A note on the vector Schwinger Model with the photon mass term: Gauge invariant reformulation, operator solution and path integral formulation

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Abstract

Vector Schwinger model is reinvestigated with the mass like term for gauge field. Phase space structure has been determined in this situation. It has been found that mass of the gauge boson acquires a generalized expression with the bare coupling constant and the parameters involved in the masslike terms for the gauge field which is in contrast with the result of a recent publication on this issue.

QED in $(1+1)$ dimension, e.g., Schwinger model [1] is a very interesting field theoretical model. It has been widely studied over the years by several authors in connection with the mass generation, confinement aspect of fermion (quark), charge shielding etc. [2, 3, 4, 5]. Here massless fermion interact with the Abelian gauge field. Photon acquires mass via a kind of dynamical symmetry breaking and the quarks disappear from the physical spectra. The exactly solvable nature of the model leads to express this model in terms of canonical boson field. It is a remarkable feature of (1+1) dimensional exactly solvable fermion field field. A new gauge non invariant regularized version of this model has been proposed in [6]. Here we find that a one parameter class of regularization commonly used to study the chiral Schwinger model has been introduced in the vector Schwinger model. For a specific choice, i.e., for the vanishing value of the parameter the model reduces to the usual vector Schwinger model but for the other admissible value of this parameter the phase space structure as well as the the physical spectra gets altered remarkably. This new regularization leads to a change in the confinement scenario of the quark too. In fact, the quarks gets liberated as it was happened in the Chiral Schwinger model [7, 8, 9, 10, 11].

Recently, we find that the Schwinger model is studied adding masslike term for gauge field with the lagrangian at the classical level [12, 13]. This model is structurally equivalent to the model studied in [6]. The masslike term for gauge gauge field are introduced in the two models with different perspective. In [6], masslike term occurred as a one loop correction in order to remove the divergence of the fermionic determinant appeared during bosonization whereas in [12, 13], the author studied the Schwinger model with the masslike term for gauge field at the classical level. In [6], the authors had reasonably fair motivation to introduce the masslike term since it is known that in QED, a regularization gets involved when one calculates the effective action by integrating the fermions out. The ambiguity in the regularization has been exploited by different authors in different times in (1+1) dimensional QED and Chiral QED and different interesting scenarios have been resulted in [7, 8, 9, 10, 11, 14, 15, 16, 17]. The most remarkable one is the chiral Schwinger model studied by Jackiw and Rajaraman [7]. They saved the long suffering of the chiral generation of the Schwinger model due to Hagen [18] from the non-unitary problem introducing a one parameter class of regularization. In [19], the authors studied the Schwinger model introducing masslike term for the gauge field at the classical level and showed that for a particular value of the ambiguity parameter the lost gauge invariance of the so called nonconfining (anomalous) Schwinger model [6] gets restored. In [12, 13], however the authors presented a surprising and untrustworthy result adding the same masslike term at the classical level. There we find that the mass generated for the boson is $m = \sqrt{2}e$ and it does not contain the parameter involved within the masslike term of the gauge field. If we look into the the work [6], a structurally equivalent model to [12], we find that the theoretical spectrum contains a massive and a massless boson and the mass of the massive boson acquires a generalized expression with the ambiguity parameter. A massless boson is there indeed in [12, 13]. It may happen so if the added term works as gauge fixing. However the added term was not a legitimate gauge fixing term in [12, 13]. A question therefore, automatically comes how did the authors get such untrustworthy result in [12, 13]? The model is thus reinvestigated in this note.

The vector Schwinger model is described by the following generating functional.

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x \mathcal{L}_f} \tag{1}$$

with

$$\mathcal{L}_f = \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu]\psi \tag{2}$$

The effective bosonized lagrangian density obtained by integrating out the the fermion one by is

$$\mathcal{L}_B = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - e\epsilon_{\mu\nu}\partial^\nu\phi A^\mu. \tag{3}$$

If electromagnetic background is introduced with masslike term for the gauge field the model reads

$$\mathcal{L}_B = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - e\epsilon_{\mu\nu}\partial^\nu\phi A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_0 A_\mu A^\mu. \tag{4}$$

Here Lorentz indices runs over the two values 0 and 1 corresponding to the two space time dimension and the rest of the notation are standard. The antisymmetric tensor is defined with the convention $\epsilon_{01} = +1$. The coupling constant e has one mass dimension in this situation. The parameter m_0 is introduced to represent the masslike term for the gauge field at the classical level in the same way as it was done in the Thirring-wess model [20]. In [12, 13] the authors used $m_0 = ae^2$. Here we would like to mention that the authors in [12] used a regularization parameter M in the generalized bosonized lagrangian and set it to zero along with few others parameter to get vector Schwinger mode. Then again the author added a mass like term for the gauge field $\mathcal{L}_m = \frac{1}{2}ae^2A_\mu A^\mu$ and termed this a again as standard regularization parameter. It is highly confusing at this level.

