Stability of cosmological solutions in F(R) Hořava-Lifshitz gravity

Diego Sáez-Gómez
Institut de Ciencies de l'Espai (ICE-CSIC/IEEC), Campus UAB,
Facultat de Ciencies, Torre C5-Par-2a pl, E-08193 Bellaterra (Barcelona), Spain

At the present paper, it is studied cosmological solutions and its stability in the frame of $F(R)$ Hořava-Lifshitz gravity. The perturbations around general spatially flat FRW solutions are analyzed and it is showed that the stability of those solutions will depend on the kind of theory, i.e. on the form of the action $F(R)$, as well as on the parameters contained in any Hořava-Lifshitz theory due to the breaking of Lorentz invariance. The (in)stability of a given cosmic solution can restrict the models and gives new observational predictions, and can give a natural explanation on the end of inflation and radiation/matter phases. An explicit example of $F(R)$ is studied, and it is showed that the instability can produce the transition between the different epochs of the Universe history.

PACS numbers: 04.50.Kd,98.80.-k,11.10.Wx

I. INTRODUCTION

Since the observational data suggested that our Universe is nowadays expanding with acceleration, a lot of models have been proposed to explain such a phenomena, which can not be explained in the frame of General Relativity (GR) if no new terms or fields are incorporated. The main candidate for dark energy is the so-called ΛCDM model, which by means of a cosmological constant, late-time acceleration can be achieved. Nevertheless, as the cosmological constant contains several unresolved problems, like the fine tuning problem, other different proposals have been seriously considered, such that scalar or vector fields (quintessence, phantom,..) or modifications of GR as the so called $F(R)$ gravity (see Refs. [1] for reviews on unification of inflation and dark energy in modified gravity). At the same time, it is also accepted that during the early times of the Universe, there was an epoch of accelerated expansion, known as inflation, what has suggested that both epochs could be unified under the same mechanism. In the frame of $F(R)$ gravity, the unification of both epochs is easily achieved and it gives a natural explanation in terms purely of gravity (see Refs. [1–5]).

Recently a new theory of gravity that claims to be power counting renormalizable has been suggested in Ref. [6]. This new theory, already known as Hořava-Lifshitz gravity, is not Lorentz invariant, what makes it to be renormalizable, but it gives consequently a lot of problems. However, it is conjectured that the Lorentz invariance is recovered in the IR limit (see Ref. [7]). Some aspects of cosmology has already been studied in the frame of this new theory (see Ref. [8]). Nevertheless, as in the frame of General Relativity, it can not explain dark energy epoch as well as inflation without new terms or fields, remaining unresolved such a problem. An extension of the standard $F(R)$ gravity to Hořava-Lifshitz theory has been performed (see Refs. [9–12]), which seems to be also renormalizable, and it can reproduce late-time acceleration with no need of any other kind of fields or cosmological constant (see Refs. [9, 10]). Even the unification of dark energy epoch with inflation can be performed in this new class of theories, and the so-called $F(R)$ viable models, which avoid violations of the local gravity tests, can be easily extended to Hořava-Lifshitz gravity (see Ref. [11]).

At the current paper, it is studied in the frame of F(R) Hořava-Lifshitz gravity, general cosmological solutions of the type of spatially flat Friedmann-Robertson-Walker (FRW) solutions, and its stability. We specially focus on the study of stability of radiation/matter dominated eras, where the Universe expands following a power law, and de Sitter solutions, which can well model the accelerated epochs. We explore space independent perturbations around these solutions, studying the effects of the extra terms introduced in the field equations by considering $F(R)$ gravity. Also the new parameters included in the theory, due to the breaking of the Lorentz invariance, could affect to a given cosmic solution. The (in)stability of a cosmological solution gives very important information as the possible exit from one phase of the cosmological history, constraints on the kind of action $F(R)$ and future observational predictions. It is given an explicit example of an $F(R)$ action, where a de Sitter solution is found to be instable, what can give a grateful exit from inflation and it produces an instability during the matter dominated epoch at large times, making a phase transition.

The paper is organized as follows. In the next section, $F(R)$ Hořava-Lifshitz gravity is briefly introduced, and the cosmological equations are obtained. Sect. III is devoted to the analysis of general spatially flat FRW solutions and the perturbations around those solutions, calculating the general equation for the perturbations in the linear approach. Then, as an example it is studied the class of solutions described by a scale factor that depends on a power of time, which is the class of solutions described by radiation or matter in the context of GR. In Appendix A, de Sitter solutions are also studied in detail. In Sect. IV an explicit example of $F(R)$ is studied, where is analyzed one of the so-called viable models, which can unify dark energy and inflationary epochs. Finnally some discussions and conclusions are provided in the last section..

