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Brane resolution in heterotic theory through BF term

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ABSTRACT: We have studied the resolution of a naked singularity of a conifold in heterotic theory by a BF topological defect living in a 5-brane. The singularity is removed due to Chern-Simons action that changes the Bianchi identity for the $\mathcal{H}_3$ 3-form. Following the previous analysis of Cvetic, Lü and Pope [1] where they have studied the resolution through an instanton defect, we have taken a conifold over an *Eguchi-Hanson* manifold and a harmonic self-dual 2-form related with F_2 to solve the differential equation for the warp factor. Since the $\mathcal{H}_3$ field is related to torsion in the extra manifold, we can interpret this conifold as one with torsion. Using the so called BF term we have found a solution with the same properties of the instanton such that the conifold is smoothed out and has a torsion that diverges in IR regime and vanish in UV regime.

KEYWORDS: Brane resolution, Kalb-Ramond field, Braneworlds.

Contents

1. Introduction

Conifolds are manifolds of the form $C^{2n} = R^+ \times X^{2n-1}$, where X^{2n-1} is a manifold topologically equivalent to a S^{2n-1} [2], with a metric of the form $ds^2(C^{2n}) = dr^2 + r^2 ds^2(X^{2n-1})$. It is a kind of orbifold generated by the action of the group Z_n over R^n and therefore has a naked singularity in $r = 0$ due to a fixed point of the action of the group [3]. This point is important because we can transform a kind of Calabi-Yau space into another one just collapsing a cycle and so generating a conifold inside the extra dimensions, process called conifold transitions [4].

Another importance of conifolds lies in extensions of AdS-CFT correspondence [5]. In fact, Klebanov and Strassler [6] have studied a extension of this theory using the manifold $\mathcal{M}^{10} = M^4 \times C^6$ that asymptotically is $AdS^5 \times X^5$ instead of $AdS_5 \times S^5$, both in the type II-B string theory. Supposing X^5 Ricci-flat, they analyzed the flux of renormalization group and concluded that the conformal field theory of dual fields is preserved only if the naked singularity is smooth out. To do it, they have added fractional branes whose flux of gauge fields due the Chern-Simons or anomaly term, removes the singularity in a way previously studied by Candelas and de la Ossa[2] called deformation. The deformation procedure geometrically deforms the quadric that defines the conifold. Pando-Zayas and Tseytlin [7] have found another way of smooth the conifold introducing an asymmetry in X^5. This does not alter the quadric but introduces a resolution parameter and so it is called resolution.

Cvetic, Lü and Pope [1] have studied the resolution of a cone defined over an Eguchi-Hanson space in a heterotic string theory. They have chosen a warped product of a 5-brane with an Eguchi-Hanson space and taken an instanton defect living in Eguchi-Hanson space. The choise of an Eguchi-Hanson space is due this manifold be a solution of Einstein equations of the Kähler Ricci-flat type and also because it has a self-dual Riemann curvature

[8, 9, 10, 11]. The instanton defect appears naturally in the anomaly action in the supergravity regime altering the Bianchi identity for the 3-form of Kalb-Ramond $\mathcal{H}_3$ field that can be understood as a magnet flux through the transverse space. It also plays an important role in AdS/QCD, where it is holographically related with barions [12, 13]. On the other hand, using the transgression of Bianchi identity, it is found a differential equation for the warp factor depending on F_2. Also, a self-dual harmonic 2-form L_2 such that $F_2 = mL_2$ it is found. Then if L_2 is L^2 normalizable the singularity is taken off [1]. The L_2 exists because the transverse space is Ricci-flat and it has a covariantly constant spinor. Therefore, L_2 has a closed fundamental 2-form that also is a Kähler form [14].

