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Hypercharge Flux, Exotics, and Anomaly Cancellation in F-theory GUTs

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We sharpen constraints related to hypercharge flux in F-theory GUTs that possess $U(1)$ symmetries and argue that they arise as a consequence of 4-dimensional anomaly cancellation. This gives a physical explanation for restrictions that were observed in spectral cover models while demonstrating that those restrictions are not tied to any particular formalism.

1 Introduction

Most approaches to F-theory GUTs [1–5] make crucial use of two important ingredients. The first is the presence of $U(1)$ symmetries, which typically originate from some global E_8 structure that has been broken down to $SU(5)_{\text{GUT}}$. Symmetries of this type can be used to protect against proton decay [6–10] as well as to motivate certain scenarios for how supersymmetry breaking is mediated to the Standard Model [11, 12]. The second important ingredient is "hypercharge flux", which provides an elegant mechanism for breaking the GUT group while addressing the doublet-triplet splitting problem [3, 4]. In explicit constructions based on spectral cover techniques [13], these two ingredients appear to be interrelated [7, 9]; models with a particular set of $U(1)$ symmetries in that setting exhibit tight constraints on how "hypercharge flux" can be distributed among the matter curves where charged fields localize [7].

The goal of this note is to understand the nature and source of these constraints. This issue is particularly pertinent in light of the recent paper [14], which appeared while the present work and that of [15] were in progress. The authors of [14] point out that methods used to construct F-theory GUTs in the current literature may be too restrictive. One therefore wonders whether the constraints that have been seen so far carry an intrinsic physical meaning or represent artifacts of a particular formalism. In this note, we sharpen the constraints following a crucial observation of Dudas and Palti [16] and suggest a physical origin for them in terms of 4-dimensional anomaly cancellation. We believe that this argument clarifies the physics of all restrictions that have appeared in spectral cover constructions and demonstrates their applicability to F-theory GUT models in general. Among the implications for phenomenology, our argument implies the existence of charged exotics in a certain class of phenomenologically-motivated F-theory GUT models that combine the flavor scenario of [17–19] with "hypercharge flux" and the existence of a $U(1)_{PQ}$ symmetry.

2 Dudas-Palti Relations and their Consequences

In an interesting recent paper, Dudas and Palti [16] noticed a simple pattern in the distribution of "hypercharge flux" in a set of spectral cover models. It is not hard to prove this relation for generic models built from spectral covers and we do this in the upcoming paper [15]. More intriguing, however, is that despite its initial formulation in the spectral cover language [16], the observation of Dudas and Palti can be written in a manner that does not make any explicit reference to spectral covers at all. To do this, consider an $SU(5)$ F-theory GUT model with an extra $U(1)$ symmetry and let q_X denote the common charge of $\mathbf{10}$ or $\overline{\mathbf{5}}$ fields on a matter curve $\Sigma^{(x)}$. In that case, the Dudas-Palti (DP) observation can be written as the simple statement that

$$\sum_{\mathbf{10} \text{ matter curves, } a} q_a \int_{\Sigma^{(a)}_{\mathbf{10}}} F_Y = \sum_{\overline{\mathbf{5}} \text{ matter curves, } i} q_i \int_{\Sigma^{(i)}_{\overline{\mathbf{5}}}} F_Y \tag{2.1}$$

where F_Y is a "hypercharge flux" that is chosen to ensure that the $U(1)_Y$ gauge boson remains massless. A relation this simple should have a physical origin and, in the present note, we will argue that it is a consequence of 4-dimensional anomaly cancellation. Before addressing this, however, let us make a few important remarks.