II. FRAMEWORK

In this section, modified Hořava-Lifshitz $F(R)$ gravity is briefly reviewed [9–12]. We start by writing a general metric in the so-called ADM decomposition in a $3+1$ spacetime (for more details see [13], [14] and references therein),

$$ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,, \tag{1}$$

where $i, j = 1, 2, 3$, N is the so-called lapse variable, and N^i is the shift 3-vector. In standard general relativity (GR), the Ricci scalar can be written in terms of this metric, and yields

$$R = K_{ij}K^{ij} - K^2 + R^{(3)} + 2\nabla_\mu(n^\mu \nabla_\nu n^\nu - n^\nu \nabla_\nu n^\mu)\,, \tag{2}$$

here $K = g^{ij}K_{ij}$, K_{ij} is the extrinsic curvature, $R^{(3)}$ is the spatial scalar curvature, and n^μ a unit vector perpendicular to a hypersurface of constant time. The extrinsic curvature K_{ij} is defined as

$$K_{ij} = \frac{1}{2N}\left(\dot{g}^{(3)}_{ij} - \nabla^{(3)}_i N_j - \nabla^{(3)}_j N_i\right)\,. \tag{3}$$

In the original model [6], the lapse variable N is taken to be just time-dependent, so that the projectability condition holds and by using the foliation-preserving diffeomorphisms (6), it can be fixed to be $N = 1$. As pointed out in [15], imposing the projectability condition may cause problems with Newton's law in the Hořava gravity. On the other hand, Hamiltonian analysis shows that the non-projectable $F(R)$-model is inconsistent[16]). For the non-projectable case, the Newton law could be restored (while keeping stability) by the "healthy" extension of the original Hořava gravity of Ref. [15].

The action for standard $F(R)$ gravity can be written as

$$S = \int d^4x \sqrt{g^{(3)}} N F(R)\,. \tag{4}$$

Gravity of Ref. [6] is assumed to have different scaling properties of the space and time coordinates

$$x^i = bx^i\,, \quad t = b^z t\,, \tag{5}$$

where z is a dynamical critical exponent that renders the theory renormalizable for $z = 3$ in $3+1$ spacetime dimensions [6] (For a proposal of covariant renormalizable gravity with dynamical Lorentz symmetry breaking, see [17]). GR is recovered when $z = 1$. The scaling properties (5) render the theory invariant only under the so-called foliation-preserving diffeomorphisms:

$$\delta x^i = \zeta(x^i, t)\,, \quad \delta t = f(t)\,. \tag{6}$$

It has been pointed that, in the IR limit, the full diffeomorphisms are recovered, although the mechanism for this transition is not physically clear. The action considered here was introduced in Ref. [9],

$$S = \frac{1}{2\kappa^2}\int dt d^3x \sqrt{g^{(3)}} N F(\tilde{R})\,, \quad \tilde{R} = K_{ij}K^{ij} - \lambda K^2 + R^{(3)} + 2\mu\nabla_\mu(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu) - L^{(3)}(g^{(3)}_{ij})\,, \tag{7}$$

where κ is the dimensionless gravitational coupling, and where, two new constants λ and μ appear, which account for the violation of the full diffeomorphism transformations. A degenerate version of the above $F(R)$-theory with $\mu = 0$ has been proposed and studied in Ref. [12]. Note that in the original Hořava gravity theory [6], the third term in the expression for $\tilde{R}$ can be omitted, as it becomes a total derivative. The term $L^{(3)}(g^{(3)}_{ij})$ is chosen to be [6]

$$L^{(3)}(g^{(3)}_{ij}) = E^{ij}G_{ijkl}E^{kl}\,, \tag{8}$$

where G_{ijkl} is the generalized De Witt metric, namely

$$G^{ijkl} = \frac{1}{2}\left(g^{ik}g^{jl} + g^{il}g^{jk}\right)\,. \tag{9}$$

In Ref. [6], the expression for E_{ij} is constructed to satisfy the "detailed balance principle" in order to restrict the number of free parameters of the theory. This is defined through variation of an action

$$\sqrt{g^{(3)}}E^{ij} = \frac{\delta W[g_{kl}]}{\delta g_{ij}} , \tag{10}$$

where the form of $W[g_{kl}]$ is given in Ref. [18] for $z = 2$ and $z = 3$. Other forms for $L^{(3)}(g^{(3)}_{ij})$ have been suggested that abandons the detailed balance condition but still render the theory power-counting renormalizable (see Ref. [10]).

We are interested in the study of (accelerating) cosmological solutions for the theory described by action (7). Spatially-flat FRW metric is assumed

$$ds^2 = -N^2dt^2 + a^2(t)\sum_{i=1}^{3}\left(dx^i\right)^2 . \tag{11}$$

If we also assume the projectability condition, N can be taken to be just time-dependent and, by using the foliation-preserving diffeomorphisms (6), it can be fixed to be unity, $N = 1$. When we do not assume the projectability condition, N depends on both the time and spatial coordinates, first. Then, just as an assumption of the solution, N is taken to be unity.