Let us now proceed to study the phase space structure of the model. To this end it is necessary to calculate the the momenta corresponding to the field A_0, A_1 and ϕ. From the standard definition of the momentum we obtain

$$\pi_0 = 0, \tag{5}$$

$$\pi_1 = F_{01}, \tag{6}$$

$$\pi_\phi = \dot{\phi} - eA_1, \tag{7}$$

where π_0, π_1 and π_ϕ are the momenta corresponding to the field A_0, A_1 and ϕ. Using the equations (5), (6) and (7), the Hamiltonian density are calculated:

$$\mathcal{H} = \frac{1}{2}(\pi_\phi + eA_1)^2 + \frac{1}{2}\pi_1^2 + \frac{1}{2}\phi'^2 + \pi_1 A_0' - eA_0\phi' - \frac{1}{2}m_0(A_0^2 - A_1^2). \tag{8}$$

Note that $\omega = \pi_0 \approx 0$, is the familiar primary constraint of the theory. The preservation of the constraint ω requires $[\omega(x), H(y)] = 0$, which leads to the Gauss' law as a secondary constraint:

$$\tilde{\omega} = \pi_1' + e\phi' + m_0 A_1 \approx 0. \tag{9}$$

The constraints (5) and (9) form a second class set. Treating (5) and (9) as strong condition one can eliminate A_0 and obtain the reduced Hamiltonian density as follows.

$$\mathcal{H}_r = \frac{1}{2}(\pi_\phi + eA_1)^2 + \frac{1}{2m_0}(\pi_1' + e\phi')^2 + \frac{1}{2}({\pi_1}^2 + \phi'^2)^2 + \frac{1}{2}m_0 A_1^2. \tag{10}$$

According to the Dirac's prescription [21] of quantizing a theory with second class constraint the Poisson brackets becomes inadequate for this situation. This type of systems however remain consistent with the Dirac brackets [21]. It is straightforward to show that the Dirac brackets between the fields describing the reduced hamiltonian (10) remain canonical. Using the canonical

Dirac brackets the following first order equations of motion are found out from the reduced Hamiltonian density (10).

$$\dot{A}_1 = \pi_1 - \frac{1}{m_0}(\pi_1'' + e\phi''), \tag{11}$$

$$\dot{\phi} = \pi_\phi + eA_1, \tag{12}$$

$$\dot{\pi}_\phi = (1 + \frac{e^2}{m_0})\phi'' + \frac{e}{m_0}\pi_1'', \tag{13}$$

$$\dot{\pi}_1 = -e\pi_\phi - (m_0 + e^2)A_1. \tag{14}$$

Note that in [12], all the equations of motion except the equation of motion corresponding to equation (13) were identical. In [12], the calculational mistake started from that erroneous equation of motion. A little algebra converts the above first order equations (11), (12), (13) and (14) into the following second order equations:

$$(\Box + (m_0 + e^2)\pi_1 = 0, \tag{15}$$

$$\Box[\pi_1 + \frac{e}{m_0 + e^2}\phi] = 0. \tag{16}$$

Equation (15) describes a massive boson field with mass $m = \sqrt{m_0 + e^2}$ whereas equation (16) describes a massless scalar field. The result clearly shows that the mass acquires a generalized expression with the parameter involved in the masslike term at the classical level as it can be expected from the result of the paper [6]. This boson can be identified with the photon that acquired mass via a dynamical symmetry breaking. The massless boson (16) is equivalent to the massless fermion in (1+1) dimension. So fermion here does not confine. It remains free as it has been found in chiral Schwinger model [7, 8, 9, 10, 11] and the so called nonconfining Schwinger model [6, 22]. In this context we should mention that there is some confusion in the literature regarding the conclusion concerning confinement and de-confinement scenario of fermion but the result in this context is considered to be more or less standard now [2, 6, 11, 14, 15, 22]. The nature of the theoretical spectrum becomes more transparent if we calculate the fermionic propagator to which we now tern.

