From Torsion for Spinors to Weak Forces for Leptons

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Abstract

We consider a geometric approach to field theory in which torsion is present beside gravity and also electrodynamics for the matter field equations, and we develop the consequences of the torsion-spin coupling for the spinor fields; in particular we focus on the specific interactions arising among fermions: we show that these interactions have the structure of the weak interactions acting among leptons. We discuss the implications for the standard model of the fundamental interactions between elementary fields in the perspective of the unification in particle physics.

Introduction

In the most general tensorial dynamics in the spacetime, the most general tensorial connection is not symmetric in the two lower indices and when the connection is assumed to be metric compatible it possesses both torsional and metric degrees of freedom; in the most general spinorial dynamics in the spacetime, the most general spinorial connection for which the gamma matrices are constant is not only given by the spinorial connection obtained from the tetrad form of the tensorial connection but also by an abelian gauge field: in this way torsion is accounted beside gravity within the most general tensorial connection, and they are both accounted beside electrodynamics within the most general spinorial connection, as discussed in [1]. In following this reasoning it is clear that not only torsion and gravitation but also electrodynamics actually have a geometric character. However, the fact that these fields have the same character does not mean that they have the same features; in fact on the one hand we have that within the connection, the metric enters in terms of the ordinary derivatives and so their vanishing depends on the spacetime frame we are employing, the gauge field enters algebraically but still its vanishing depends on the gauge frame we are using, but torsion never vanishes according to any choice of frame: as a consequence of these properties, we have a version of the principle of equivalence for gravitation and electrodynamics, and torsion can be separated away from the torsionless connection, within the most general connection. Therefore when in the most general covariant derivative the differential part is separated from torsion, it is possible to describe gravitation as the effect of the curvature of the spacetime metric and electrodynamics as the effect of the curvature of the gauge field while torsion provides additional constraints through a non-linear potential. This non-linear potential provides effective mass term and autointeraction for the fermion; of course in the case of many fermions it provides not only effective mass term and autointeraction for each fermion but also mutual interactions among all fermions involved: further these reciprocal mutual interactions are responsible for fermionic repulsive forces between two fermion fields, as it has been discussed in references [2] and [3] and in [4] and [5]. So it is clear how this most general dynamics gives actually rise to repulsive forces for any couple of fermion; then these repulsive forces taking place for any couple of fermions must be present in particular for a pair of leptons, and this would lead to their straightforward identification with the weak interactions for leptons themselves.

On the other hand however, the Fermi interactions have since long been recognized to be low-energy limit of the weak interactions that arise after their separation from the electrodynamic interactions following the symmetry breaking in the Weinberg-Salam standard model, and so in the standard model the weak interactions have to be considered as fundamental. The basic properties of the standard model are that the breakdown of a symmetry is needed to separate the weak interactions from the electrodynamic interactions by assigning mass to the former and leaving massless the latter; the problem with this mechanism is three-fold: from a theoretical point of view, the mass generation does not only occur for the mediators of the weak interaction but also for some of the matter fields that are subject to this force, and this is a theoretical issue, because the mass is assigned to the mediators of the weak interaction by transferring to them an additional degree of freedom whereas the matter fields already have all the degrees of freedom needed; from a phenomenological point of view, the generation of the masses of the massive fields is accompanied by the generation of a cosmological constant given by $4\Lambda=-\lambda^{2}v^{4}$, and this is a phenomenological issue, because it gives a negative value of the cosmological constant; from an experimental point of view, the generation of the mass of the massive fields and the cosmological constant of the universe is achieved through the introduction of an additional scalar field, and this is an experimental issue, because this scalar Higgs boson has never been observed. To be fair it is clear that the fact that the Higgs boson has not been discovered does not mean that it will never be discovered, the fact that we have obtained a negative cosmological constant does not imply that we will never be able to find another mechanism providing a positive cosmological constant so that after their compensation is accomplished the result will be a positive cosmological constant, and the fact that there is a discrimination in the way in which matter and mediators get their messes may be solved by fermiophobic-Higgs modifications of the standard model; but nevertheless it is not unthinkable that such fermiophobic-Higgs modifications are impossible, or that another mechanism of cosmological constant generation is not definable, or that the Higgs itself does not exist, and so it is important to find models in which these problems do not even appear. Obviously the most direct way in which these problems are avoided altogether is to look for Higgsless models in which the cosmological constant and the masses of the matter fields are already present in the model; it is not a problem under a theoretical viewpoint to build such models because after all both the cosmological constant and the masses of the matter fields may be thought as integration constants once the system of field equations is obtained by integration from the conservation laws given by the Jacobi-Bianchi identities: the problem is rather that the mediators of the weak interactions would be massive and the mediator of the electrodynamic interaction is massless and so unification would be impossible. Now, that the weak and electrodynamic interactions are kept separated is actually not that bad since their being structured in the $SU(2)\times U(1)$ group does not really unify them in the first place and furthermore because in our prescription the electrodynamic potential is accommodated into the most general spinorial connection already; the problem would rather be where to accommodate the weak interactions, and this may be not a problem as well because in our prescription if it is true that the most general spinorial connection maybe be decomposed into the electrodynamic potential and the most general tensorial connection, it is also true that the most general tensorial connection is decomposable in terms of the gravitational potentials and torsion: and the torsional potential would then be the only candidate in which the weak interactions find place in a natural way.