For the metric (11), the scalar $\tilde{R}$ is given by

$$\tilde{R} = \frac{3(1 - 3\lambda + 6\mu)H^2}{N^2} + \frac{6\mu}{N}\frac{d}{dt}\left(\frac{H}{N}\right) . \tag{12}$$

For the action (7), and assuming the FRW metric (12), the second FRW equation can be obtained by varying the action with respect to the spatial metric $g^{(3)}_{ij}$, which yields

$$0 = F(\tilde{R}) - 2(1 - 3\lambda + 3\mu)\left(\dot{H} + 3H^2\right)F'(\tilde{R}) - 2(1 - 3\lambda)\dot{\tilde{R}}F''(\tilde{R}) + 2\mu\left(\dot{\tilde{R}}^2F^{(3)}(\tilde{R}) + \ddot{\tilde{R}}F''(\tilde{R})\right) + \kappa^2p_m , \tag{13}$$

here $\kappa^2 = 16\pi G$, p_m is the pressure of a perfect fluid that fills the Universe, and $N = 1$. Note that this equation becomes the usual second FRW equation for convenient $F(R)$ gravity (4), by setting the constants $\lambda = \mu = 1$. When we assume the projectability condition, variation over N of the action (7) yields the following global constraint

$$0 = \int d^3x\left[F(\tilde{R}) - 6(1 - 3\lambda + 3\mu)H^2 - 6\mu\dot{H} + 6\mu H\dot{\tilde{R}}F''(\tilde{R}) - \kappa^2\rho_m\right] . \tag{14}$$

Now, using the ordinary conservation equation for the matter fluid $\dot{\rho}_m + 3H(\rho_m + p_m) = 0$, and integrating Eq. (13),

$$0 = F(\tilde{R}) - 6\left[(1 - 3\lambda + 3\mu)H^2 + \mu\dot{H}\right]F'(\tilde{R}) + 6\mu H\dot{\tilde{R}}F''(\tilde{R}) - \kappa^2\rho_m - \frac{C}{a^3} , \tag{15}$$

where C is an integration constant, taken to be zero, according to the constraint equation (14). If we do not assume the projectability condition, we can directly obtain (15), which corresponds to the first FRW equation, by variation over N. Hence, starting from a given $F(\tilde{R})$ function, and solving Eqs. (13) and (14), a cosmological solution can be obtained.

III. COSMOLOGICAL SOLUTIONS AND ITS STABILITY IN $F(\tilde{R})$ GRAVITY

In this section, we are interested to study the stability of general cosmological solutions in the frame of $F(\tilde{R})$ Hořava-Lifshitz gravity, with special attention to those cosmological solutions that model the history of the Universe, as de Sitter or power law solutions. It is well known that in standard F(R) gravity, any cosmological solution can be reproduced by reconstructing the function of the Ricci scalar $F(R)$ (see Ref. [5]). As it was showed in [11], dark energy and even the unification with the inflationary epoch can be reproduced in this new frame of $F(\tilde{R})$ Hořava-Lifshitz theories. The stability of those solutions plays a crucial role in order to get the transition from one cosmological phase to another.

A. Stability of general flat FRW cosmological solutions

Let us start by studying a general spatially flat FRW metric (11). We will focus below specially on de Sitter and power law solutions of the type $a(t) \propto t^m$ as dark energy epoch and the radiation/matter dominated eras are governed by this class of cosmological solutions respectively, so that the implications of the extra geometrical terms coming from $F(\tilde{R})$ could be determinant for the stability and transition during those epochs. We assume a general solution,

$$H(t) = h(t) \ . \tag{16}$$

Then, the scalar curvature $\tilde{R}$ yields,

$$\tilde{R}_h(t) = 3(1-3\lambda+6\mu)h^2(t) + 6\mu\dot{h}(t) \ . \tag{17}$$

Assuming that solution (16) is satisfied by a particular choice of $F(\tilde{R})$, the FRW equation (15) has to be fulfilled,

$$0 = F(\tilde{R}_h) - 6\left[(1-3\lambda+3\mu)h^2 + \mu\dot{h}\right]F'(\tilde{R}_h) + 6\mu h\dot{\tilde{R}}_h F''(\tilde{R}_h) - \kappa^2\rho_m \ , \tag{18}$$

where the matter fluid is taken to be a perfect fluid with equation of state $p_m = w_m\rho_m$, with w_m constant. By the energy conservation equation $\dot{\rho}_m + 3h(1+w_m)\rho_m = 0$, the evolution of the matter energy density can be expressed in terms of the given solution $h(t)$ as,

