Near Horizon Limits of Massless BTZ and Their CFT Duals

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Abstract

We consider the massless BTZ black hole and show that it is possible to take its "near horizon" limit in two distinct ways. The first one leads to a null self-dual orbifold of AdS_3 and the second to a singular space-like $AdS_3/\mathbb{Z}_K$ orbifold in the large K limit, the "pinching orbifold". We show that from the dual 2d CFT viewpoint, the null orbifold corresponds to the $p^+ = 0$ sector of the Discrete Light-Cone Quantization (DLCQ) of the 2d CFT where a chiral sector of the CFT is decoupled, while the pinching orbifold corresponds to taking an infinite mass gap limit in both the right and left sectors of the 2d CFT, essentially leaving us with the states $L_0 = \bar{L}_0 = \frac{c}{24}$ only. In the latter case, one can combine the near horizon limit with sending the 3d Planck length ℓ_P to zero, or equivalently the dual 2d CFT central charge c to infinity. We provide preliminary evidence that in that case some nontrivial dynamics may survive the limit.

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1   Introduction

Despite the progress made in understanding the statistical mechanical origin of black hole thermodynamics, the identification of the underlying microscopic degrees of freedom for generic black holes remains an open problem. In most examples in which such an identification was achieved, the black hole possesses an AdS$_3$ throat in its near-horizon limit, and the microscopic degrees of freedom are captured by a two-dimensional conformal field theory. In some other cases, the near horizon geometries resemble but are not quite AdS$_3$, and it is clearly important to understand the precise meaning of such geometries, and their connection to the degrees of freedom of two-dimensional field theories. Motivated by this, we decided to analyse and interpret the various possible near-horizon limits that one can take in the case of the BTZ black hole [1].

In [2] this analysis was initiated by considering extremal BTZ black holes. In particular it was noted that (i) the spacelike circle at the causal boundary of the extremal BTZ geometry

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becomes a light-like circle at the horizon and, (ii) moving from the boundary to the horizon is exactly like boosting the spacelike circle so that it becomes approximately null. Next, it was noted that the dual 2d CFT resides on a $\mathbb{R} \times S^1$ boundary, and that this boosting to the speed of light exactly matches Seiberg's definition [3] of the Discrete Light-Cone Quantization (DLCQ) of the 2d CFT. Since the eigenvalues of left and right excitation operators $L_0 - c/24$ and $\bar{L}_0 - c/24$ are scaled with opposite boost factors (if one of them is scaled up by the factor γ the other is scaled by γ^{-1}), the boosting will create an infinite mass gap in one of the sectors, say the $\bar{L}_0$ sector, if we intend to keep the mass scale of the L_0 sector finite. That is, in the DLCQ description, due to the infinite mass gap, we cannot excite the $\bar{L}_0$ sector and it is to be set to its ground state with $\bar{L}_0 = \frac{c}{24}$. On the other hand, the L_0 sector which has a finite mass gap can be arbitrarily excited. In other words, in the DLCQ description a non-chiral 2d CFT reduces to a chiral sector.

Since the DLCQ prescription emerges when approaching the horizon, the near-horizon geometry of the extremal BTZ black hole should be the holographic dual of the DLCQ of the 2d CFT. As was shown [2], the near-horizon geometry of extremal BTZ is the so-called spacelike self-dual AdS_3 orbifold [4]. It contains an AdS_2 factor which cannot be excited and which is the geometric manifestation of the frozen $\bar{L}_0$ sector. The excitations in the dynamical L_0 sector crucially involve the remaining third space-time coordinate.

In [5] it was proposed that chiral 2d CFT's may provide us with the "dual CFT" description of near horizon extremal 4d Kerr black hole, and this idea has subsequently been extended to various other types of near-extremal black holes. This proposal was given preliminary backing through an asymptotic symmetry group analysis and a successful computation of the black hole entropy using Cardy's formula. However, it is not clear if this chiral CFT captures any of the dynamics around an extremal black hole [6]. In particular, for the simplest extremal black hole, i.e. extremal BTZ, in [2] we showed that the dual description of the self-dual AdS_3 geometry that appears in the near horizon limit is indeed a thermal state in a chiral 2d CFT with temperature determined by the light-cone momentum p^+ of the dual DLCQ CFT. Consistently, the light-cone momentum p^+ is also the only free parameter of the self-dual AdS_3 geometry. Because the right-movers are frozen very little dynamics is left in this case.

One issue which was left open in the above analysis was the geometry representing the "ground state", i.e. the $p^+ = 0$ sector, of the DLCQ chiral CFT. Naively, this geometry should be obtainable from a suitable near-horizon limit of massless BTZ, and as we show

below this is indeed the case. We will show that it is the null self-dual AdS$_3$ orbifold whose metric is given in (2.9) which is dual to the ground state of the chiral CFT, whereas the spacelike self-dual AdS$_3$ orbifold (2.3) corresponds to a generic state with $p^+ \neq 0$ [2].

The null self-dual orbifold was recently found in a near-horizon limit in [7], but is not the only geometry which has appeared in the literature in the study of near-horizon limits of extremal vanishing horizon black holes. A different geometry, which we will refer to as the pinching orbifold of AdS$_3$, has also appeared several times starting with the work of [8], and more recently in [9, 10, 11, 12]. Its geometry is exactly the same as that of $M = 0$ BTZ, except that the periodicity of the asymptotic S^1 is changed from 2π to $2\pi\epsilon$, the near-horizon limit being the $\epsilon \to 0$ limit. We will review the fairly straightforward near-horizon limit of the $M = 0$ BTZ black hole which produces this pinching orbifold and show that in this limit both sectors of the 2d CFT are frozen rather than just one.