Recently, Carlevaro and Isräel [15] have studied the resolution of a cone in a more wide scenario. They have taken a non-warped product of a Minkowski space with a conformal 6-conifold over a $T^{1,1}$ arbitrary base space in a heterotic string theory with a non-vanishing 3-form $\mathcal{H}_3$. Since we can define a torsion form from $\mathcal{H}_3$, this conifold is not a Ricci-flat space. So, in order to determine the geometry they solved the Strominger equation [16], necessary condition for supersymmetric invariance. This equation couple the exterior derivative of a 2-fundamental form and a 3-holomorfic form to the dilaton. So, this manifold has a varying dilaton that makes the string coupling not a constant. Assuming the conifold is asymptotically Ricci-flat, a gauge bundle is Abelian and taking the double scaling limit [17] where the string coupling constant goes to zero and the ratio $\frac{g_s \alpha'}{a}$ is fixed, the authors have found analytically and numerically a solution that is a torsional analogue of solutions of Eguchi-Hanson space in six dimensions. Since the $\mathcal{H}_3$ diverges only at origin the torsion have a IR singularity and vanish at infinity.

Torsional manifolds have being extensively studied because, on one hand, it can be viewed as spinning branes that are solutions extending the black branes to include spin (Euclidean Kerr solution) and this generates torsion [18]. On another hand, torsional manifolds appear on study of compactifications with non-vanishing fluxes that explain the stabilization of moduli fields, necessary condition for deduce the coupling constants of standard model [19]. Furthermore, torsion effects also have importance in brane worlds scenarios where they alter the area of event horizon of four dimensional black holes [20].

Motivated by these two issues, namely, the resolution of conifolds and the inclusion of torsion in extra dimensions, both due to instanton $F_2 \wedge F_2$ term, we have studied the resolution of conifold due a topological defect generated by a $B \wedge F$ term instead of $F \wedge F$ one. On one hand this change is possible because both are 4-forms metric independent. Further, the $B \wedge F$ belongs to the same Chern Class of $F \wedge F$ that appears in the anomaly term in type II string theories. Since both B_2 and F_2 belong to Neveu-Schwarz sector and heterotic supergravity does not have Ramond-Ramond fields, we propose to make the same shift $\mathcal{F}_2 \longrightarrow B_2 + kF_2$ in Chern-Simons action and study the effects over the transgression of Bianchi identity [3]. On another hand, the BF theory is interesting by itself because it is an extension of 3D Chern-Simons theory to four dimensions and so it is a theory for topological gravity [21]-[25]. Further, since 3D Chern-Simons theory describes the topological models for fractional statistics for planar systems, the BF theory can do the same for three space dimensions where on has interesting applications in condensed matter [27, 26]. Following the steps of Cvetic, Lü and Pope [1], we take a conformal conifold over

an Eguchi-Hanson space, with the same ansatz previously used for a L_2. Using a well known ansatz for B_2 form we obtain a differential equation for the warp factor. Now, we can tuning the strength of B_2 and F_2 to find a solution that resolves the conifold and in addition has the same torsional properties of the resolved conifold due the instanton term.

This work is organized as follows: in section II we obtain the BF and instanton term from the general Chern-Simons action and we present the transgression of Bianchi identity. We study the general characteristics of four dimensional Euclidean Ricci-flat spaces in section III whereas in section IV we review the resolution using the instanton term only over an Eguchi-Hanson conifold as done in [1] discussing the behavior of warp factor and the torsion. In section V, we obtain a resolved solution with the same features of instanton using the BF term instead of instanton one and finally, our conclusions are presented in the section VI.

2. BF term in heterotic theory

Consider the bosonic sector of heterotic theory in the supergravity regime whose action is given by [3, 4, 23]:

$$S_{10}^{het} = S_{NS} + S_{YM} + S_{CS}, \tag{2.1}$$

where S_{NS} is the Neveu-Schwarz action for the curvature R, for the dilaton field (ϕ) and for the NS-NS field strength $(\mathcal{H}_3)$ and is expressed by

$$S_{NS} = \frac{1}{2\kappa^2} \int_{M_{10}} d^{10}x \sqrt{-g} e^{-2\phi} (R + 4||\nabla\phi||^2 - \frac{1}{2}||\mathcal{H}_3||^2). \tag{2.2}$$

S_{YM} is an action of Yang-Mills type of form

$$S_{YM} = \frac{1}{2g^2} \int_{M_{10}} tr(\star_{10} R_2 \wedge R_2 + \star_{10} F_2 \wedge F_2), \tag{2.3}$$

where the R_2 is the curvature 2-form and the F_2 is the field strength for the A_1 gauge field. This term gives gravitational correction of higher order and introduces an instanton defect.