First, one might wonder if (2.1) encodes all of the constraints on the distribution of "hypercharge flux". There is, of course, another set of relations that encode the cancellation of 4-dimensional gauge anomalies of the MSSM groups[1]

$$\begin{aligned} 0 &= 5[c_1] + 3 \sum_{\mathbf{10} \text{ matter curves,} i} [\Sigma^{(i)}_{\mathbf{10}}] - \sum_{\overline{\mathbf{5}} \text{ matter curves,} a} [\Sigma^{(a)}_{\overline{\mathbf{5}}}] \\ 0 &= \sum_{\overline{\mathbf{5}} \text{ matter curves,} i} \int_{\Sigma^{(i)}_{\overline{\mathbf{5}}}} F_Y \\ 0 &= \sum_{\mathbf{10} \text{ matter curves,} a} \int_{\Sigma^{(a)}_{\mathbf{10}}} F_Y \end{aligned} \tag{2.2}$$

These have been known for quite a while [4, 13, 20]. The first was derived using a "stringy" anomaly cancellation argument [13] and can also be understood from a 4-dimensional point of view as encoding cancellation of the $SU(3)^3$ anomaly in the presence of an internal "hypercharge flux". The others restrict the way that "hypercharge flux" is distributed along the matter curves and are always satisfied when F_Y is constructed from a $(1,1)$-form ω_Y that is globally trivial [3,4]. We suspect that (2.1) and (2.2) represent the only constraints because it appears that one can use spectral covers to construct, at least in principle[2], all distributions of "hypercharge flux" that satisfy them in that setting [15].

In light of this, we should correct some misstatements that were made in [7]. There, it was claimed that the presence of "hypercharge flux" on $\overline{\mathbf{5}}$ matter curves automatically implied that "hypercharge flux" must thread some **10** matter curves as well. The DP relations (2.1) do not forbid a configuration in which "hypercharge flux" threads only $\overline{\mathbf{5}}$ curves, though, and it is possible to construct spectral covers that do precisely this [15].

[1] Here, $[c_1]$ is the homology class of the anti-canonical curve of the GUT divisor.

[2] By this, we mean that it is possible to construct suitable spectral covers modulo a few assumptions that must be checked on a case-by-case basis. In the course of building a model, one must introduce new objects that are holomorphic sections of a given set of bundles. One must always make sure that all of these bundles truly admit holomorphic sections and this will depend on the choice of GUT divisor and normal bundle.

Finally, let us comment on implications of the DP relations (2.1) for F-theory model building. To realize a very attractive approach to flavor hierarchies [17–19][3], one would like to engineer all three generations of the **10** on one matter curve and all three generations of the $\overline{\mathbf{5}}$ on a second matter curve. The Higgs fields then lie on distinct matter curves, $\Sigma_{\overline{\mathbf{5}}}^{(H_u)}$ and $\Sigma_{\overline{\mathbf{5}}}^{(H_d)}$, which carry +1 and -1 units of "hypercharge flux", respectively, in order to lift the triplets [3,4]. Crucial to this scenario is that "hypercharge flux" not be allowed to thread any curve Σ other than $\Sigma_{\overline{\mathbf{5}}}^{(H_u)}$ and $\Sigma_{\overline{\mathbf{5}}}^{(H_d)}$; if it did, one would obtain massless matter fields on Σ that do not comprise a complete GUT multiplet. As one assumes that the standard model fields are engineered as complete GUT multiplets, the threading of "hypercharge flux" through such a Σ will necessarily introduce new charged exotics into the spectrum [7].

If we wish to combine this scenario with a $U(1)$ symmetry, the DP relations (2.1) imply that the charges q_{H_u} and q_{H_d} associated to the matter curves $\overline{\mathbf{5}}^{(H_u)}$ and $\overline{\mathbf{5}}^{(H_d)}$ must satisfy

$$q_{H_u} - q_{H_d} = 0 \tag{2.3}$$

We must be careful, though, because the doublet H_u comes from a **5** rather than a $\overline{\mathbf{5}}$. This means that its charge is actually $-q_{H_u}$ so, writing (2.3) in terms of the actual H_u and H_d charges we get

$$Q(H_u) + Q(H_d) = 0 \tag{2.4}$$

What type of $U(1)$ symmetry can this be? Because all **10**'s ($\overline{\mathbf{5}}$'s) are engineered on a single curve, all of them must carry a common charge. The only $U(1)$ symmetry of this type that commutes with $SU(5)$, satisfies (2.4), and preserves the MSSM superpotential is the famous $U(1)_\chi$, which is the linear combination of $U(1)_Y$ and $U(1)_{B-L}$ that enters naturally in $SO(10)$ unification models. We see that PQ symmetries, broadly defined as $U(1)$'s for which (2.4) does not hold, cannot be combined with the desired distribution of hypercharge flux. This means that if we insist on realizing all 3 generations of **10**'s ($\overline{\mathbf{5}}$'s) on a single matter curve, the presence of $U(1)_{PQ}$ implies the existence of additional charged matter fields that do not come in complete GUT multiplets in accord with a claim of [7].