Since the massless BTZ black hole has a naked singularity, it is not unexpected that both the null self-dual orbifold, as well as the pinching orbifold, are not good reliable gravitational backgrounds. Nevertheless, we hope our observations help to clarify the physical interpretation of the appearance of these two distinct singular geometries arising in different near horizon limits.

In the case of the pinching orbifold, as one goes to lower and lower energies, both the left and right moving sectors lose their dynamics because the dual CFT has a mass gap. For CFT's with an AdS$_3$ dual this mass gap typically is of the order of $1/c$. One could therefore expect that one might be able to retain some dynamics by combining a near-horizon limit with a $c \to \infty$ limit. We will present a preliminary analysis of this possibility in section 4, after discussing the various near-horizon limits in section 2 and their dual interpretation in section 3. We conclude with some discussion of our results and provide a comprehensive review of near-horizon limits of BTZ geometries in the appendix.

In [13], we will discuss various generalisations of all these near-horizon limits to other black holes, in particular to two R-charge AdS$_5$ black holes, following up on previous work in [9, 14].

Note added: While developing these ideas, we became aware of the work [15] which has some overlap with our discussion regarding the null self-dual orbifold and its interpretation as the $p^+ = 0$ sector of the DLCQ of the 2d CFT.

2 Near horizon (low energy) limits of massless BTZ

Near horizon limits can generically be interpreted as low energy limits. There may exist different inequivalent low energy limits to be considered in a given physical system, and consequently, one may expect the existence of more than a single near horizon limit when realising these notions holographically. If so, these different limits may focus on different physics of the same theory.

We will study this possibility for the massless BTZ black hole. Since this has a naked singularity, one expects its near horizon geometry to be singular too. Keeping the radius of AdS_3 ℓ_3 fixed, and remembering there are no bulk degrees of freedom, the resulting geometry must be a quotient of AdS_3.[1]

In the following we partially review some set of well known relations between massless BTZ black holes, extremal BTZ black holes and their near horizon limits that will allow us to get the appropriate physical intuition to correctly interpret the two different inequivalent near horizon limits that exist for massless BTZ, one leading to the *null self-dual* orbifold and the second to a singular geometry that we shall refer to as a *pinching orbifold*.

2.1 Null self-dual AdS_3 orbifold as the near horizon limit of massless BTZ

Consider an extremal BTZ black hole

$$ds^2 = -\frac{(r^2 - r_h^2)^2}{r^2\,\ell_3^2}\,dt^2 + \frac{\ell_3^2\,r^2}{(r^2 - r_h^2)^2}\,dr^2 + r^2\left(d\phi - \frac{r_h^2}{r^2}\frac{dt}{\ell_3}\right)^2, \tag{2.1}$$

and take the near horizon limit

$$r^2 = r_h^2 + \epsilon\rho\,, \qquad t = \frac{\tau}{\epsilon}\,, \qquad \varphi = \phi - \frac{\tau}{\epsilon\,\ell_3} \qquad \epsilon \to 0\,. \tag{2.2}$$

This gives rise to the spacelike self-dual orbifold [2] [2]

$$\begin{aligned}
ds^2 &= \frac{\ell_3^2}{4}\frac{d\rho^2}{\rho^2} - \frac{\rho^2}{r_h^2}\frac{d\tau^2}{\ell_3^2} + r_h^2\left(d\varphi + \frac{\rho}{r_h^2}\frac{d\tau}{\ell_3}\right)^2 \\
&= \frac{\ell_3^2}{4}\frac{d\rho^2}{\rho^2} + 2\frac{\rho}{\ell_3}d\tau d\varphi + r_h^2 d\varphi^2\,.
\end{aligned} \tag{2.3}$$

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[1]The set of inequivalent abelian quotients of AdS_3 appears already in [16]. A complete analysis on d=3 self-dual quotients along with their supersymmetry appears in [4]. This was extended to higher dimensions in [17, 18], including a complete supersymmetry analysis [18] and geometrical properties of the causally well-behaved ones in [19], where it was already mentioned that the null self-dual orbifold should play a role in any DLCQ formulation of the dual field theory.

[2]The importance of this geometry for the physics of extremal black holes was already emphasized some time ago in [20, 21].

Its causal boundary is a null cylinder parameterised by the null non-compact τ direction and the null circle in the φ direction. The φ direction at generic ρ is a spacelike circle, becoming null at the boundary, while τ is always null. It was discussed in [2] that the self-dual AdS$_3$ orbifold geometry is dual to a chiral CFT which is what remains of a generic 2d CFT in the DLCQ description; this chiral CFT resides on the null cylinder causal boundary of the self-dual orbifold. Once the periodicity of the φ direction is fixed to 2π, the only parameter of the self-dual orbifold metric, r_h, is then related to the value of the light-cone momentum p^+ defining the DLCQ sector:

$$p^+ = \frac{c}{6}\left(\frac{r_h}{\ell_3}\right)^2 . \tag{2.4}$$

To obtain (2.4) we have used the fact that $p^+ = L_0 - \frac{c}{24}$ and that the mass M or angular momentum J of the original extremal BTZ and L_0 are related as[3]

$$M\ell_3 = J = p^+ = \frac{r_h^2}{4\ell_3 G_3} , \qquad c = \frac{3\ell_3}{2G_3} . \tag{2.6}$$

As discussed in [2] the self-dual orbifold is a thermal state in the DLCQ of the 2d CFT at temperature[4]

$$T_{\text{self-dual}} = \frac{r_h}{\pi\ell_3} = \sqrt{\frac{6p^+}{\pi^2 c}} . \tag{2.8}$$