To see if this is possible and in this case how it can be done, a necessary condition is that once the field equations are written down the torsional potentials have to be rearranged in such a way that the weak interaction phenomenology is reproduced; in this paper we will show that this is indeed possible by finding the way in which within the matter field equations the torsional potentials are formally equivalent to the weak interaction terms.

1 Torsion for Spinors

To begin with we write the most general matter field equations with torsional potentials for the spinor fields as

$$i\gamma^{\mu}D_{\mu}e-m_{e}e=0 \tag{1}$$

$$i\gamma^{\mu}D_{\mu}\nu=0 \tag{2}$$

where the electron e has mass m_e and charge q_e and the neutrino ν is massless and neutral as usual. Notice that the weak interactions have been left out.

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In these matter field equations the most general covariant derivatives D_{μ} has torsional contributions that can be separated away leaving the simplest covariant derivatives ∇_{μ} as in the torsionless case but with additional torsional contributions in terms of potentials for the spinors as

$$i\gamma^{\mu}\nabla_{\mu}e-m_{e}e-\tfrac{3}{16}\overline{e}\gamma_{\mu}e\gamma^{\mu}e-\tfrac{3}{16}\overline{\nu}\gamma_{\mu}\nu\gamma^{\mu}\gamma e=0 \tag{3}$$

$$i\gamma^{\mu}\nabla_{\mu}\nu-\tfrac{3}{16}\overline{e}\gamma_{\mu}\gamma e\gamma^{\mu}\nu=0 \tag{4}$$

in which the spinorial potentials represent mass terms or autointeraction of the spinor with itself and interactions of each spinor with all the others. These extra interactions have now to be written in the form of the weak interactions.

By employing geometrical identities such as Fierz rearrangements it is possible to see that the torsional potentials for the spinor fields can be rearranged in a form with the structure of the weak interactions terms as

$$i\gamma^{\mu}\nabla_{\mu}e-m_{e}e-\tfrac{3}{8}(\cos\theta)^{2}\overline{e}\gamma_{\mu}e\gamma^{\mu}e+q_{e}\tan\theta Z_{\mu}\gamma^{\mu}e- \\ -\tfrac{g}{2\cos\theta}Z_{\mu}\gamma^{\mu}e_{L}+\tfrac{g}{\sqrt{2}}W^{*}_{\mu}\gamma^{\mu}\nu=0 \tag{5}$$

$$i\gamma^{\mu}\nabla_{\mu}\nu+\tfrac{g}{2\cos\theta}Z_{\mu}\gamma^{\mu}\nu+\tfrac{g}{\sqrt{2}}W_{\mu}\gamma^{\mu}e_{L}=0 \tag{6}$$

in which there has been a shift in the coupling of the autointeraction of the spinor with itself and the interactions of each spinor with all the others have been written in the form of the weak interactions as we wanted.

