynamical equations at leading order in supermembrane tension

The above equations can be formally solved by

$$\begin{aligned} \psi^\alpha &= \psi_0^\alpha + \frac{1}{32} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\phi}}{\phi}} (\hat{\theta} \tilde{\sigma}^a)^\alpha \partial_a G_0(x - \hat{x}) + \\ &\quad + \frac{1}{8 \cdot 4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta} \sigma^d \tilde{\sigma}^a)^\alpha \partial_a G_0(x - \hat{x}) + \mathcal{O}(f^3), \end{aligned} \quad (5.14)$$

$$\begin{aligned} \phi(x) &= \phi_0(x) + \frac{1}{64} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\phi}}{\phi}} \left(G_0(x - \hat{x}) + i(\hat{\theta} \sigma^a \hat{\theta}) \partial_a G_0(x - \hat{x}) \right) - \\ &\quad - \frac{1}{64} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \sigma_{bc} \hat{\theta} G_0(x - \hat{x}) - \frac{i}{64} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d G_0(x - \hat{x}) + \\ &\quad + \mathcal{O}(f^4), \end{aligned} \quad (5.15)$$

where ψ_0^α and $\phi_0(x)$ are solutions of the free equations and $G_0(x - \hat{x})$ is the Green function of the free $D = 4$ Klein-Gordon operator $\square := \partial_a \partial^a$,

$$\square \phi_0(x) = 0, \quad \sigma_{\alpha\dot{\alpha}}^a \partial_a \psi_0^\alpha = 0 \quad \square G_0(x - \hat{x}) = \delta^4(x - \hat{x}). \quad (5.16)$$

Eqs. (5.14) and (5.15) give only formal solutions as far as the pull-back of the phase of the complex scalar superfield enters their *r.h.s.* through $\sqrt{\frac{\hat{\phi}}{\phi}}$ multipliers in the integrands.

Assuming the solution of the homogeneous equation to be real, $\phi_0(x) = (\phi_0(x))^*$ one can solve Eqs. (5.12) and (5.13) in the first order in the supermembrane tension T (this is set to unity in our equations above and below, but can be easily restored by $\int d^3\xi \sqrt{g} \mapsto T \int d^3\xi \sqrt{g}$ and $\int_{W^3} \mapsto T \int_{W^3}$). This reads (setting back $T = 1$)

$$\begin{aligned} \psi^\alpha &= \psi_0^\alpha + \frac{1}{32} \int d^3\xi \sqrt{g} (\hat{\theta}\tilde{\sigma}^a)^\alpha \partial_a G_0(x - \hat{x}) + \\ &+ \frac{1}{8 \cdot 4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta}\sigma^d \tilde{\sigma}^a)^\alpha \partial_a G_0(x - \hat{x}) + \mathcal{O}(f^3), \end{aligned} \quad (5.17)$$

$$\begin{aligned} \phi(x) &= \phi_0(x) + \frac{1}{64} \int d^3\xi \sqrt{g} \left(G_0(x - \hat{x}) + i(\hat{\theta}\sigma^a \hat{\theta}) \partial_a G_0(x - \hat{x}) \right) + \\ &- \frac{1}{64} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \sigma_{bc} \hat{\theta} G_0(x - \hat{x}) - \frac{i}{64} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d G_0(x - \hat{x}) + \\ &+ \mathcal{O}(f^4), \quad \phi_0(x) = (\phi_0(x))^*. \end{aligned} \quad (5.18)$$

The contribution of higher order in string tension would include the product of distributions (of the type $G_0(x - \hat{x}(\xi_1)) \delta^4(x - \hat{x}(\xi_2))$) and their accounting requires a careful study of a classical counterpart of the renormalization procedure, similar to the one developed for the radiation reaction problem [23] and for general relativity [24]. The generalization of such a technique for the case of p-brane has been developed in very recent [25].

6 Conclusion and discussion

In this manuscript we studied the interaction of the four dimensional supermembrane with dynamical scalar multiplet. We have derived the complete set of the superfield equations of motion for this interacting system in the case when scalar multiplet is described by chiral superfield of special form, namely expressed through the real pre-potential superfield. Such a *special* scalar multiplet has a simple dual description in terms of 3-form potential.