The $r_h \to 0$ limit of the spacelike self-dual orbifold sends the temperature (2.8) to zero and yields the metric

$$ds^2 = r^2 dx^+ dx^- + \ell_3^2 \frac{dr^2}{r^2} , \qquad x^- \sim x^- + 2\pi , \tag{2.9}$$

where we have conveniently renamed $\rho = r^2$, φ as x^- and $\tau = 2\ell_3 x^+$. Notice the causal character of the compact direction x^- has changed, from an everywhere spacelike direction (except at the boundary) to an everywhere null direction. By construction, this corresponds to the null self-dual orbifold. The latter has the same boundary as (2.3), and so it could be viewed as belonging to the same semiclassical Hilbert space that one could construct through

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[3]In our conventions the ADM mass and angular momentum of a generic BTZ geometry with inner and outer horizons $r_\pm$ are given by

$$M\ell_3 = \frac{r_+^2 + r_-^2}{8G_3\ell_3} , \qquad J = \frac{r_+ r_-}{4G_3\ell_3} . \tag{2.5}$$

[4]The left and right temperatures are given for a generic BTZ is given by

$$T_L = \frac{r_+ + r_-}{2\pi\ell_3} , \qquad T_R = \frac{r_+ - r_-}{2\pi\ell_3} \tag{2.7}$$

and the Hawking temperature of the BTZ black hole T_H is $\frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R}$.

a Brown-Henneaux type analysis [22]. Thus, the null orbifold should correspond to a state in the DLCQ CFT. Since the $r_h \to 0$ limit corresponds to $p^+ \to 0$, it is natural to relate the null orbifold to the $p^+ = 0$ sector of the DLCQ CFT, which is the most natural identification to make especially when working in some Poincaré patch description of AdS$_3$.

Instead of taking the vanishing horizon limit after the near horizon limit, we can study the near horizon limit of the massless BTZ black hole. This singular black hole is

$$ds^2 = r^2 d\tilde{x}^+ d\tilde{x}^- + \ell_3^2 \frac{dr^2}{r^2} \qquad \tilde{x}^\pm = \phi \pm t \, , \qquad \phi \sim \phi + 2\pi \, . \tag{2.10}$$

Consider the near horizon limit

$$r = \epsilon\rho \, , \qquad \tilde{x}^- = x^- \, , \qquad \tilde{x}^+ = \frac{x^+}{\epsilon^2} \, , \qquad \epsilon \to 0 \, . \tag{2.11}$$

The lightlike direction $\tilde{x}^+$ effectively decompactifies, for the same reason as for the spacelike self-dual orbifold, while x^- remains compact $x^- \sim x^- + 2\pi$. Thus, the near horizon limit (2.11) of a massless BTZ black hole is the null self-dual AdS$_3$ orbifold (2.9).

Exciting the null self-dual orbifold : If this proposal is correct, the spacelike self-dual orbifold (2.3) should be viewed as an excitation over the null orbifold

$$ds^2 = \frac{\ell_3^2}{z^2} \left(dx^+ dx^- + dz^2 \right) \, , \qquad x^- \sim x^- + 2\pi \, , \tag{2.12}$$

In particular, injecting some chiral momentum into the system keeping its causal null cylinder boundary should correspond to the spacelike self-dual orbifold. This is achieved by adding some wave to the conformally flat metric

$$ds^2 = \frac{\ell_3^2}{z^2} \left[dx^+ dx^- + kz^2(dx^-)^2 + dz^2 \right] \, , \tag{2.13}$$

Since there are no propagating degrees of freedom in d=3, the latter is locally AdS$_3$, and it is indeed isometric to the spacelike self-dual orbifold (2.3), with r_h^2 being replaced with $k\ell_3^2$. Recalling (2.4), this corresponds to a DLCQ sector with light-cone momentum

$$p^+ = \frac{c}{6} k \, . \tag{2.14}$$

This observation is consistent with the interpretation given above based on [2]. Indeed, among the Brown-Henneaux diffeomorphisms, there is only one chiral sector that preserves the null boundary structure shared by both orbifolds. The spacelike self-dual one is obtained after the finite action of one of these diffeomorphisms belonging to this same chiral sector.

All these observations are consistent with the well-known fact that extremal BTZ is a chiral excitation above the massless BTZ black hole [23, 24]. To see this, consider the massless BTZ black hole (2.10) and add some chiral momentum as above

$$ds^2 = \tilde{r}^2 \left(d\tilde{x}^+ d\tilde{x}^- + k \frac{\ell_3^2}{\tilde{r}^2} (d\tilde{x}^-)^2 \right) + \ell_3^2 \frac{d\tilde{r}^2}{\tilde{r}^2} \,. \tag{2.15}$$

The latter is equivalent to the extremal massive BTZ black hole (2.1) (with $\tilde{r}^2 = r^2 - r_h^2$).

Summary: The near horizon limit (2.11) of massless BTZ is the null AdS$_3$ orbifold. The latter corresponds to a thermal state of vanishing temperature in the dual 2d chiral CFT. Thus it describes the $p^+ = 0$ sector of the DLCQ CFT. Exciting the null orbifold by the addition of chiral momentum generates the spacelike self-dual orbifold, which appears as the near horizon limit of extremal BTZ. This set of relations is summarized in the diagram 2.16.

$$
\begin{array}{ccc}
\text{Massless BTZ} & \xrightarrow{\text{+ momentum}} & \text{Extremal BTZ} \\
\Big\downarrow \text{near horizon} & & \Big\downarrow \text{near horizon} \\
\text{Null orbifold} & \xrightarrow{\text{+ momentum}} & \text{Self-dual orbifold}
\end{array}
\tag{2.16}
$$

Notice this full discussion corresponds to the construction of non-relativistic gravity duals in d=3. The near horizon limit implements the low energy limit involved in the non-relativistic limit. In d=3, the lack of bulk propagating degrees of freedom manifests itself in a set of different global identifications characterising the different chiral momentum sectors of the DLCQ CFT. These different quotients realise the correct symmetries preserved after the non-relativistic limit of the original d=2 CFT. The only non-singular non-relativistic gravity dual corresponds to the sector of large p^+, as is customary in gravity/gauge theory dualities and Matrix theory[25, 26].[5]