$$\rho_{mh} = \rho_0 \mathrm{e}^{-3(1+w_m)\int h(t)dt} \ . \tag{19}$$

being ρ_0 an integration constant. We are interested to study the perturbations around the solution $h(t)$. For that, we expand in powers of $\tilde{R}$, the function $F(\tilde{R})$ around $\tilde{R}_h$ given by (17), what yields,

$$F(\tilde{R}) = F_h + F'_h(\tilde{R}-\tilde{R}_h) + \frac{F''_h}{2}(\tilde{R}-\tilde{R}_h)^2 + \frac{F^{(3)}_h}{6}(\tilde{R}-\tilde{R}_h)^3 + O(\tilde{R}-\tilde{R}_h)^4 \ , \tag{20}$$

where the derivatives of the function $F(\tilde{R})$ are evaluated in R_h given by (17). Note that there will be also matter perturbations, which will contribute to the stability of the solution, inducing a mode on the perturbation. We can write the perturbations as,

$$H(t) = h(t) + \delta(t) \ , \quad \rho_m \simeq \rho_{mh}(1+\delta_m(t)) \ . \tag{21}$$

Hence, by introducing the above quantities in the FRW equation, the perturbation equation in the linear approach yields,

$$\ddot{\delta} + b\dot{\delta} + \omega^2\delta = \frac{\kappa^2\rho_{mh}}{36\mu^2 hF''_h}\delta_m \ , \tag{22}$$

where,

$$b = -\frac{h'}{h} - \frac{1-3\lambda+3\mu}{\mu}h + \frac{1-3\lambda+6\mu}{\mu} + 6\left((1-3\lambda+6\mu)h\dot{h} + \mu\ddot{h}\right)\frac{F^{(3)}_h}{F''_h} \ ,$$

$$\omega^2 = \left[1-3\lambda+6\mu-2(1-3\lambda+3\mu)h\right]\frac{F'_h}{6\mu^2 hF''_h}$$

$$+ (1-3\lambda+6\mu)\left(\frac{-1+3\lambda-3\mu}{\mu^2}h + \frac{-1+h}{\mu h}\dot{h}\right) + \frac{\ddot{h}}{h} + 6(1-3\lambda+6\mu)\left(\frac{1-3\lambda+6\mu}{\mu}\dot{h}h + \ddot{h}\right)\frac{F^{(3)}_h}{F''_h} \ . \tag{23}$$

In this case the solution for $\delta(t)$ can be splitted in two branches, one corresponding to the homogeneous part of the equation (22), whose solution will depend on the background theory, i.e. on $F(\tilde{R})$ and its derivatives, and the other one corresponding to the particular solution of the eq. (22), which represent the solution induced by the matter perturbation, δ_m. Then, the complete solution can be written as,

$$\delta(t) = \delta_{homg}(t) + \delta_{inh}(t) \ . \tag{24}$$

We are interested on the perturbations induced by the function $F(\tilde{R})$ and its derivatives, such that we focus on the homogeneous solution, δ_{homg}. By a first qualitative analysis, we can see that the homogeneous part of equation (22) yields exponential or damped oscillating perturbations. The form of the perturbations will depend completely on the form of the function $F(\tilde{R})$ and its derivatives evaluated on R_h. Nevertheless, we could assume some constraints to obtain some qualitative information. Let us consider the cases,

- The trivial case given by $F'_h = F''_h = F^{(3)}_h = 0$ makes the perturbations tend to zero $\delta(t) = 0$, and any cosmic solution will be stable.
- For $F'_h \neq 0$ and $F''_h, F^{(3)}_h \to 0$, in (23) the term which becomes important is,

$$\omega^2 \sim [1 - 3\lambda + 6\mu - 2(1 - 3\lambda + 3\mu)h] \frac{F'_h}{6\mu^2 h F''_h} \ . \tag{25}$$

  And depending on the model $F(\tilde{R})$ and the solution $h(t)$, the stability can be easily studied.
- For $F'_h, F''_h \to 0$ but $F^{(3)}_h \neq 0$, looking at (23), we can see that the perturbations will depend on the value of the last term in the coefficients A and B, which yield

$$b \sim 6 \left((1 - 3\lambda + 6\mu)h\dot{h} + \mu\ddot{h} \right) \frac{F^{(3)}_h}{F''_h} \ , \quad \omega^2 \sim 6(1 - 3\lambda + 6\mu) \left(\frac{1 - 3\lambda + 6\mu}{\mu} \dot{h}h + \ddot{h} \right) \frac{F^{(3)}_h}{F''_h} \ . \tag{26}$$

  And the stability will depend on the sign of these terms, being stable when both of them are greater than zero, what gives a damped oscillating perturbation that decays.