3 Dudas-Palti Relations from Anomaly Cancellation

Let us now turn to the physical origin of the DP relations (2.1). To define "hypercharge flux", we utilize a $(1,1)$-form ω_Y that satisfies a special condition: it is nontrivial on the GUT divisor but trivializes in the bulk 3-fold that comprises the base of our elliptically fibered Calabi-Yau [3,4]. This condition is important because it guarantees that the hypercharge

[3] Alternative approaches to flavor include [21–25].

gauge boson will remain massless when our "hypercharge flux" is turned on. At a more practical level, though, it affects the integrals of ω_Y over the matter curves of our geometry in a way that ensures the cancellation of MSSM gauge anomalies.

We would like to ask if such an ω_Y exhibits additional properties in a geometry that engineers some bulk $U(1)$ symmetries in addition to $SU(5)_{\rm GUT}$[4]. To investigate this, let us use ω_Y to construct a flux that is purely in the $U(1)_Y$ direction and consider what happens when we turn on this flux *and nothing else.* Our flux will induce a nontrivial spectrum but, by construction, it cannot give rise to any gauge anomalies[5]. Of particular interest to us are mixed anomalies with insertions of both MSSM and $U(1)$ currents as these get contributions only from chiral fields that localize on matter curves in the GUT divisor. We will show that the condition (2.1) simply expresses a set of nontrivial relations that the $(1,1)$-form ω_Y must satisfy in order for these 4-dimensional mixed gauge anomalies to cancel.

Before proceeding, though, it is important to distinguish the flux that we are turning on here from what is usually referred to as "hypercharge flux" in the literature. The latter represents a flux that, for instance, couples only to the doublets in a $\overline{\mathbf{5}}$ but not to the triplets. As such, it is really best thought of as a combination of a pure $U(1)_Y$ flux and a flux for some $U(1)$'s from the bulk [4]. Here we want to be very careful that our flux, which we are using as a sort of probe to study the properties of ω_Y, lies purely along the $U(1)_Y$ direction.

To make things completely explicit, we use ω_Y to define a line bundle $\mathcal{L}_Y$ on the GUT 7-branes that defines a nontrivial $U(1)_Y$ background. We normalize that background so that all fields of the MSSM are sections of the integer quantized gauge bundles listed below

$SU(5)$	$SU(3)\times SU(2)\times U(1)_Y$	Bundle
$\mathbf{10}$	$(\mathbf{1},\mathbf{1})_{+1}$	$\mathcal{L}_Y^6$
	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\mathcal{L}_Y$
	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	$\mathcal{L}_Y^{-4}$
$\overline{\mathbf{5}}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\mathcal{L}_Y^2$
	$(\mathbf{1},\mathbf{2})_{-1/2}$	$\mathcal{L}_Y^{-3}$

(3.1)

[4]Engineering global $U(1)$'s of this type can be very subtle [26] but recent results indicate how this can be done in practice [27,28].

[5]A subtlety arises here because some F-theory compactifications cannot be globally consistent unless a bulk G-flux is added, for instances to satisfy the quantization condition [29]. Even though such a G-flux will typically induce an anomalous spectrum with respect to our $U(1)$, we claim that any "$U(1)_Y$-dependent" contributions to the anomalies must vanish in the sense that the total anomalies do not change when the $U(1)_Y$ flux is scaled by an integer N. To see this, recall that $U(1)$ gauge anomalies in the 4-dimensional theory are cancelled by an exchange of fields $\hat{C}_0/\hat{C}_2$ that descend from the RR 4-form C_4 and couple as $\int d^4x\,\left(\hat{C}_2\wedge F+\hat{C}_0\wedge F_{MSSM}\wedge F_{MSSM}+\ldots\right)$. The fields $\hat{C}_0$ and $\hat{C}_2$ are related by the self-duality relation of C_4 and their 4-dimensional couplings descend from the bulk interaction $\int C_4\wedge G\wedge G$. Because ω_Y is globally trivial, it does not play any role in the emergence of these couplings from dimensional reduction so they must be independent of the rescaling N, along with any anomalies that they cancel.