2.2   The pinching AdS$_3$ orbifold as the near horizon limit of massless BTZ

The above discussion is coherent and completes the description of chiral excitations in 2d CFTs. But, it was apparent that when taking the near horizon of the massless BTZ black hole (2.10), there was an inequivalent near horizon limit that we could have considered

$$r = \epsilon \rho \,, \qquad \hat{x}^\pm = \epsilon \, x^\pm \,. \tag{2.17}$$

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[5]In higher dimensions, the easiest construction of non-relativistic gravity duals to DLCQ CFTs is also achieved by exciting momentum through the addition of some wave propagating in the bulk, which in $d > 3$ is indeed dynamical [28, 29]. We would like to emphasize that such construction had already been used in [23, 24].

The resulting geometry, like *all* geometries we are considering, is locally AdS$_3$

$$ds^2 = \rho^2 d\hat{x}^+ d\hat{x}^- + \ell_3^2 \frac{d\rho^2}{\rho^2} \qquad \hat{x}^\pm \sim \hat{x}^\pm + 2\pi\epsilon\,. \tag{2.18}$$

We will refer to is as a pinching AdS$_3$ orbifold due to its interpretation as AdS$_3/\mathbb{Z}_{1/\epsilon}$ where the quotient acts on the spacelike S^1 of AdS$_3$. By construction, its causal boundary is a *pinching cylinder*, $R \times S^1/\mathbb{Z}_{1/\epsilon}$. This geometry is singular, but it is *not* equivalent to the null self-dual orbifold. In the next section, we will interpret it as a low energy limit in which *all* excitations in both chiral sectors are decoupled. Thus, different near horizon limits capture very different physics.

3   Low energy IR limits of 2d non-chiral CFTs

All black hole geometries considered in the previous section are interpreted as thermal states in some dual 2d non-chiral CFT with finite central c. This theory is formulated on a cylinder of radius R

$$ds^2 = -dt^2 + R^2 d\phi^2 = -du\, dv \quad ; \quad u = t - R\phi,\ v = t + R\phi \tag{3.1}$$

where ϕ is a circle, i.e. $\phi \sim \phi + 2\pi$. After quantization, the eigenvalues of the momentum operators conjugate to u and v, P^v and P^u, respectively, are

$$P^v = \left(h + n - \frac{c}{24}\right)\frac{1}{R}, \qquad P^u = \left(h - \frac{c}{24}\right)\frac{1}{R}, \qquad n \in \mathbb{Z} \tag{3.2}$$

where n denotes the quantized momentum along the S^1. Assuming unitarity, implies $h \geq 0$ and $h + n \geq 0$. These are related to the eigenvalues of the standard operators $L_0, \bar{L}_0$ used in radial quantization on the plane by $\bar{L}_0 = h + n$ and $L_0 = h$. We will assume that the 2d CFT is non-singular, and therefore, that its spectrum is discrete.

We now interpret the two near horizon limits studied in the previous section.

1. The null orbifold has the same boundary structure as the spacelike self-dual orbifold and corresponds to the $r_h \to 0$ limit of the latter. The first feature freezes out the right moving sector to $h = 0$ (or $\bar{L}_0 = \frac{c}{24}$). To see this we note that taking the near horizon limit in the dual 2d CFT amounts to taking the mass gap of the right moving sector to infinity while keeping the gap in the left moving sector finite [2]. The second corresponds to setting the temperature to zero, and consequently, the momentum $p^+ = 0$.

2. The pinching orbifold corresponds to sending the radius R of the limiting boundary cylinder to zero, $R \sim \epsilon \to 0$. This generates an infinite gap in both chiral sectors of the initial 2d CFT. The only surviving finite excitations are those corresponding to $h = n = 0$. Thus, given a CFT with a fixed central charge c, this limit freezes out (decouples) both left and right moving sectors, leaving us with the Hilbert space:

$$\mathcal{H} = \{|c/24\rangle_L \otimes |c/24\rangle_R\}. \tag{3.3}$$

This CFT limit parallels the pinching orbifold introduced in 2.2. To see this, relabel $R = \epsilon \ell$ and write the cylinder metric as

$$ds^2 = -dt^2 + \ell^2 d\psi^2, \qquad \phi = \frac{\psi}{\epsilon}. \tag{3.4}$$

We can indeed interpret the latter as the pinching cylinder $R \times S^1/\mathbb{Z}_{1/\epsilon}$.

The above discussion gives evidence for the claim that different near horizon limits describe different low energy limits in the dual 2d CFT. In particular, given a non-singular unitary 2d CFT with finite central charge c, the null self-dual orbifold describes the $p^+ = 0$ sector of the DLCQ of the original CFT whereas the pinching AdS$_3$ orbifold decouples both chiral sectors. The first has a non-trivial Hilbert space, whereas the second contains *no dynamics*, but just the degeneracy of the quantum states $L_0 = \bar{L}_0 = c/24$. This conclusion is generic unless we scale the central charge of the CFT while taking the low energy limit as we will now discuss.

4 The pinching orbifolds and double scaling limits

Sending $R \to 0$ or $c \to \infty$ has opposite effects for the spectrum of the 2d CFT. The first generates an infinite gap, whereas the second reduces it, generating a continuous spectrum in the limit. It is natural to wonder whether a double scaling limit $R \to 0$, $c \to \infty$, $cR =$ fixed could keep some nontrivial dynamics after the low energy limit discussed in the previous section.