Although the interacting systems of dynamical D=4 $\mathcal{N} = 1$ supergravity and superstring have been studied some years ago [7], the superfield equations of motion for the dynamical system including supermembrane have not been known before. Furthermore, this paper seems to be the first study of superfield equations for interacting system involving matter (not supergravity) superfields and a supersymmetric extended object.

Furthermore, although the supermembrane in flat superspace and in the background of $\mathcal{N} = 1$, $D = 4$ supergravity is well known [2, 3], to our best knowledge, the action of supermembrane in a chiral superfield background, Eq. (3.13), was not written before.

The characteristic coupling of the scalar field to the membrane which appears in the bosonic limit of the action (3.13) is similar to the coupling presented in [15], but not identical to this. The difference is that the counterpart of our arbitrary holomorphic function $K(\phi)$ of complex field ϕ ($\partial_{\bar{\phi}} K := \partial K / \partial \bar{\phi} = 0$) is replaced in [15] by certain covariantly chiral function \mathcal{L} which is the product of holomorphic function, superpotential $W(\phi)$, and of the exponent of the Kähler potential $\mathcal{K}(\phi, \bar{\phi})$ ($\mathcal{D}_{\bar{\phi}} \mathcal{L} = 0$ with suitable covariant derivative $\mathcal{D}_{\bar{\phi}} = \partial_{\bar{\phi}} + \dots$). In contrast, we do not find any relation of $K(\phi)$ with superpotential $W(\Phi)$ and Kähler potential \mathcal{K} of the most general scalar multiplet action.

This may be related to that we are studying the supermembrane in flat superspace with only the scalar supermultiplet background, requiring only the global supersymmetry

and the κ -symmetry of the action, while the superfield counterpart of the membrane action of [15] should appear as a part of the interacting superfield action including the superfield supergravity, scalar supermultiplets and supermembrane. The construction of such an action and deriving superfield equations from it will be the subject of our future study.

A complete but gauge fixed version of such an action, obtained on the line of [8]-[9], should then include the pure bosonic membrane coupled to spacetime fields of the supergravity and matter multiplets, possesses invariance under the local supersymmetry $1/2$ of which should be broken on the membrane worldvolume by the condition identifying the pull-back of the local supersymmetry parameter with the κ -symmetry. Just such a (hypothetical) gauge fixed action would produce the equations of [15] invariant under the local supersymmetry restricted by a κ -symmetry-like projection condition on the membrane worldvolume. Such a relation of the supersymmetry with κ -symmetry, characteristic for [15] but absent in our dynamical system, might be the reason of that the counterpart of $K(\phi)$ is expressed through the superpotential and Kähler potential in [15] while remains arbitrary holomorphic function in our case.

In Sec. 5 we have specified our bulk superfield equations for the simplest case when the bulk part of the action is given by the free kinetic term (with Kähler potential $\mathcal{K} = \Phi\bar{\Phi}$ and vanishing superpotential, $W(\Phi) = 0$), extracted from these the equations for spacetime, component fields and presented a solution of the dynamical equations for physical fields in the leading order on supermembrane tension. A search for solution of the superfield equations with a nontrivial superpotential is an interesting direction for future study.

Acknowledgments

One of the authors (I.B.) thanks Dima Sorokin, Ulf Lindström and Martin Roček for useful conversations, Warren Siegel for useful comments and communications and the organizers of the VIII Simons Workshop, especially Martin Roček, for their kind hospitality in Stony Brook where a part of this work was done. The partial support by the research grants FIS2008-1980 from the Spanish MICINN and by the Basque Government Research Group Grant ITT559-10 is greatly acknowledged.