However, in general it can be showed that the equation (22) can not be analytically resolved for arbitrary solutions and actions, and numerical analysis is needed. Nevertheless, by imposing some restrictions on $F(\tilde{R})$ as above, some qualitative information can be obtained. In order to study this question deeper, some specific solutions $h(t)$ are studied below as well as an explicit example of $F(\tilde{R})$.

B. Stability of radiation/matter eras: Power law solutions

At this section it is considered an important class of cosmological solutions, the power law solutions described by,

$$H(t) = \frac{m}{t} \quad \to \quad a(t) \propto t^m \ . \tag{27}$$

In the context of General Relativity, this class of solutions are generated by a perfect fluid with EoS parameter $w = -1 + \frac{2}{3m}$, such that the matter/radiation dominated epochs can be approximately described by this class of solutions (27). Let us study the stability for this solution, and how the inclusion of extra terms in the action as well as the new parameters included in the theory (λ, μ) may affect the solution (27). As in the above section, the perturbation equation (22) can not be in general analytically resolved, although under some restrictions, we can obtain some qualitative information on the stability of the solution. If we assumed that $F(\tilde{R})$ does not deviate from the linear action during radiation/matter dominated epochs, its derivatives can be restricted to be neglected $F''_h, F^{(3)}_h \sim 0$ (as they should become important just during dark energy epoch or inflation), and in such a case, the coefficient in front of $\delta(t)$ in the eq. (22)can be approximated as,

$$\omega^2 \sim [1 - 3\lambda + 6\mu - 2(1 - 3\lambda + 3\mu)h(t)] \frac{F'_h}{6\mu^2 h(t) F''_h} \ . \tag{28}$$

As it can be seen the value of the frequency depends on the time, such that the stability may change along the phase described by the solution (27). Then, for small t, we have that $\omega^2 \sim -2(1 - 3\lambda + 3\mu)\frac{6\mu^2 F'_h}{F''_h}$ and if it is assumed $\lambda \sim \mu$, the perturbations will grow exponentially when $\frac{F'_h}{F''_h} > 0$, and the solution becomes unstable. While for large t, the frequency can be approximated as $\omega^2 \sim (1 - 3\lambda + 6\mu)\frac{F'_h}{6\mu^2 h(t) F''_h}$ and the instability will be large if $\frac{F'_h}{F''_h} < 0$, producing a phase transition.

IV. EXAMPLE OF A VIABLE $F(\tilde{R})$ MODEL

Let us consider an explicit model of $F(\tilde{R})$ gravity in order to apply the analysis about the stability performed above. We are interested to study here the stability of radiation/matter dominated eras as well as de Sitter solutions. The model considered here was proposed in Ref. [2], studied in Ref. [3] in the context of standard gravity and generalized to Hořava-Lifshitz gravity in Ref. [11]. The action is defined as,

$$F(\tilde{R}) = \chi\tilde{R} + \frac{\tilde{R}^n(\alpha\tilde{R}^n - \beta)}{1+\gamma\tilde{R}^n}\,, \tag{29}$$

where $(\chi, \alpha, \beta, \gamma)$ is a set of constant parameters of the theory. In Ref. [2], it has been showed that this model can well reproduced late-time acceleration with no need of a cosmological constant or any kind of exotic field as well as inflation, unifying under the same mechanism both accelerated epochs of the Universe history. For simplicity here it is assumed $n = 2$. The radiation/matter dominated epochs, which can be described by the class of solutions given in (27), the second term in (29) could play an essential role in the exit of such phase. Then, it is interesting to study the possible effect of those extra geometrical terms during those epochs in order to produce the transition to the accelerated era. By assuming the solution (27), and following the steps described in the above section, the stability will be affected by the derivatives of the function (29) evaluated in $h(t) = m/t$. As the interesting point comes for large times, the derivatives if $F(\tilde{R})$ can be approximated in the limit for large time t as,

$$F'_h \to \chi\ , \quad F''_h \to -2\beta\ , \quad F^{(3)}_h \to 0\ . \tag{30}$$

Here, we are assuming that $0 < \beta << 1$, such that by the analysis performed above, we can conclude that the linear perturbations will grow exponentially, and the radiation/matter dominated phase becomes unstable for large times producing the transition to another different phase. Then, the above function can explain perfectly the end of matter dominated epoch with no need to have a cosmological constant.