To achieve this we should have some non-zero entropy. Recall that the geometric entropy of classical BTZ black holes, given by the Bekenstein-Hawking area law, is exactly given by the semi-classical Cardy formula

$$S = \frac{2\pi r_+}{4G_3} = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} + 2\pi\sqrt{\frac{c}{6}\left(\bar{L}_0 - \frac{c}{24}\right)}, \tag{4.1}$$

where we have used the standard correspondence[6]

$$L_0 - \frac{c}{24} = \frac{M\ell_3 + J}{2} = \frac{c}{24}\left(\frac{r_+ + r_-}{\ell_3}\right)^2, \quad \bar{L}_0 - \frac{c}{24} = \frac{M\ell_3 - J}{2} = \frac{c}{24}\left(\frac{r_+ - r_-}{\ell_3}\right)^2. \quad (4.2)$$

The first equality in (4.1) suggests that if together with taking a nearly-massless BTZ, i.e. $r_\pm \to \epsilon r_\pm$ with $\epsilon \to 0$, we also scale $G_3 \to \epsilon G_3$, the entropy does not change and *remains finite*. The second equality suggests the scaling $c \to c/\epsilon$ (where we used the Brown-Henneaux formula $c = 3\ell_3/(2G_3)$ [22]), and then (4.2) imply $L_0 - \frac{c}{24} \to \epsilon(L_0 - \frac{c}{24})$ and $\bar{L}_0 - \frac{c}{24} \to \epsilon(\bar{L}_0 - \frac{c}{24})$.

In what follows we first discuss the geometric argument in terms of BTZ metrics to connect the above scalings to the pinching orbifold. Afterwards we discuss the meaning of the limit in the dual field theory, and finally we mention some subtleties involved in the latter.

4.1 The gravity perspective

Before introducing the actual double scaling limit in gravity, let us review an interesting observation. Consider the BTZ metric

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2\left(d\phi - \frac{r_+ r_-}{r^2}\frac{dt}{\ell_3}\right)^2, \quad (4.3)$$

where

$$F(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell_3^2 r^2} \quad \text{and} \quad \phi \sim \phi + 2\pi.$$

The λ-transformation

$$r = \lambda\rho, \quad r_\pm = \lambda\rho_\pm, \quad t = \lambda^{-1}\tau, \quad \phi = \lambda^{-1}\psi, \quad (4.4)$$

formally keeps the form of the metric invariant as in (4.3), but the periodicity in ψ is $2\pi\lambda$. Note this was not simply a coordinate transformation, as the parameters $r_\pm$ also changed. Denoting a BTZ black hole with charges $\{M, J\}$ and periodicity $2\pi\alpha$ by $\mathrm{BTZ}(M, J; 2\pi\alpha)$, we learn that in classical gravity

$$\mathrm{BTZ}(M\lambda^2, J\lambda^2; 2\pi) \equiv \mathrm{BTZ}(M, J; 2\pi\lambda), \quad (4.5)$$

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[6]We remark that the property that the entropy is only a function of $c(L_0 - \frac{c}{24})$ and $\bar{c}(\bar{L}_0 - \frac{\bar{c}}{24})$ products goes beyond the simple saddle point analysis of Cardy and is valid up to exponentially suppressed contributions [30].

where we used (2.5). Notice the *entropy* remains *invariant* under this λ-transformation whereas the Hawking temperature gets scaled by λ^{-1} *cf.* (2.7). Both solutions in (4.5) belong to the *same gravity theory* since both $\{\ell_3, G_3\}$ remained unchanged. Thus, the central charge did not transform and consequently, we did not yet achieve the goal set at the beginning of this section.

The double scaling limit : If we want to be left with a finite entropy after taking the near horizon of a nearly massless BTZ black hole, all we have to do is to combine the near horizon limit described previously with an scaling of the 3d Newton's constant as

$$r_\pm = \epsilon \rho_\pm, \qquad r = \epsilon \rho_+ + \epsilon \rho, \qquad t = \epsilon^{-1} \tau, \qquad \phi = \epsilon^{-1} \psi, \qquad G_3 = \epsilon \tilde{G}_3 \qquad \epsilon \to 0. \quad (4.6)$$

This is nothing but a λ-transformation (4.4) with $\lambda = \epsilon$, accompanied by the Newton's constant extra scaling. The latter is the responsible to keep the entropy of the nearly massless BTZ finite. As shown in the appendix, the corresponding near horizon geometry, which is insensitive to the rescaling of G_3, is a *pinching orbifold*. The scaling in Newton's constant does scale the central charge as we wanted, i.e. $c \to c/\epsilon$, at the price of changing the actual conformal field theory description of the system, as we discuss next.

4.2 The 2d CFT perspective

As discussed the equivalent of the scaling (4.6) in terms of 2d CFT quantities is to rescale the central charge by a factor of $\epsilon^{-1} \equiv K$, and $L_0 - c/24$ and $\bar{L}_0 - c/24$ by a factor of $\epsilon = K^{-1}$. As this scaling changes the central charge it takes us to a different 2d CFT. One can gain some intuition about these different 2d CFT's by thinking about the CFT dual to the D1-D5 system. This 2d CFT can be described by a 2d sigma model with $N = (4,4)$ supersymmetry with a target which can be thought of as a suitable symmetric product $\text{Sym}^{N_1 N_5}(\mathcal{M}_4)$. Rescaling the central charge is like rescaling $N_1 N_5$. Therefore, we expect a relation of the form

$$\text{CFT}_{\text{new}} \approx \text{Sym}^K(\text{CFT}_{\text{old}}). \tag{4.7}$$

This clearly only makes sense when $K = 1/\epsilon$ is an integer, and in that case the new CFT has a long string sector which is directly inherited from the old theory. The Virasoro algebra related to this long string sector can be directly related to that of the original CFT using standard orbifold technology. Explicitly, given the Virasoro algebra

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}n(n^2 - 1)\delta_{n+m} \tag{4.8}$$

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consider the subalgebra with generators[7]

$$l_n \equiv \frac{1}{K} L_{nK}, \quad n \neq 0, \qquad l_0 \equiv \frac{1}{K}(L_0 - \frac{c}{24}) + \frac{c}{24}K. \qquad (4.9)$$

It is then straightforward to see that l_n also form a Virasoro algebra with central charge $c' = cK$ and that the spectrum of l_0 has a spacing of $1/K$ compared to that of L_0.