Appendix A: Notation, conventions and some useful formulae

We use mostly minus Minkowski metric $\eta^{ab} = \text{diag}(1, -1, -1, -1)$ and complex Weyl spinor notation. $D = 4$ vector and spinor indices are denoted by symbols from the beginning of Latin and Greek alphabets, $a, b, c = 0, 1, 2, 3$, $\alpha, \beta, \gamma = 1, 2$, $\dot{\alpha}, \dot{\beta}, \dot{\gamma} = 1, 2$. In particular, the coordinates of the target $D = 4$, $\mathcal{N} = 1$ superspace $\Sigma^{(4|4)}$ are denoted, respectively, by x^a and θ^α , $\bar{\theta}^{\dot{\alpha}} = (\theta^\alpha)^*$. The star $*$ denotes complex conjugation of bosonic variables and involution on Grassmann algebra (see [18] and refs. therein); in practical terms this implies that $(\theta^\alpha \hat{\theta}^\beta)^* = \hat{\theta}^{\dot{\beta}} \bar{\theta}^{\dot{\alpha}} = -\bar{\theta}^{\dot{\alpha}} \hat{\theta}^{\dot{\beta}}$. Then, to keep the plus sign in $(\theta^2)^* = \bar{\theta}^2$ with $(\theta^2) := \theta^\alpha \theta_\alpha$, we have to define $\bar{\theta}^2 := \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}$. The consistency of the Grassmann algebra involution that $(\partial_\alpha)^* \equiv \left(\frac{\partial}{\partial \theta^\alpha}\right)^* = -\partial_{\dot{\alpha}} \equiv -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}$. Then, the covariant spinor derivative defined in (2.6) are related by $(D_\alpha)^* = -\bar{D}_{\dot{\alpha}}$. Then $\bar{D}\bar{D} := \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} = (DD)^*$ where $DD := D^\alpha D_\alpha$.

The contraction the spinorial indices are raised and lowered by the unit antisymmetric tensors $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha} = i\sigma^2 \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\epsilon^{\dot{\alpha}\dot{\beta}} = -\epsilon^{\dot{\beta}\dot{\alpha}}$ and their inverse $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$, $\epsilon_{\dot{\alpha}\dot{\beta}} = -\epsilon_{\dot{\beta}\dot{\alpha}}$. This implies $\theta_\alpha = \epsilon_{\alpha\beta} \theta^\beta$, $\theta^\alpha = \epsilon^{\alpha\beta} \theta_\beta$, etc. However, to get $\partial^\alpha \theta_\beta = \delta_\beta^\alpha$ simultaneously with $\partial_\alpha \theta^\beta = \delta_\alpha^\beta$ we have to assume that for the derivatives over the fermionic variables $\partial^\alpha = -\epsilon^{\alpha\beta} \partial_\beta$ so that, when we rise the spinorial index of the covariant fermionic derivative (2.6), we arrive at $D^\alpha := \epsilon^{\alpha\beta} D_\beta = -\partial^\alpha - i(\bar{\theta} \tilde{\sigma}^a)^\alpha \partial_a$.

The list of properties of relativistic Pauli matrices $\sigma_{\beta\dot{\alpha}}^a = \epsilon_{\beta\alpha} \epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\sigma}^{a\dot{\beta}\alpha}$ include

$$\sigma^a \tilde{\sigma}^b = \eta^{ab} + \frac{i}{2} \epsilon^{abcd} \sigma_c \tilde{\sigma}_d, \quad \tilde{\sigma}^a \sigma^b = \eta^{ab} - \frac{i}{2} \epsilon^{abcd} \tilde{\sigma}_c \sigma_d, \quad (\text{A.1})$$

$$\sigma^{ab} := \frac{1}{2} (\sigma^a \tilde{\sigma}^b - \sigma^b \tilde{\sigma}^a) = \frac{i}{2} \epsilon^{abcd} \sigma_{cd}, \quad (\text{A.2})$$

$$\tilde{\sigma}^{ab} := \frac{1}{2} (\tilde{\sigma}^a \sigma^b - \tilde{\sigma}^b \sigma^a) = -\frac{i}{2} \epsilon^{abcd} \tilde{\sigma}_{cd}, \quad (\text{A.3})$$

$$\sigma^{abc} = -i \epsilon^{abcd} \sigma_d. \quad (\text{A.4})$$

The 3-dimensional worldvolume vector indices are denoted by symbols from the middle of Latin alphabet. In particular, the local coordinates of the supermembrane worldvolume W^3 are denoted by ξ^m with $m = 0, 1, 2$. The worldvolume Hodge star operation is defined as in (3.6),

$$* \hat{E}^a := \frac{1}{2} d\xi^m \wedge d\xi^n \sqrt{g} \epsilon_{mnk} g^{kl} \hat{E}_l^a. \quad (\text{A.5})$$