The two actions above are entire world-volume or metric dependent. However, for the cancelation of anomalies we must add the anomaly or Chern-Simons action S_{CS} given by

$$S_{CS}^{het} = \int B_2 \wedge X_8(F_2, R_2), \tag{2.4}$$

where X_8 is the symmetric polynomial of eighth order both in gauge and Lorentz curvature. By the Green-Schwarz mechanism there exists a 4-form Y_4 such that

$$d\mathcal{H}_3 = Y_4 = tr(R_2 \wedge R_2 - F_2 \wedge F_2). \tag{2.5}$$

This is called a transgression of the Bianchi identity for the $\mathcal{H}_3$ field. In order to NS-NS field strength $\mathcal{H}_3$ satisfy this equation we must add to this field a topological term

$$\mathcal{H}_3 = dB_2 + \frac{l_s^2}{4} tr \left(\omega_1 \wedge d\omega_1 + \frac{2}{3}\omega_1^3 - (A_1 \wedge dA_1 + \frac{2}{3}A_1^3) \right). \tag{2.6}$$

This additional term is nothing but the Chern-Simons 3-form for the spin connection ω_1 and for the gauge field A_1.

Extremizing the action for B_2 field yields the equations:

$$d \star_{10} \mathcal{H}_3 = 0 \tag{2.7}$$

$$d\mathcal{H}_3 = tr(R_2 \wedge R_2 - F_2 \wedge F_2). \tag{2.8}$$

In (2.8) we neglect higher order terms in F_2 and R_2. These equations are essentials for the resolution of four-dimensional spaces. Integrating the previous equation we obtain

$$\chi = \int_{M_4} tr(R_2 \wedge R_2) = \int_{M_4} tr(F_2 \wedge F_2), \tag{2.9}$$

where χ is the Euler number and the integral $\int_{M_4} tr(F_2 \wedge F_2)$ is the instanton charge. Then we can interpret the resolution process as a result of the flux generated by a topological defect of instanton type living in a 4-brane. This open the question: could be possible a resolution using another topological defect living in 4-branes?

In type II string theories the Chern-Simons action for the anomaly cancelation [23] is

$$S_{CS}^{II} = \frac{T_p}{2} \int tr \left(e^{\mathcal{F}_2} \wedge C \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \right) \tag{2.10}$$

where $\mathcal{F}_2 = B_2 + kF_2$, $\kappa = 2\pi\alpha' = 2\pi\sqrt{l_s}$ [3]. Let us consider only the trivial cases where $\hat{A}(R_T) = \hat{A}(R_N) = 1$. Expanding the exponential until second order we obtain $tr(B_2 \wedge B_2 + 2\kappa B_2 \wedge F_2 + \kappa^2 F_2 \wedge F_2)$. Note that this yields three different topological defects that, if we suppose them to live in a 4-brane, we can obtain their topological charge by fluxes. We can interpret this expansion in terms of constant κ and so the $B \wedge F$ term is relevant in first order in κ, what means for a scale $l_s > l_p$, while the instanton term $F \wedge F$ is relevant for κ^2, that means $l_s >> l_p$.

Inspired in the Chern-Simons term in type II theories, let us make a replacement $F_2 \rightarrow \mathcal{F}_2 = B_2 + kF_2$ in heterotic theory. This change can be done because this field already belongs to the heterotic action. Further, although this change is based in the anomaly term in type II theories, it does not depends on the R-R fields that do not exist in the heterotic theory.