Let us now study the stability of de Sitter solutions. For more details on de Sitter solutions and its stability see Appendix A. It is known that the above model (29) may contain several de Sitter solutions (see Ref. [11] and [3]), which are the solutions of the algebraic equation (A1), which yields,

$$\tilde{R}_0 + \frac{\tilde{R}_0^n(\alpha\tilde{R}_0^n - \beta)}{1+\gamma\tilde{R}_0^n} + \frac{6H_0^2(-1+3\lambda-3\mu)\left[1 + n\alpha\gamma\tilde{R}_0^{3n-1} + \tilde{R}_0^{n-1}(2\gamma\tilde{R}_0 - n\beta) + \tilde{R}_0^{2n-1}(\gamma^2\tilde{R}_0 + 2n\alpha)\right]}{(1+\gamma\tilde{R}_0^n)^2} = 0\,. \tag{31}$$

This equation has to be resolved numerically, even for the simple case studied here $n = 2$. Nevertheless, one of the de Sitter points from the model (29) can be obtained from the minimum of the second term in (29) assuming the constraint on the parameters $\beta\gamma/\alpha \gg 1$, and it gives the de Sitter point,

$$\tilde{R}_0 \sim \left(\frac{\beta}{\alpha\gamma}\right)^{1/4}\ , \qquad F'(\tilde{R}_0) = \chi\,, \qquad F(\tilde{R}_0) = \tilde{R}_0 - 2\Lambda\ , \quad \text{where} \quad \Lambda \sim \frac{\beta}{2\gamma}\,. \tag{32}$$

Then, by evaluating the derivatives of (29) around $\tilde{R}_0$ and by the equation (A6) the perturbation δ can be calculated. Note that the stability condition for a de Sitter solution given by $\frac{F'_0}{F''_0} > 12H_0^2$ (see Appendix A), is not satisfied for this case, such that the de Sitter point (32) will not be stable. By resolving eq. (A6), the perturbations are exponential functions on the time t,

$$\delta(t) = C_1 \mathrm{e}^{a_+ t} + C_2 \mathrm{e}^{a_- t}\ , \quad \text{with} \quad a_\pm = \frac{H_0(1-3\lambda+3\mu)}{2\mu}\ . \tag{33}$$

Hence, the model (29) will be unstable around this de Sitter point, what predicts the exit from the accelerated phase in the near future of such solution, what can give a natural explanation on the end of the inflationary epoch. Although, as it has been pointed above, the theory described by (29) contains more than one different de Sitter points, given by the roots of equation (31), which may be stable.

V. DISCUSSIONS

Here we have analyzed spatially flat FRW cosmology for a non linear Hořava-Lifshitz gravity , basically an extension of the so-called standard $F(R)$ gravity as it can be seen by the FRW equations, which in the IR limit, where the parameters (λ, μ) are supposed to be reduced to the unity, the standard FRW equations for $F(R)$ gravity are recovered. The stability of this general class of solutions has been studied and it is showed that it depends mainly by the choice of the function $F(R)$ and in part on the values of the parameters (λ, μ). At late times the curvature scalar is very

small, and the main effect on the perturbation of a cosmological solution comes from the value of the derivatives of $F(R)$. It is shown that in general, the perturbation equation can not be resolved analytically, even in the linear approach. Nevertheless, under some restrictions, it yields important information, and the (in)stability of the different phases of the Universe history can be studied. It is showed that for some values of the derivatives of $F(R)$, a given solution becomes (in)stable, what gives an important restriction on the models. By analyzing an explicit example in Sect. IV, an $F(R)$ function of the class of viable models, we have showed that this kind of theories can well explain the end of matter dominated epoch, and reproduces late time acceleration. We have found that for this example, it exists a de Sitter point that becomes instable, what predicts the end of such accelerated epoch, giving a possible natural explanation on the end of inflationary epoch. However, as such action contains more de Sitter solutions, they may become stable. Then, a deeper analysis on the phase space has to be done, in order to connect the different regions that the Universe passes with this model.
Hence, the analysis performed here gives a general approach for the study of spatially flat FRW solutions in the frame of higher order Hořava-Lifshitz theories, constraining the class of functions $F(R)$ allowed by the observations, and giving a natural explanation on the end of inflation and matter dominated epochs.

Acknowledgments

I would like to thank Emilio Elizalde and Sergei Odintsov for giving me support to perform this task and useful discussions. I acknowledge an FPI fellowship from MICINN (Spain), project FIS2006-02842.

Appendix A: de Sitter solutions in $F(\tilde{R})$ gravity

Let us consider one of the simplest but most important solution in cosmology, de Sitter (dS) solution. As dark energy and even inflation can be modeled (in its simplest form) by dS solution, its stability becomes very important, specially in the case of inflation, where a grateful exit is needed to finish with such superaccelerated phase of the early Universe. In general, standard $F(R)$ gravity contains several de Sitter points, which represent critical points (see [4]). The analysis can be extended to $F(\tilde{R})$ Hořava-Lifshitz gravity, where the de Sitter solution $H(t) = H_0$, with H_0 being a constant, has to satisfy the first equation FRW equation (15),

$$0 = F(\tilde{R}_0) - 6H_0^2(1 - 3\lambda + 3\mu)F'(\tilde{R}_0)\,, \tag{A1}$$

where we have taken $C = 0$ and assumed absence of any kind of matter. The scalar $\tilde{R}$ is given in this case by,