In our conventions $d\xi^m \wedge d\xi^n \wedge d\xi^k = -\epsilon^{mnk} d^3 \xi \equiv \epsilon^{knm} d^3 \xi$ so that

$$* \hat{E}_a \wedge \hat{E}^a = -3 d^3 \xi \sqrt{g}, \quad * \hat{E}_a \wedge \delta \hat{E}^a = -d^3 \xi \sqrt{g} \hat{E}_{ma} g^{mn} \delta \hat{E}_n^a \quad (\text{A.6})$$

and

$$\delta(* \hat{E}_a \wedge \hat{E}^a) = 3 * \hat{E}_a \wedge \delta \hat{E}^a. \quad (\text{A.7})$$

The superspace generalization of Dirac delta function reads

$$\delta^8(z) := \frac{1}{16} (\theta^2)^2 (\bar{\theta}^2)^2 \delta^4(x)$$

and obeys

$$\int d^8 z \delta^8(z - \hat{z}) f(z) = f(\hat{z}), \quad \int d^8 z \delta^8(z) = \int d^4 x \bar{D}\bar{D} D D \delta^8(z) = 1$$

Appendix B: Components of the supermembrane current superfield $J(z) = J^{NG}(z) + J^{WZ}(z)$

The leading component of the Nambu-Goto current (4.44) reads

$$J^{NG}|_0 = +\frac{1}{16} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} \hat{\theta}\hat{\theta}\delta^4(x - \hat{x}) + \frac{1}{16} \sqrt{\frac{\bar{\phi}}{\phi}} \int d^3\xi \sqrt{g} \hat{\theta}\hat{\theta}\delta^4(x - \hat{x}) \quad (\text{B.1})$$

The general expression for the fermionic derivative of the Nambu-Goro current superfield is

$$\bar{D}_{\dot{\alpha}} J^{NG} = -\frac{1}{4} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\Phi}}{\hat{\bar{\Phi}}}} \bar{D}_{\dot{\alpha}} DD \delta^8(Z - \hat{Z}) . \quad (\text{B.2})$$

so that

$$\bar{D}_{\dot{\alpha}} J^{NG}|_0 = \frac{1}{8} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} \hat{\theta}_{\dot{\alpha}} \delta^4(x - \hat{x}) - \frac{i}{16} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\phi}}{\hat{\bar{\phi}}}} (\sigma^a \hat{\theta})_{\dot{\alpha}} (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) . \quad (\text{B.3})$$

Furthermore, as

$$\bar{D}\bar{D}J^{NG} = -\frac{1}{4} \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\Phi}}{\hat{\bar{\Phi}}}} \bar{D}\bar{D}DD \delta^8(Z - \hat{Z}) \quad (\text{B.4})$$

we find that

$$\begin{aligned} \bar{D}\bar{D}J^{NG}|_0 &= \\ &= \int d^3\xi \sqrt{g} \sqrt{\frac{\hat{\phi}}{\hat{\bar{\phi}}}} \left(-\frac{1}{4} \delta^4(x - \hat{x}) - \frac{i}{4} (\hat{\theta} \sigma^a \hat{\theta}) \partial_a \delta^4(x - \hat{x}) + \frac{1}{16} (\hat{\theta})^2 (\hat{\theta})^2 \square \delta^4(x - \hat{x}) \right) = \\ &= -\frac{1}{4} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} \delta^4(x - \hat{x}) - \\ &\quad - \frac{i}{4} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} (\hat{\theta} \sigma^a \hat{\theta}) \left(\partial_a \delta^4(x - \hat{x}) + \left(\frac{\partial_a \phi}{2\phi} - \frac{\partial_a \bar{\phi}}{2\bar{\phi}} \right) \delta^4(x - \hat{x}) \right) + \mathcal{O}(f^4) . \end{aligned} \quad (\text{B.5})$$

One can also write Eq. (B.5) in the equivalent but more compact form of

$$\bar{D}\bar{D}J^{NG}|_0 = -\frac{1}{4} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} \delta^4(x - \hat{x}) + \mathcal{O}(f^2)_{[\bar{D}\bar{D}J^{NG}]} + \mathcal{O}(f^4)_{[\bar{D}\bar{D}J^{NG}]} , \quad (\text{B.6})$$

where

$$\mathcal{O}(f^2)_{[\bar{D}\bar{D}J^{NG}]} = -\frac{i}{4} \sqrt{\frac{\phi}{\bar{\phi}}} \int d^3\xi \sqrt{g} (\hat{\theta} \sigma^a \hat{\theta}) \left(\partial_a \delta^4(x - \hat{x}) + \left(\frac{\partial_a \phi}{2\phi} - \frac{\partial_a \bar{\phi}}{2\bar{\phi}} \right) \delta^4(x - \hat{x}) \right) . \quad (\text{B.7})$$