Since the long string sector in an orbifold theory tends to dominate the entropy, the equivalence of the long string sector in the orbifold theory to the original CFT provides a natural explanation of the constancy of the entropy.

4.3   Some caveat comments

The previous discussion was all at the level of supergravity, and one may wonder how much of this survives when considering the full quantum theory. For special CFT's and special values of c, L_0 and $\bar{L}_0$ a much more precise statement can be made. For example, for the extremal BTZ black hole in the D1-D5 system, with $L_0 - \bar{L}_0 = N_p$, there are U-dualities which replace $N_1 N_5$ by $K N_1 N_5$ and N_p by N_p/K, as long as $N_1 N_5$ and N_p are coprime and K divides N_p. These U-dualities do not exist for arbitrarily large values of K since K cannot exceed N_p. To be able to take K to infinity we need to consider non-extremal and non-supersymmetric black holes and consider excitations with $L_0 = \bar{L}_0 \neq c/24$, since $L_0 - \bar{L}_0$ is quantized, but no suitable dualities that do the job for these systems are known.

It is also important to keep in mind that once $L_0, \bar{L}_0$ become comparable to $c/24$, the original BTZ metric is presumably no longer a good description of the system. For these values of the parameters there can be Gregory-Laflamme instabilities to localisation on some of the internal compact directions that we suppressed in our discussion[8], and in addition higher order terms in the space-time effective action may become important. It is therefore not at all clear that the $\epsilon \to 0$ limit we took will make sense in the full quantum theory.

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[7]Notice that the apparently strange transformation for the generator l_0 is due to the fact that we were working on the plane. Indeed, if we would have worked on the cylinder, the transformation is the expected one :

$$l_n^{\text{cyl}} \equiv \frac{1}{K} L_{nK}^{\text{cyl}}, \quad n \neq 0, \qquad l_0^{\text{cyl}} \equiv \frac{1}{K} L_0^{\text{cyl}}.$$

We now see that the transformation quoted on the plane makes sure the above cylinder transformation brings us back to the plane.

[8]For an explicit example, see [27].

5   Discussion and outlook

In this paper we have discussed two possible near horizon, low energy, limits of nearly massless BTZ black holes. If massless BTZ is viewed as a special case of extremal BTZ, in the near horizon limit one obtains a null self-dual AdS_3 orbifold which is a special case of the spacelike self-dual orbifold with zero momentum. In the dual 2d CFT, and in light of the discussions in [2], this means that the null orbifold is dual to the zero light-cone momentum $p^+ = 0$ sector of the DLCQ of the CFT. Whether the singular and ill-defined null orbifold metric will eventually shed any new light on the equally singular and ill-defined $p^+ = 0$ sector/state remains to be seen. Our point here was to establish the connection between the two. The second limit gave rise to the singular pinching AdS_3 orbifold, whose dual 2d CFT interpretation corresponds to decouple both chiral sectors of the theory leaving us with the states $L_0 = \bar{L}_0 = c/24$.

Another point we did not address so far is the fact that the $M = 0$ BTZ black hole does not describe the true ground state of the system, which is global AdS_3. One may wonder whether there is also a geometry which describes the "near-horizon" limit of geometries with $L_0 < c/24$, i.e. conical defect metrics. The natural candidate is to take (2.3) and to continue r_h^2 to negative values. This will rise to timelike orbifolds with closed timelike curves. We would normally discard such geometries as unphysical and leave a further understanding of these to future work.

We discussed that the nearly massless BTZ black hole can also be viewed as a BTZ black hole with parametrically finite mass and angular momentum over a pinching AdS_3 orbifold. The natural units to measure the inner and outer horizon radii $r_\pm$ of a BTZ black hole are the AdS_3 radius units ℓ_3. Therefore, it is intuitively expected that the smallness of the radii could be resolved if one used a different set of units. In our case 3d Newton constant G_3 is the other natural scale in the problem. In this units and in a suitable $G_3 \to 0$ limit, such that $r_\pm/G_3$ remains finite, radii will remain finite. We used this to argue the existence of a double scaling limit under which non-trivial degrees of freedom survive.

Non-extremal excitations and Rindler physics : It is natural to consider whether similar considerations would provide new insights for non-extremal BTZ black holes. In this case, there may also be different near horizon limits that one could take, but if we consider

$$r = r_+ + \epsilon\, \frac{1}{2} \frac{r_+^2 - r_-^2}{\ell_3^2\, r_+}\, y^2 \,, \quad \frac{t}{\ell_3} = \frac{r_+\, \ell_3}{r_+^2 - r_-^2}\, \tau \,, \quad \phi = \frac{r_-\, \ell_3}{r_+^2 - r_-^2}\, \tau + \frac{\sqrt{\epsilon}}{r_+}\, \psi \qquad \epsilon \to 0 \qquad (5.1)$$

the resulting metric is :

$$ds^2 \to \epsilon \left(y^2 \, d\tau^2 + dy^2 + d\psi^2 \right) \qquad \psi \sim \psi + 2\pi \, \frac{r_+}{\sqrt{\epsilon}} \, . \qquad (5.2)$$

The overall ϵ scaling can be interpreted as a limit where $\ell_p \to 0$, i.e. $\ell_p = \sqrt{\epsilon} \, \ell'_p$. Notice that in Planck units, the periodicity of the angular variable ψ is *finite*. Thus, the near horizon limit is two dimensional Rindler space times an S^1. Applying the Bekenstein-Hawking formula to the area of the Rindler horizon at $y = 0$, one is able to reproduce the entropy of the original non-extremal black hole.