By doing this change the Yang-Mills term in the action turns to be:

$$S_{YM} = \int_{M_{10}} tr(\star_{10}R_2 \wedge R_2 + \star_{10}B_2 \wedge B_2 + 2k \star_{10} F_2 \wedge B_2 + k^2 \star_{10} F_2 \wedge F_2). \tag{2.11}$$

The 4-form Y_4 should also be altered for

$$Y_4 = tr(R_2 \wedge R_2 - 2kB_2 \wedge F_2 - k^2 F_2 \wedge F_2). \tag{2.12}$$

Hence the NS-NS or Kalb-Ramond field strength must satisfies

$$d\mathcal{H}_3 = tr(R_2 \wedge R_2 - 2kB_2 \wedge F_2 - k^2 F_2 \wedge F_2). \tag{2.13}$$

Then we must add another topological term for the $\mathcal{H}_3$ of the form

$$\mathcal{H}_3 = dB_2 + \frac{l_s^2}{4}tr\left(\omega_1 \wedge d\omega_1 + \frac{2}{3}\omega_1^3 - (A_1 \wedge dA_1 + \frac{2}{3}A_1^3) - B_2 \wedge A_1\right). \tag{2.14}$$

This changes yields to new equations for $\mathcal{H}_3$, namely

$$d \star_{10} \mathcal{H}_3 = X_8 - \star_{10} B_2 - \star_{10} F_2 \tag{2.15}$$
$$d\mathcal{H}_3 = tr(R_2 \wedge R_2 - kdB_2 \wedge A_1 - 2kB_2 \wedge F_2 - k^2 F_2 \wedge F_2). \tag{2.16}$$

Since we want to study the resolution through a BF term and compare with the instanton term approach let us consider only the $B_2 \wedge F_2$ and $F_2 \wedge F_2$ terms in anomaly term. Further, we will disregard the $\star_{10}R_2 \wedge R_2$ and $\star_{10}B_2 \wedge B_2$ terms. So we will study solutions of the system of equations

$$d \star_{10} \mathcal{H}_3 = X_8 - \star_{10} B_2 - \star_{10} F_2 \tag{2.17}$$
$$d\mathcal{H}_3 = tr(-2kB_2 \wedge F_2 - k^2 F_2 \wedge F_2). \tag{2.18}$$

The transgression of Bianchi identity represents the magnetic flux due a instanton and a BF topological defect. The 2-form field strength usually take values on $SU(32)$ or $E_8 \times E_8$ gauge group but we will consider only an Abelian $U(1)$ case for simplicity.

3. Resolution over four-dimensional transverse spaces

As previously done by Cvetic, Lü and Pope [1], let us take the bulk as the warped product between a D5 brane and a Ricci-flat manifold:

$$\mathcal{M}^{10} = R^{1,5} \times M^4, \tag{3.1}$$

where M^4 is a conifold over a base space with SU(2) holonomy group. Taking the coordinates (θ, ϕ, ψ), we will study a metric:

$$ds_{10}^2 = H^{-\frac{1}{4}}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{\frac{3}{4}}(r)ds_4^2, \tag{3.2}$$

where

$$ds_4^2 = \alpha^2(r)dr^2 + r^2(\beta^2(r)\sigma^2 + \gamma^2(r)(d\theta^2 + sen^2\theta d\phi^2)) \tag{3.3}$$

$$\sigma = d\psi + \cos\theta d\phi. \tag{3.4}$$

The vielbeins of this metric are:

$$e^1 = \alpha dr , \ e^2 = r\beta\sigma$$
$$e^3 = r\gamma d\theta , \ e^4 = r\gamma sen\theta d\phi. \tag{3.5}$$