The leading term of the Wess–Zumino (WZ) contribution to the supermembrane current reads

$$J^{WZ}(x) := J^{WZ}(Z)|_0 = \frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \hat{\theta} \sigma^d \hat{\theta} \delta^4(x - \hat{x}) -$$

$$\begin{aligned}
& -\frac{1}{16} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta}^\alpha \sigma_{bc\alpha}{}^\beta \hat{\theta}_\beta \hat{\theta} \hat{\theta} \delta^4(x - \hat{x}) - \\
& -\frac{1}{16} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta}^{\dot{\alpha}} \tilde{\sigma}_{bc}{}^{\dot{\beta}}{}_{\dot{\alpha}} \hat{\theta}_{\dot{\beta}} \hat{\theta} \hat{\theta} \delta^4(x - \hat{x}) + \\
& +\frac{i}{8} \int_{W^3} \hat{E}^c \wedge d\hat{\theta}^\alpha \wedge d\hat{\theta}^{\dot{\alpha}} \sigma_{c\alpha\dot{\alpha}} \hat{\theta} \hat{\theta} \hat{\theta} \delta^4(x - \hat{x}) = \\
& = \frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \hat{\theta} \sigma^d \hat{\theta} \delta^4(x - \hat{x}) + \mathcal{O}(f^4). \quad (\text{B.8})
\end{aligned}$$

Its closest fermionic partners read

$$\begin{aligned}
D_\alpha J^{WZ}|_0 &= -\frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\sigma^d \hat{\theta})_\alpha \delta^4(x - \hat{x}) + \\
& +\frac{1}{16} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \left(2(d\hat{\theta} \tilde{\sigma}_{bc} \hat{\theta}) \hat{\theta}_\alpha + d\hat{\theta}^\beta \sigma_{bc\beta\alpha} (\hat{\theta})^2 \right) \delta^4(x - \hat{x}) + \\
& +\frac{i}{96} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\sigma^{de} \hat{\theta})_\alpha (\hat{\theta})^2 \partial_e \delta^4(x - \hat{x}) + \\
& +\frac{i}{32} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta}^{\dot{\alpha}} (\sigma^a \tilde{\sigma}_{bc})_{\alpha\dot{\alpha}} (\hat{\theta})^2 (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) - \\
& -\frac{i}{4} \int_{W^3} \hat{E}^c \wedge d\hat{\theta}^\beta \wedge d\hat{\theta}^{\dot{\beta}} \sigma_{c\beta\dot{\beta}} \hat{\theta}_\alpha (\hat{\theta})^2 \delta^4(x - \hat{x}) = \\
& = -\frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\sigma^d \hat{\theta})_\alpha \delta^4(x - \hat{x}) + \mathcal{O}(f^3). \quad (\text{B.9})
\end{aligned}$$

$$\begin{aligned}
\bar{D}_{\dot{\alpha}} J^{WZ}|_0 &= \frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta} \sigma^d)_{\dot{\alpha}} \delta^4(x - \hat{x}) - \\
& -\frac{1}{16} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \left(2(d\hat{\theta} \sigma_{bc} \hat{\theta}) \hat{\theta}_{\dot{\alpha}} + (d\hat{\theta} \tilde{\sigma}_{bc})_{\dot{\alpha}} (\hat{\theta})^2 \right) \delta^4(x - \hat{x}) - \\
& -\frac{i}{96} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta} \tilde{\sigma}^{de})_{\dot{\alpha}} (\hat{\theta})^2 \partial_e \delta^4(x - \hat{x}) - \\
& -\frac{i}{32} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge (d\hat{\theta} \sigma_{bc} \sigma^a)_{\dot{\alpha}} (\hat{\theta})^2 (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) + \\
& +\frac{i}{4} \int_{W^3} \hat{E}^c \wedge d\hat{\theta}^\beta \wedge d\hat{\theta}^{\dot{\beta}} \sigma_{c\beta\dot{\beta}} \hat{\theta}_{\dot{\alpha}} (\hat{\theta})^2 \delta^4(x - \hat{x}) = \\
& = \frac{1}{48} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta} \sigma^d)_{\dot{\alpha}} \delta^4(x - \hat{x}) + \mathcal{O}(f^3). \quad (\text{B.10})
\end{aligned}$$