The rescaling of the 3d Planck length, or equivalently, the 3d Newton constant, can perhaps once more be interpreted as passing to the long string sector of an orbifold theory, and it would be very interesting if this could be used to shed further light on a possible holographic dual description of Rindler space and hence a generic non-extremal black hole.

Relevance for the microscopics of extremal black holes : One of our motivations to consider the vanishing mass BTZ black hole was as a prototype of an extremal vanishing horizon black hole (EVH). Given the difficulties to prove/disprove the Kerr/CFT proposal, it is natural to move in the moduli space of extremal solutions to find points where local AdS$_3$ throats appear. It was already noted in [8] that such (singular) throats exist for d=5 extremal Kerr. In fact, the singular geometry reported there is an example of the embedding of our pinching orbifold in d=5. More examples of such near extremal vanishing black holes in AdS$_5$ and AdS$_4$ backgrounds were studied in [9, 14]. More recently, they have also appeared in [7, 10, 11, 12]. The analysis and conclusions reported here are therefore relevant to understand the different physics involved in the different near horizon geometries emerging in these singular situations. We will take first steps in this direction in an upcoming publication [13].

Acknowledgements

A   Near-extremal vanishing horizon limits for BTZ black holes

In this appendix, we discuss the geometries obtained as near horizon limits of (near-)extremal vanishing horizon BTZ black holes as a function of the speed at which the horizon and the non-extremality vanish. As discussed in the main text for massless BTZ, we already know of the existence of two different ways of taking these limits : one giving rise to the null self-dual orbifold and a second one giving rise to the pinching orbifold. We will show that there exist pinching orbifold versions for all vanishing horizon limits in AdS$_3$. Interestingly, we show that due to the self-dual nature of the null orbifold, the latter does not extend to arbitrary near-extremal vanishing horizon limits.

A.1   Pinching orbifolds as near horizons of near-extremal vanishing horizon BTZ

We discuss the different extremal vanishing horizon limits for the family of BTZ black holes

$$ds^2(L) = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{L^2 r^2}dt^2 + \frac{L^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2 \left(d\phi - \frac{r_+ r_-}{Lr^2}dt\right)^2 \quad (A.1)$$

in an asymptotically AdS_3 spacetime of radius of curvature L.

To discuss the different extremal vanishing horizon limits, we introduce the notation

$$r_\pm^2 = r_\star^2 \pm \delta r_\star^2 \qquad r_\star = \epsilon^\alpha \rho_\star, \quad \delta r_\star = \epsilon^\gamma \delta\rho_\star \qquad (A.2)$$

with $\gamma \geq \alpha$. Thus, α and γ control how fast the horizon and non-extremality vanish, respectively. We define the near-horizon limit as

$$\begin{aligned} r &= r_\star + \epsilon^\beta y \equiv \sigma\, \epsilon^{\min(\alpha,\beta)}, & (A.3) \\ r + r_\star &= 2r_\star + \epsilon^\beta y \equiv \hat{\sigma}\, \epsilon^{\min(\alpha,\beta)}, & (A.4) \\ t &= \epsilon^{-\beta}\tau, & (A.5) \\ \phi &= \epsilon^{-\min(\alpha,\beta)}\psi + \frac{r_+ r_-}{Lr_\star^2}t, & \epsilon \to 0. & (A.6) \end{aligned}$$

The resulting finite metric is

$$ds_{\mathrm{nh}}^2(L) = -\frac{(y^2\hat{\sigma}^2 - \epsilon^{2\kappa}\delta\rho_\star^4)}{L^2\sigma^2}d\tau^2 + \frac{L^2\sigma^2}{(y^2\hat{\sigma}^2 - \epsilon^{2\kappa}\delta\rho_\star^4)} + \sigma^2\left(d\psi + \frac{y\hat{\sigma}}{L\sigma^2}\mathcal{B}\,d\tau\right)^2 \quad (A.7)$$

where

$$\mathcal{B} = \sqrt{1 - \epsilon^{4(\gamma-\alpha)}\frac{\delta\rho_\star^4}{\rho_\star^4}}, \qquad \kappa = 2\gamma - \beta - \min(\alpha, \beta). \quad (A.8)$$

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What is important to stress is that if the horizon vanishes ($\alpha > 0$) and we consider a near horizon limit ($\beta > 0$), the resulting metric is an orbifold of AdS$_3$, and it will always be a pinched version of a known quotient given the ϵ dependent periodicity $\psi \sim \psi + 2\pi\,\epsilon^{\min(\alpha,\,\beta)}$.

When $\gamma \geq \alpha > 0$, κ, $\beta \geq 0$, one distinguishes different possibilities:

1. *Pinching Self-dual orbifold* corresponds to $\beta > \alpha > 0$, $\kappa > 0$, i.e. $\gamma > \alpha$. Thus,

$$\sigma = \frac{\hat{\sigma}}{2} = \rho_* , \quad \mathcal{B} = 1 ,$$

   and the near-horizon metric is

$$ds^2(L) = -4\frac{y^2}{L^2}d\tau^2 + \frac{L^2}{4}\frac{dy^2}{y^2} + \rho_*^2(d\psi + 2\frac{y}{L\rho_*}d\tau)^2 . \tag{A.9}$$

   The three dimensional part of (A.9) is a spacelike self-dual AdS$_3$ orbifold, except for the fact that ψ has periodicity $2\pi\epsilon^\alpha$.