Following Carlevaro and Isräel [15], we can take a set of complexified vielbeins of the form

$$E^1 = e^1 + ie^2 \ , \ E^2 = e^3 + ie^4, \tag{3.6}$$

and we can define the fundamental 2-form J_2 by

$$\begin{aligned} J_2 &= \frac{i}{2} \sum_{a=1}^{2} A_a E^a \wedge \bar{E}^a \\ &= A_1 e^1 \wedge e^2 + A_2 e^3 \wedge e^4, \end{aligned} \tag{3.7}$$

where A_1 and A_2 are constants. Therefore

$$J_2 \wedge J_2 = A_1 A_2 e^1 \wedge e^2 \wedge e^3 \wedge e^4. \tag{3.8}$$

As $J_2 \wedge J_2$ is proportional to the volume of M_4 then it satisfies the Strominger equation [16]:

$$d(e^{-2\phi} J_2 \wedge J_2) = 0. \tag{3.9}$$

Since

$$dJ_2 = \left(A_1 r \alpha \beta + 2r A_2 \gamma (\gamma + r\gamma') \right) sen\theta dr \wedge d\theta \wedge d\phi, \tag{3.10}$$

the condition for M^4 be a Kähler manifold is

$$dJ_2 = 0 \Leftrightarrow A_1 r \alpha \beta + A_2 2\gamma(\gamma' + r\gamma) = 0. \tag{3.11}$$

We can follow [1] and take $A_1 = -A_2 = 1$, so

$$J_2 = e^1 \wedge e^2 - e^3 \wedge e^4. \tag{3.12}$$

In this background, let us consider only the actions of the instanton term, what means $\star_{10} B_2 = \star_{10} F_2 = 0$. Therefore,

$$d(\star_{10} \mathcal{H}_3) = 0. \tag{3.13}$$

Now, let us search for solutions of the form [1]:

$$e^{-2\phi} \star_{10} \mathcal{H}_3 = dH^{-1} \wedge d^6 x. \tag{3.14}$$

Following the steps done in [1] we can relate the 2-form gauge F_2 with a self-dual 2-form L_2 as

$$\begin{aligned} F_2 &= mL_2 \\ &= mr^n (e^1 \wedge e^2 + e^3 \wedge e^4). \end{aligned} \tag{3.15}$$

In the ansatz above we have supposed a spherical symmetry and a power radial dependence only, whose exponent we will determine using the condition that M^4 is asymptotically Euclidean and that L_2 is a squared-integrable form [1, ?]. Then using the ansatz for $\mathcal{H}_3$ and F_2 the Bianchi identity yields the differential equation for the warp factor

$$\triangle_4 H(r) = -m^2 r^{2n}. \tag{3.16}$$

This is a Poisson like equation for the warp factor where the Laplacian is evaluated over the M^4 and so

$$\frac{d}{dr}\left(\frac{r^3 \beta \gamma^2}{\alpha} H'(r)\right) - m^2 r^{2n+3} \alpha \beta \gamma^2 = 0. \tag{3.17}$$

Although the metric factors that determine the initial manifold have singularities, we can find solutions of equation above that for a specific power n it does not have singularity anymore.

Let us now study the equation for the warp factor in a particular example of a Ricci-flat manifold.

4. Resolution over Eguchi-Hanson spaces

As done in [1], let us choose M^4 as the Eguchi-Hanson space. The factors of metric are:

$$\alpha^2 = W(r)^{-1} \ , \ \beta^2 = \frac{1}{4} W(r)$$
$$\gamma^2 = \frac{1}{4} \ , \ W(r) = 1 - \frac{a^4}{r^4}, \tag{4.1}$$

where $r \geq a$. This manifold is asymptotically locally Euclidean (ALE) with topology of the form $S^2/\mathbb{Z}_2$ or $\mathbb{C}^2/\mathbb{Z}_2$ and near $r = a$ the topology is $\mathbb{R}^2 \times S^2$, with a bolt singularity. Such factors satisfy the equation for closure of fundamental form J_2, so the Eguchi-Hanson space has a Kähler form given above. The differential equation for warp factor is

$$(r^3 W(r) H'(r))' = -m^2 r^{2n+3}, \tag{4.2}$$

whose solution is

$$H(r) = C_2 + \frac{m^2}{(2n+4)} \int \frac{r'^{2n+5}}{(r'^4 - a^4)} dr' + C_1 \int \frac{r'}{(r'^4 - a^4)} dr'. \tag{4.3}$$