3 Weak Forces for Leptons

Now to see that this the spinor field equations we have obtained are the field equations of the standard model we need to define

$$Z_{\mu}=\frac{3\cos\theta}{8g(\sin\theta)^{2}}\left[\frac{1}{2}\left(\overline{e}_{L}\gamma^{\nu}e_{L}-\overline{\nu}\gamma^{\nu}\nu\right)-(\sin\theta)^{2}\overline{e}\gamma_{\mu}e\right] \tag{7}$$

$$W_{\mu}=\frac{3\sqrt{2}\left[1-4(\sin\theta)^{2}\right]}{32g(\sin\theta)^{2}}\left(\overline{e}_{L}\gamma^{\nu}\nu\right) \tag{8}$$

so that it is in terms of the electron and neutrino that the mediators of the weak interactions are built: notice that according to this procedure, since there is no possible mixing between the electron and the neutrino it follows that there is no gauge transformation for the mediators of the weak interactions as well. The lack of mixing between electron and neutrino is due to the fact that the first is massive and the last is massless and similarly the lack of gauge transformations for the mediators of the weak interactions with the photon is due to the fact that the former are massive and the latter is massless: to see that this is actually the case it is enough to mention the fact that both Z^2 and W^2 are negative so that these two fields have the 3 degrees of freedom that correspond to massive vector particles; moreover it is possible to see that they are partially conserved axial currents verifying

$$\nabla_{\mu}Z^{\mu}=-\frac{3\cot\theta}{16}\frac{m_e}{q_e}\left(i\overline{e}\gamma e\right) \tag{9}$$

$$\nabla_{\mu}W^{\mu}=-\frac{3\sqrt{2}\left[1-4(\sin\theta)^{2}\right]}{32\sin\theta}\frac{m_e}{q_e}\left(i\overline{e}\gamma\nu\right) \tag{10}$$

as soon as the system of field equations is considered. Notice that to maintain the correct sign on the right then the θ angle is to be smaller than $\frac{\pi}{6}$ radiants compatible with the Weinberg angle constraints that are known experimentally.

Conclusion

In what we have done so far, we have proven that it is possible to consider the field equations for the spinors and see that the spinorial interactions can be written in the form of the weak interactions; this form is obtained by taking the electron and neutrino field to construct the weak vector bosons and as a consequence of this fact the vector bosons already have the amount of degrees of freedom they need to be massive: so the Higgs field is not necessary.

In this way then, a Higssless standard model has been constructed: this circumstance carries a conceptual problem given by the fact that the electrodynamic interactions and the weak forces are not separated apart from a unified gauge interaction of fundamental features by breaking its symmeytry because no such symmetry ever existed at all; what we have here is rather the situation for which the electrodynamic interactions and the weak forces are essentially different things, the former being accommodated into the most general spinorial connection as a gauge interaction of fundamental character while the latter arising as an effect of the torsion present in the spinor field equations. Under this point of view then, we have that the issue of unification is not solved but circumvented: in fact it is not by breaking a symmetry but by never assuming that symmetry at all that we can explain the asymmetry we observe; that is electrodynamics and weak forces are not different interactions that were once unified into a fundamental interaction but they are different interactions with electrodynamics having that fundamental essence while the weak forces being the result of the more general torsional dynamics for the fermionic fields.

This is nevertheless some sort of unification, because electrodynamics as a gauge field and torsion as what takes place within the connection in an inevitable way are such that only when they are both included can the connection be the most general connection for the covariant derivatives of the matter fields.

Now, the effective problem this approach may face is the smallness of the effects due to the spin coupling: limits are usually very stringent, as discussed in references [6], [7] and [8]; however, the constraining bounds are placed in terms of vacuum large scale models that do not apply to the context of the present discussion. On the other hand, we have that the non-linearities in the matter field equations are supposed to be manifest at weak scales: that these effects may be relevant at these energies due to an energy-dependent coupling is not a new idea, and references can be found for instance in [3] and [5]; nevertheless, we do not know yet whether this energy-dependent coupling is a viable mechanism as such. Therefore an extensive further research about the issue of the energy scales at which these effects should become evident has to be done.

References

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