To calculate fermion propagator one needs to work with the original fermionic model. The calculation is analogous to the so called nonconfining Schwinger model [6]. The same calculaton for chiral Schwinger model is available in [9, 10]. The effective action obtained by integrating out ϕ from the bosonized action (4) is

$$S_{eff} = \int d^2x \frac{1}{2}[A_\mu(x) M^{\mu\nu} A_\nu(x)], \tag{17}$$

where,

$$M^{\mu\nu} = m_0 g^{\mu\nu} - \frac{\Box + e^2}{\Box} \tilde{\partial}^\mu \tilde{\partial}^\nu. \tag{18}$$

Here we have used the standard notation $\tilde{\partial}^\mu = \epsilon^{\mu\nu}\partial_\nu$. The gauge field propagator is just the inverse of $M^{\mu\nu}$ and it is found to be

$$\Delta_{\mu\nu}(x-y) = \frac{1}{m_0}[g_{\mu\nu} + \frac{\Box + e^2}{\Box(\Box + (m_0 + e^2))} \tilde{\partial}_\mu \tilde{\partial}_\nu] \delta(x-y). \tag{19}$$

Note that the two poles of propagator are found at the expected positions. One at zero and another at $m_0 + e^2$ indicating a massive and a massless excitations.

Setting an Ansatz, for the Green function of the Dirac operator $(i\partial\!\!\!/ - gA\!\!\!/)$, enable us to construct the propagator of the original fermion ψ. The conventional construction of the Ansatz is

$$G(x, y; A) = e^{ie(\Phi(x) - \Phi(y))} S_F(x-y), \tag{20}$$

where S_F is the free, massless fermion propagator and Φ is determined when the Ansatz (20) is plugged into the equation for the Green function. From the standard construction the Green function can be written down as

$$G(x, y; A) = e^{ie \int d^2z A^\mu(z) J_\mu(z)} S_F(x-y), \tag{21}$$

where the *current* J_μ has the following expression.

$$J_\mu = (\partial^z_\mu + \gamma_5 \tilde{\partial}^z_\mu)(D_F(z-x) - D_F(z-y)). \tag{22}$$

Here D_F is the propagator of a massless free scalar field. Such propagators have to be infra-red regularized in two dimensions [2]:

$$D_F(x) = -\frac{i}{4\pi} ln(-\mu^2 x^2 + i0), \tag{23}$$

where μ is the infra-red regulator mass.

Finally we obtain the fermion propagator by functionally integrating $G(x,y;A)$ over the gauge field:

$$\begin{aligned} S'_F &= \int \mathcal{D}Ae^{\frac{i}{2}\int d^2z\ (A_\mu(z)M^{\mu\nu}A_\nu(z)+2eA_\mu J^\mu)} S_F(x-y) \\ &= \mathcal{N}\exp[\frac{D_F}{\frac{m_0^2+m_0e^2}{e^4}} + \frac{\Delta_F(m^2=m_0+e^2)}{\frac{m_0+e^2}{e^2}}]\ \ S_F. \end{aligned} \tag{24}$$

Here Δ_F is the propagator of a massive free scalar field and $\mathcal{N}$ is a wave function renormalization factor.

Both the results therefore confirms strongly that the mass acquires a generalized expression with bare coupling constant and the parameter involved in the masslike term for the gauge field. It is known that in vector Schwinger model this type of parameter free mass generates. In fact, in the bosonized vector Schwinger model no such parameter exists if gauge fixing is not introduced at the lagrangian level. In this context, note that setting $a=0$, in the bosonized lagrangian of the so called nonconfining Schwinger model [6] one can obtain vector Schwinger model [1].

So from our result it can be concluded that it is the calculational error that led the authors of [12, 13] to land in to this type of untrustworthy result. We have pointed out in the body of this paper where the the actual error started. Actually, the use of that erroneous equation corresponding to the equation (13) led them to reach to the wrong expression of the mass term for photon. The fermionic propagator presented in [12, 13] also carried the same error. In equation(24) of this note the correct expression fermionic operator is calculated.

As a concluding remark we would like to mention that the term $\frac{1}{2}ae^2A^\mu A_\mu$ is not a legitimate gauge fixing term. With this term the vector Schwinger model turns into the so called nonconfining Schwinger model [6, 22]. Introduction of proper gauge fixing term like $\frac{\alpha}{2e^2}(\partial_\mu A^\mu)^2$ in the starting lagrangian of vector Schwinger model however leads to a result that corresponds to parameter free mass term for the photon though there exists a parameter α in the starting lagrangian.

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