Then, as far as

$$\begin{aligned}
DDJ^{WZ}(Z) &= 2i \int_{W^3} \hat{E}^c \wedge \hat{E}^\alpha \wedge \hat{E}^{\dot{\alpha}} \sigma_{c\alpha\dot{\alpha}} DD \delta^8(Z - \hat{Z}) + \\
& -\frac{1}{2} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^{\dot{\alpha}} \tilde{\sigma}_{bc}{}^{\dot{\beta}}{}_{\dot{\alpha}} \left(\bar{D}_{\dot{\beta}} DD + 4i\sigma_{\dot{\beta}\dot{\beta}}^a \partial_a D^\beta \right) \delta^8(Z - \hat{Z}) + \\
& +\frac{i}{6} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \partial^d DD \delta^8(Z - \hat{Z}), \quad (\text{B.11}) \\
\bar{D}\bar{D}J^{WZ}(Z) &= 2i \int_{W^3} \hat{E}^c \wedge \hat{E}^\alpha \wedge \hat{E}^{\dot{\alpha}} \sigma_{c\alpha\dot{\alpha}} \bar{D}\bar{D} \delta^8(Z - \hat{Z}) +
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \sigma_{bc\alpha}{}^\beta \left(D_\beta \bar{D} \bar{D} - 4i \sigma_{\beta\dot{\beta}}^a \partial_a \bar{D}^{\dot{\beta}} \right) \delta^8(Z - \hat{Z}) - \\
& - \frac{i}{6} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \partial^d \bar{D} \bar{D} \delta^8(Z - \hat{Z}) , \tag{B.12}
\end{aligned}$$

one finds that

$$\begin{aligned}
DDJ^{WZ}|_0 &= \frac{1}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \tilde{\sigma}_{bc} \hat{\theta} \delta^4(x - \hat{x}) - \frac{i}{4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d \delta^4(x - \hat{x}) - \\
& - \frac{i}{2} \int_{W^3} \hat{E}^c \wedge d\hat{\theta}^\alpha \wedge d\hat{\theta}^{\dot{\alpha}} \sigma_{c\alpha\dot{\alpha}} (\hat{\theta})^2 \delta^4(x - \hat{x}) + \\
& + \frac{i}{8} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta}^{\dot{\alpha}} (\hat{\theta} \sigma^a \tilde{\sigma}_{bc})_{\dot{\alpha}} (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) = \\
& = \frac{1}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \tilde{\sigma}_{bc} \hat{\theta} \delta^4(x - \hat{x}) - \\
& - \frac{i}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d \delta^4(x - \hat{x}) + \mathcal{O}(f^4) . \tag{B.13}
\end{aligned}$$

$$\begin{aligned}
\bar{D}\bar{D}J^{WZ}|_0 &= \frac{1}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \sigma_{bc} \hat{\theta} \delta^4(x - \hat{x}) + \frac{i}{4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d \delta^4(x - \hat{x}) - \\
& - \frac{i}{2} \int_{W^3} \hat{E}^c \wedge d\hat{\theta}^\alpha \wedge d\hat{\theta}^{\dot{\alpha}} \sigma_{c\alpha\dot{\alpha}} (\hat{\theta})^2 \delta^4(x - \hat{x}) - \\
& - \frac{i}{8} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \sigma_{bc} \sigma^a \hat{\theta} (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) = \\
& = \frac{1}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge d\hat{\theta} \sigma_{bc} \hat{\theta} \delta^4(x - \hat{x}) + \\
& + \frac{i}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} (\hat{\theta})^2 \partial^d \delta^4(x - \hat{x}) + \mathcal{O}(f^4) . \tag{B.14}
\end{aligned}$$