2. *Pinching near self-dual orbifold* corresponds to $\beta > \alpha > 0$, $\kappa = 0$. In this case $2\gamma = \alpha + \beta$ and the metric has the same form as (A.9) except for the fact that y^2 is replaced by $y^2 - \frac{\delta\rho_*^2}{4\rho_*^2}$ in the two dimensional base spanned by $\{\tau, y\}$. These are the pinching analogues of the so-called AdS$_2$ black holes, for a recent discussion of the latter see [31, 32].

3. *Pinching Massless BTZ* corresponds to $\alpha > \beta > 0$. In this case $\kappa > 0$ and

$$\sigma = \hat{\sigma} = y, \quad \mathcal{B} = 1 ,$$

   The near-horizon metric becomes

$$\begin{aligned}
ds^2(L) &= -\frac{y^2}{L^2}d\tau^2 + L^2\frac{dy^2}{y^2} + y^2(d\psi + d\tau/L)^2 \\
&= L^2\frac{dy^2}{y^2} - y^2dx^+dx^- ,
\end{aligned} \tag{A.10}$$

   where $x^- = \psi$ and $x^+ = 2\tau/L + \psi$. This metric is that of a massless BTZ black hole (2.10), except for the fact that now $x^- \sim x^- + 2\pi\epsilon^\beta$.

4. *Pinching Extremal BTZ* corresponds to $\alpha = \beta > 0$, $\kappa > 0$. In this case $\gamma > \alpha$ and

$$\sigma = \rho_* + y, \quad \hat{\sigma} = 2\rho_* + y, \quad \mathcal{B} = 1 .$$

   The near-horizon metric is

$$ds^2(L) = -\frac{(\sigma^2 - \rho_*^2)^2}{\sigma^2 L^2}d\tau^2 + L^2\frac{\sigma^2 d\sigma^2}{(\sigma^2 - \rho_*^2)^2} + \sigma^2(d\psi + \frac{\sigma^2 - \rho_*^2}{\sigma^2}d\tau)^2 \tag{A.11}$$

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5. *Pinching Non-extremal BTZ* corresponds to $\alpha = \beta > 0$, $\kappa = 0$. In this case $\gamma = \alpha$ and

$$\sigma = \rho_* + y, \quad \hat{\sigma} = 2\rho_* + y, \quad \mathcal{B} = \left(1 - \frac{\delta\rho_*^4}{\rho_*^4}\right)^{1/2},$$

for which the near-horizon metric reduces to

$$ds^2 = -h(\sigma^2)d\tau^2 + \frac{d\sigma^2}{h(\sigma^2)} + \sigma^2(d\psi + \mathcal{B}\frac{\sigma^2 - \rho_*^2}{\sigma^2}d\tau)^2 \tag{A.12}$$

where

$$h(\sigma^2) = \frac{(\sigma^2 - \rho_*^2 - \delta\rho_*^2)(\sigma^2 - \rho_*^2 + \delta\rho_*^2)}{\sigma^2 L^2}.$$

Thus, for all allowed near-extremal vanishing horizon BTZ black holes, there exists a pinched orbifold near horizon geometry.

A.2 Null orbifold limits

Let us consider an extremal BTZ black hole

$$ds^2 = -\frac{(r^2 - r_+^2)^2}{r^2 L^2}dt^2 + \frac{L^2 r^2}{(r^2 - r_+^2)^2}dr^2 + r^2\left(d\phi - \frac{r_+^2}{r^2}\frac{dt}{L}\right)^2, \tag{A.13}$$

with horizon scaling to zero as $r_+ = \epsilon^\alpha \rho_+$ and study the near horizon limit

$$r^2 = \epsilon^{2\alpha}\rho_+^2 + \epsilon\rho, \qquad \epsilon \to 0. \tag{A.14}$$

1. If $2\alpha > 1$, this reduces to the massless BTZ calculation discussed in the main text. Thus, we do get a null-self-dual orbifold if we scale one of the lightline coordinates.

2. If $2\alpha < 1$, we complete the near horizon limit (A.14) with

$$\phi = \varphi + \frac{\tau}{\epsilon L}, \qquad t = \frac{\tau}{\epsilon}. \tag{A.15}$$

As for the vanishing horizon limit of the spacelike self-dual orbifold discussed in the main text, the quadratic piece in $d\varphi^2$ vanishes in the limit, dt^2 pieces cancel and we are only left with the cross-term $d\varphi\, d\tau$, which is the signature of the null self-dual orbifold, since $\varphi \sim \varphi + 2\pi$.

3. If $2\alpha = 1$, we also complete the near horizon limit (A.14) with

$$\phi = \varphi + \frac{\tau}{\epsilon L}, \qquad t = \frac{\tau}{\epsilon}. \tag{A.16}$$

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In this case, it is natural to work wit the new radial coordinate $\sigma = \rho_+ + \rho$. The resulting metric is a null self-dual orbifold

$$ds^2 = \frac{L^2}{4(\sigma - \rho_+^2)^2}\, d\sigma^2 + 2(\sigma - \rho_+^2)\, d\varphi\, \frac{d\tau}{L}\,. \tag{A.17}$$

It is interesting to study whether this null self-dual orbifold extends to *near-extremal* vanishing horizon limits. This point will be further discussed in [13]. Here, we will just consider the particular near horizon, vanishing horizon, scalings

$$r^2 = \epsilon\rho\,, \quad r_\pm^2 = \epsilon\rho_\pm\,, \quad \epsilon \to 0\,. \tag{A.18}$$

It can be shown that whenever $\rho_+ \neq \rho_-$, it is *not* possible to find a finite non-degenerate metric by scaling time t and shifting the angular variable ϕ while keeping its periodicity independent of ϵ to avoid the generation of any of the pinching orbifolds discussed previously. In other words, if the amount of non-extremality is large enough, the null self-dual structure emerging in the near horizon disappears.

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