$$\tilde{R}_0 = 3(1 - 3\lambda + 6\mu)H_0^2\ . \tag{A2}$$

Then, the positive roots of equation (A1) are the de Sitter points allowed by a particular choice of an $F(\tilde{R})$ function. Assuming de Sitter solution, we can write $F(\tilde{R})$ around $\tilde{R}_0$ as a series of powers of the scalar $\tilde{R}$,

$$F(\tilde{R}) = F_0 + F_0'(\tilde{R} - \tilde{R}_0) + \frac{F_0''}{2}(\tilde{R} - \tilde{R}_0)^2 + \frac{F_0^{(3)}}{6}(\tilde{R} - \tilde{R}_0)^3 + O(\tilde{R}^4)\ . \tag{A3}$$

Here, the primes denote derivative respect $\tilde{R}$ while the subscript 0 means that the function $F(\tilde{R})$ and its derivatives are evaluated in $\tilde{R}_0$. Then, we can perturb the solution writing the Hubble parameter as,

$$H(t) = H_0 + \delta(t)\ . \tag{A4}$$

Using the function $F(\tilde{R})$ evaluated around a given dS solution (A3), and the perturbed solution (A4) in the first FRW equation (15), the equation for the perturbation yields,

$$0 = \frac{1}{2}F_0 - 3H_0^2(1 - 3\lambda + 3\mu) - 3H_0\left[\left((1 - 3\lambda)F_0' + 6F_0''H_0^2(-1 + 3\lambda - 6\mu)(-1 + 3\lambda - 3\mu)\right)\delta(t)\right.$$
$$\left. +6F_0''\mu H_0(-1 + 3\lambda - 3\mu)\dot{\delta}(t) - 12F_0''\mu^2\ddot{\delta}(t)\right]\ . \tag{A5}$$

Here we have taken the linear approach on δ and its derivatives. Note that the first two terms in the equation (A5) can be dropped because of the equation (A1), which is assumed to be satisfied, and equation (A5) can be written in a more convenient way as,

$$\ddot{\delta}(t)+\frac{H_0(1-3\lambda+9\mu)}{2\mu}\dot{\delta}(t)+\frac{1}{12\mu^2}\left[(3\lambda-1)\frac{F_0'}{F_0''}-6H_0^2(1-3\lambda+6\mu)(1-3\lambda+3\mu)\right]\delta(t)=0\,. \tag{A6}$$

Then, the perturbations on the dS solution will depend completely on the model, specifically on the derivatives of the $F(\tilde{R})$ function, as well as on the parameters of the theory (λ,μ). Note that the instability will be large if the term in front of $\delta(t)$ is negative, as the perturbations will increase exponentially, while if we have a positive frequency, the perturbations will behave as a damped harmonic oscillator. During dark energy epoch, as the scalar curvature is very small, the IR limit of the theory can be assumed, where GR is recovered, and in such a case we have $\lambda=\mu\sim 1$, and the frequency ω^2 will depend completely on the value of $\frac{F_0'}{F_0''}$. In order to avoid large instabilities during the dark energy phase, the condition $\frac{F_0'}{F_0''}>12H_0^2$ has to be imposed. Nevertheless, during the inflationary epoch, where the scalar curvature is large, the IR limit is not a convenient approach, and the perturbations will depend also on the values of (λ,μ). Although if we assume a very small F_0'', the first term in front of the frequency ω^2 in the equation (A6) will dominate and if $\lambda>1/3$, the stability of the solution will depend on the sign of $\frac{F_0'}{F_0''}$, being stable when such a coefficient is positive.

---

[1] S. Nojiri, S. D. Odintsov, arXiv:1011.0544 [gr-qc], arXiv:1008.4275 [hep-th], eConf **C0602061**, 06 (2006) [Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007)] [arXiv:hep-th/0601213];
S. Capozziello, M. De Laurentis and V. Faraoni, arXiv:0909.4672 [gr-qc].

[2] S. Nojiri and S. D. Odintsov, Phys. Rev. D **77**, 026007 (2008) [arXiv:0710.1738 [hep-th]].

[3] E. Elizalde and D. Sáez-Gómez, Phys. Rev. D **80**, 044030 (2009) [arXiv:0903.2732 [hep-th]].

[4] G. Cognola, E. Elizalde, S. D. Odintsov, P. Tretyakov, and S Zerbini, Phys. Rev. D **79**, 044001 (2009) [arXiv:0810.4989 [gr-qc]].

[5] S. Nojiri, S. D. Odintsov, D. Sáez-Gómez, Phys. Lett. B **681**, 74 (2009) [arXiv:0908.1269 [hep-th]].

[6] P. Horava, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].

[7] P. Horava, Phys. Rev. D **82**, 064027 (2010) [arXiv:1007.2410 [hep-th]].