We now determine the contributions to mixed gauge anomalies that arise from the chiral spectrum on a generic $\mathbf{10}$ or $\overline{\mathbf{5}}$ matter curve. To obtain (2.1) it will be sufficient to consider anomalies of the type $G_{SM}^2 \times U(1)$, where G_{SM} denotes a Standard Model gauge group.

Consider first the contribution from fields that localize on a $\mathbf{10}$ curve, $\Sigma_{\mathbf{10}}^{(a)}$, which carry a $U(1)$ charge q_a. Denoting the $U(1)_Y$ flux there by N_a

$$N_a = \int_{\Sigma_{\mathbf{10}}^{(a)}} c_1(\mathcal{L}) = \int_{\Sigma_{\mathbf{10}}^{(a)}} \omega_Y \tag{3.2}$$

we find the following contributions to mixed $G_{SM}^2 \times U(1)$ anomalies

Multiplet	Chirality	$SU(3)^2 \times U(1)$	$SU(2)^2 \times U(1)$	$U(1)_Y^2 \times U(1)$
$(\mathbf{1},\mathbf{1})_{+1}$	$6N_a$	0	0	$6q_aN_a$
$(\mathbf{3},\mathbf{2})_{+1/6}$	N_a	$2q_aN_a$	$3q_aN_a$	$q_aN_a/6$
$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	$-4N_a$	$-4q_aN_a$	0	$-16q_aN_a/3$
Total		$-2q_aN_a$	$3q_aN_a$	$5q_aN_a/6$

(3.3)

Note that a negative chirality means that we obtain zero modes of the conjugate multiplet, which carry an opposite $U(1)$ charge. We now do the same thing for fields on a $\overline{\mathbf{5}}^{(i)}$ curve that carry $U(1)$ charge q_i. Letting N_i denote the $U(1)_Y$ flux

$$N_i = \int_{\Sigma_{\overline{\mathbf{5}}^{(i)}}} c_1(\mathcal{L}) = \int_{\Sigma_{\overline{\mathbf{5}}^{(i)}}} \omega_Y \tag{3.4}$$

we find

Multiplet	Chirality	$SU(3)^2 \times U(1)$	$SU(2)^2 \times U(1)$	$U(1)_Y^2 \times U(1)$
$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$2N_i$	$2q_iN_i$	0	$2q_iN_i/3$
$(\mathbf{1},\mathbf{2})_{-1/2}$	$-3N_i$	0	$-3q_iN_i$	$-3q_iN_i/2$
Total		$2q_iN_i$	$-3q_iN_i$	$-5q_iN_i/6$

(3.5)

From this, we see that cancellation of all of $G_{SM}^2 \times U(1)$ anomalies implies ω_Y must satisfy

$$\sum_{\mathbf{10}\text{ matter curves },a} q_a \int_{\Sigma_{\mathbf{10}}^{(a)}} \omega_Y = \sum_{\overline{\mathbf{5}}\text{ matter curves },i} q_i \int_{\Sigma_{\overline{\mathbf{5}}}^{(i)}} \omega_Y \tag{3.6}$$

When ω_Y is used to construct conventional "hypercharge flux" in F-theory GUT models, this is nothing other than the DP relations (2.1). It is easy to check that pure MSSM anomalies cancel provided (2.2) holds while other mixed anomalies, as well as the $U(1)^3$ anomaly, vanish without giving rise to any additional constraints. Our analysis here has been very specialized, focusing only on gauge anomalies in models with an $SU(5)$ gauge group. Even though the story seems less constrained than in 6-dimensions [30–32], it would nevertheless be very interesting to study the implications of anomaly cancellation more generally in 4-dimensional F-theory GUT models.

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