To analyze the structure of the auxiliary field equation one needs also to know

$$[D_\alpha, \bar{D}_{\dot{\beta}}] J^{WZ}|_0 = -\frac{1}{4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \sigma_{\alpha\dot{\beta}}^d \delta^4(x - \hat{x}) + \mathcal{O}(f^2) , \tag{B.15}$$

where

$$\begin{aligned}
\mathcal{O}(f^2) &= \frac{i}{4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a \epsilon_{abcd} \left((\sigma^{de} \hat{\theta})_\alpha \hat{\theta}_{\dot{\beta}} + \hat{\theta}_\alpha (\hat{\theta} \tilde{\sigma}^{de})_{\dot{\beta}} \right) \partial_e \delta^4(x - \hat{x}) + \\
& + \frac{1}{4} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \left((\sigma_{bc} d\hat{\theta})_\alpha \hat{\theta}_{\dot{\beta}} - (d\hat{\theta} \tilde{\sigma}_{bc})_{\dot{\beta}} \hat{\theta}_\alpha \right) \delta^4(x - \hat{x}) + \mathcal{O}(f^4) , \tag{B.16} \\
\mathcal{O}(f^4) &= \frac{1}{2 \cdot 4!} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge \hat{E}^a (\hat{\theta})^2 (\hat{\theta})^2 \epsilon_{abcd} \left(\sigma_{\alpha\dot{\beta}}^d \square \delta^4(x - \hat{x}) - \sigma_{\alpha\dot{\beta}}^e \partial_e \partial^d \delta^4(x - \hat{x}) \right) +
\end{aligned}$$

$$\begin{aligned}
& + \frac{i}{8} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge (d\hat{\theta} \sigma_{bc} \sigma^a)_{\hat{\beta}} \hat{\theta}_{\alpha} (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) + \\
& + \frac{i}{8} \int_{W^3} \hat{E}^c \wedge \hat{E}^b \wedge (d\hat{\theta} \tilde{\sigma}_{bc} \tilde{\sigma}^a)_{\alpha} \hat{\theta}_{\hat{\beta}} (\hat{\theta})^2 \partial_a \delta^4(x - \hat{x}) - \\
& - i \int_{W^3} \hat{E}^c \wedge d\hat{\theta}^{\gamma} \wedge d\hat{\theta}^{\dot{\gamma}} \sigma_{c\dot{\gamma}} \hat{\theta}_{\alpha} \hat{\theta}_{\hat{\beta}} \delta^4(x - \hat{x}) .
\end{aligned} \tag{B.17}$$

References

- [1] C.M. Hull and P. K. Townsend, *Unity of superstring dualities*, Nucl. Phys. **B438**, 109-137 (1995) [hep-th/9410167];
E. Witten, *String theory dynamics in various dimensions*, Nucl. Phys. **B443**, 85 -126 (1995) [hep-th/9503124];
J.H. Schwarz, *Second Superstring Revolution*, Nucl.Phys.Proc.Suppl. **B55**, 1-32 (1997) (hep-th/9607067);
P.K. Townsend, *Four Lectures on M–theory*, In :*Summer School High energy physics and cosmology, Trieste 1996* (Eds. E. Gava, A. Masiero, K.S. Narain, S. Randjbar-Daemi, Q. Shafi), The ICTP Series in Theoretical Physics, Vol. 13, World Scientific, 1997, Singapore, pp. 385-438 [hep-th/9612121].
- [2] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Supermembranes and eleven-dimensional supergravity*, Phys. Lett. **B189**, 75 (1987); *Properties of the eleven-dimensional supermembrane theory*, Ann. Phys. (NY) **185**, 330 (1988).
- [3] A. Achucarro, J. P. Gauntlett, K. Itoh and P. K. Townsend, *World volume supersymmetry from space-time supersymmetry of the four–dimensional supermembrane*, Nucl. Phys. **B314**, 129–157 (1989).
- [4] J. A. De Azcarraga and P. K. Townsend, *Superspace Geometry And Classification Of Supersymmetric Extended Objects*, Phys. Rev. Lett. **62** (1989) 2579.
- [5] J.A. de Azcárraga, J.P. Gauntlett, J.M. Izquierdo and P.K. Townsend, *Topological extensions of the supersymmetry algebra for extended objects*, Phys. Rev. Lett. **63**, 213 (1989).
- [6] Igor A. Bandos, José A. de Azcárraga, Jose M. Izquierdo and Jerzy Lukierski, *D = 4 supergravity dynamically coupled to a massless superparticle in a superfield Lagrangian approach*, Phys. Rev. **D67**, 065003 (2003) [hep-th/0207139].
- [7] I. A. Bandos and J. M. Isidro, *D=4 supergravity dynamically coupled to superstring in a superfield Lagrangian approach*, Phys. Rev. D **69**, 085009 (2004) [hep-th/0308102].
- [8] I.A. Bandos, J.A. de Azcárraga and J.M. Izquierdo, *Supergravity interacting with bosonic p–branes and local supersymmetry*, Phys. Rev. **D65**, 105010 (2002) [hep-th/0112207].