[8] T. Takahashi and J. Soda, Phys. Rev. Lett. **102**, 231301 (2009) [arXiv:0904.0554 [hep-th]];
E. Kiritsis and G. Kofinas, Nucl. Phys. B **821**, 467 (2009) [arXiv:0904.1334 [hep-th]];
R. Brandenberger, Phys. Rev. D **80**, 043516 (2009) [arXiv:0904.2835 [hep-th]];
S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, Phys. Lett. B **679**, 6 (2009) [arXiv:0905.0055 [hep-th]];
T. P. Sotiriou, M. Visser and S. Weinfurtner, JHEP **0910**, 033 (2009) [arXiv:0905.2798 [hep-th]];
E. N. Saridakis, Eur. Phys. J. C **67**, 229 (2010) [arXiv:0905.3532 [hep-th]];
M. Minamitsuji, Phys. Lett. B **684**, 194 (2010) [arXiv:0905.3892 [astro-ph.CO]];
G. Calcagni, Phys. Rev. D **81**, 044006 (2010) [arXiv:0905.3740 [hep-th]];
A. Wang and Y. Wu, JCAP **0907**, 012 (2009) [arXiv:0905.4117 [hep-th]];
M. i. Park, JHEP **0909**, 123 (2009) [arXiv:0905.4480 [hep-th]];
S. Nojiri and S. D. Odintsov, Phys. Rev. D **81**, 043001 (2010) [arXiv:0905.4213 [hep-th]];
M. Jamil, E. N. Saridakis and M. R. Setare, Phys. Lett. B **679**, 172 (2009) [arXiv:0906.2847 [hep-th]];
M. i. Park, JCAP **1001**, 001 (2010) [arXiv:0906.4275 [hep-th]];
C. Bogdanos and E. N. Saridakis, Class. Quant. Grav. **27**, 075005 (2010) [arXiv:0907.1636 [hep-th]];
C. G. Boehmer and F. S. N. Lobo, arXiv:0909.3986 [gr-qc];
I. Bakas, F. Bourliot, D. Lust and M. Petropoulos, Class. Quant. Grav. **27**, 045013 (2010) [arXiv:0911.2665 [hep-th]];
G. Calcagni, JHEP **0909**, 112 (2009) [arXiv:0904.0829 [hep-th]];
S. Carloni, E. Elizalde and P. J. Silva, Class. Quant. Grav. **27**, 045004 (2010) [arXiv:0909.2219 [hep-th]];
X. Gao, Y. Wang, W. Xue and R. Brandenberger, JCAP **1002**, 020 (2010) [arXiv:0911.3196 [hep-th]];
Y. S. Myung, Y. W. Kim, W. S. Son and Y. J. Park, arXiv:0911.2525 [gr-qc];
E. J. Son and W. Kim, arXiv:1003.3055 [hep-th];
A. Wang, arXiv:1003.5152 [hep-th];
A. Ali, S. Dutta, E. N. Saridakis and A. A. Sen, arXiv:1004.2474 [astro-ph.CO];
S. Mukohyama, arXiv:1007.5199 [hep-th].

[9] M. Chaichian, S. Nojiri, S. D. Odintsov, M. Oksanen and A. Tureanu, Class. Quantum Grav. **27**, 185021 (2010) [arXiv:1001.4102 [hep-th]];

[10] S. Carloni, M. Chaichian, S. Nojiri, S. D. Odintsov, M. Oksanen and A. Tureanu, Phys. Rev. D **82**, 065020 (2010) [arXiv:1003.3925 [hep-th]];

[11] E. Elizalde, S. Nojiri, S. D. Odintsov and D. Sáez-Gómez, *Unifying inflation with dark energy in modfied $F(R)$ Hořava-Lifshitz gravity*, arXiv:1006.3387
[12] J. Kluson, Phys. Rev. D **81**, 064028 (2010) [arXiv:0910.5852 [hep-th]];
J. Kluson, arXiv:1002.4859 [hep-th].
[13] R. L. Arnowitt, S. Deser and C. W. Misner, arxiv:gr-qc/0405109; C. Gao, Phys. Lett. B **684**, 85 (2010) [arXiv:0905.0310 [astro-ph.CO]].
[14] C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, W. H. Freeman and Company, 1973, San Francisco.
[15] D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. **104**, 181302 (2010) [arXiv:0909.3525 [hep-th]].
[16] M. Chaichian, M. Oksanen, A. Tureanu, arXiv:1006.3235 [hep-th].
[17] S. Nojiri and S. D. Odintsov, Phys. Lett. B **691**, 60 (2010) [arXiv:1004.3613 [hep-th]].
[18] P. Horava, JHEP **0903**, 020 (2009) [arXiv:0812.4287 [hep-th]].