- [9] I. Bandos and J. A. de Azcarraga, *Dirac equation for the supermembrane in a background with fluxes from a component description of the $D = 11$ supergravity-supermembrane interacting system*, JHEP **0509**, 064 (2005) [hep-th/0507197] and refs. therein.
- [10] V. P. Akulov, D. V. Volkov and V. A. Soroka, “Gauge fields on superspaces with different holonomy groups,” JETP Lett. **22** (1975) 187 [Pisma Zh. Eksp. Teor. Fiz. **22** (1975) 396]. V. P. Akulov, D. V. Volkov and V. A. Soroka, “On the general covariant theory of gauge fields in superspace,” Theor. Math. Phys. **31**, 285 (1977) [Teor. Mat. Fiz. **31**, 12 (1977)];
- [11] M. F. Sohnius and P. C. West, “An Alternative Minimal Off-Shell Version Of $N=1$ Supergravity,” Phys. Lett. B **105**, 353 (1981); M. Sohnius and P. C. West, “The Tensor Calculus And Matter Coupling Of The Alternative Minimal Auxiliary Field Formulation Of $N=1$ Supergravity,” Nucl. Phys. B **198**, 493 (1982).
- [12] K. S. Stelle and P. C. West, “Minimal Auxiliary Fields For Supergravity,” Phys. Lett. B **74**, 330 (1978); S. Ferrara and P. van Nieuwenhuizen, “The Auxiliary Fields Of Supergravity,” Phys. Lett. B **74**, 333 (1978).
- [13] J. Wess and B. Zumino, “Superspace Formulation Of Supergravity,” Phys. Lett. **B66** (1977) 361; J. Wess and B. Zumino, “Superfield Lagrangian For Supergravity,” Phys. Lett. B **74**, 51 (1978).
- [14] S. J. J. Gates, “Super p form gauge superfields,” Nucl. Phys. **B184**, 381 (1981).
- [15] M. Hubscher, P. Meessen and T. Ortin, “Domain walls and instantons in $N=1$, $d=4$ supergravity,” JHEP **1006**, 001 (2010) [arXiv:0912.3672 [hep-th]].
- [16] S. J. J. Gates and W. Siegel, “Variant Superfield Representations,” Nucl. Phys. B **187**, 389 (1981).
- [17] D. V. Volkov and V. P. Akulov, *Possible universal neutrino interaction*, JETP Lett. **16**, 438 (1972); *Is the neutrino a Goldstone particle?*, Phys. Lett. **B46**, 109 (1973).
- [18] V. I. Ogievetsky and L. Mezincescu, “Symmetries Between Bosons And Fermions And Superfields,” Usp. Fiz. Nauk **117**, 637-683 (1975).
- [19] J. Wess and J. Bagger, “Supersymmetry and supergravity,” Princeton, USA: Univ. Pr. 1992, 259 pp.
- [20] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, “Superspace, or one thousand and one lessons in supersymmetry,” Front. Phys. **58**, 1 (1983) [arXiv:hep-th/0108200].
- [21] D. P. Sorokin, V. I. Tkach and D. V. Volkov, *Superparticles, twistors and Siegel symmetry*, Mod. Phys. Lett. **A4** (1989) 901-908.
- [22] J. Iliopoulos and B. Zumino, “Broken supergauge symmetry and renormalization,” Nucl. Phys. B **76**, 310 (1974).

- [23] P. O. Kazinski, S. L. Lyakhovich and A. A. Sharapov, “Radiation reaction and renormalization in classical electrodynamics of point particle in any dimension,” *Phys. Rev. D* **66**, 025017 (2002) [arXiv:hep-th/0201046]; P. O. Kazinski and A. A. Sharapov, “Radiation reaction for a massless charged particle,” *Class. Quant. Grav.* **20**, 2715 (2003) [arXiv:hep-th/0212286] and refs. therein.
- [24] L. Infeld and J. Plebanski, *Motion and Relativity*, Pergamon Press, Warszawa 1960.
- [25] K. Lechner, “Ultraviolet singularities in classical brane theory,” arXiv:1011.3746 [hep-th].