As we are interested in solutions asymptotically Euclidean we can choose $C_2 = 1$ and must choose the power n such that $\lim_{r \to \infty} H(r) = 1$. If $n \geq 0$, the first integral does not have an upper bound and so diverges at infinity. For $n = -1$, the manifold has a singularity at $r = a$ and diverge at infinity. For $n = -2$, we have the warp factor defined in the range $0 \leq r \leq a$ only, having a singularity at $r = a$. For $n \leq -3$, $H(r)$ is defined only for $a \leq r \leq \infty$, with $\lim_{r \to \infty} H(r) = 1$. However, for $n = -3$ we still have a singularity in $r = a$, but for $n \leq -4$, i.e., $2n + 5 = -3$, as done in [1], the power of r in the integrand is $p \leq -3$ and so the solution is:
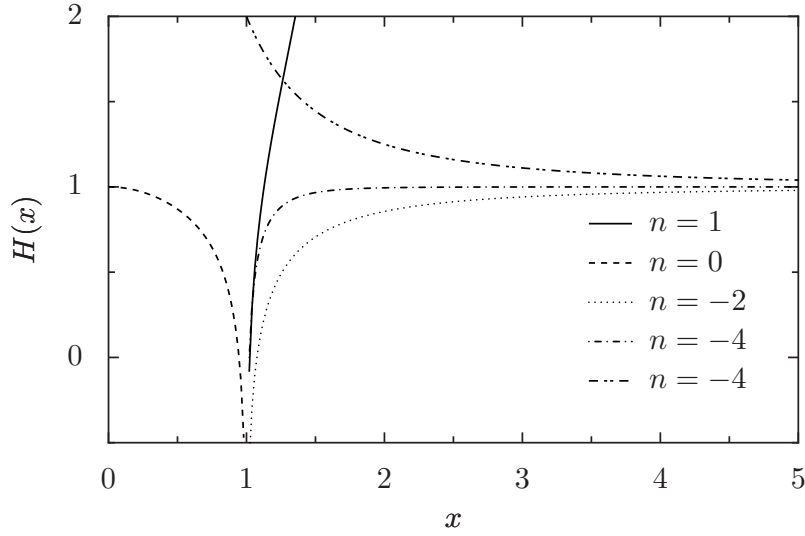
Figure 1: Graphic of warp factor for various powers n for $a = 1$. For $n > 0$ the warp factor diverges both at $r = 1$ as at infinity. For $a = 0$ there is only interior solution diverging at $a = 1$ too. For negatives powers we have reached a well behaved solution only for $n \geq -4$ when we can find a integration constant that cancels the logarithmic divergence.

$$H(r) = 1 + \frac{m^2 + a^4 b}{4a^6} \ln \frac{(r^2 - a^2)}{(r^2 + a^2)} + \frac{m^2}{2a^4 r^2}. \tag{4.4}$$

There is a logarithmic divergence at $r = a$ but we can choose a particular integration constant, $b = -\frac{m^2}{a^4}$ and

$$H(r) = 1 + \frac{m^2}{2a^4 r^2}, \tag{4.5}$$

so that the naked singularity in $r = a$ is taken off. In Fig. (4) we have plotted the warp factor for various powers.

Using the ansatz for $\mathcal{H}_3$ field and the warp factor above we obtain:

$$\mathcal{H}_3 = -\frac{m^2}{2a^4} \frac{1}{r^3} W(r)^{\frac{1}{2}} e^\theta \wedge e^\phi \wedge e^\sigma. \tag{4.6}$$

This solution is quite similar to that found by Carlevaro and Isräel [15] in six dimensions. From this field we can define the 2-form of torsion:

$$T^a = \frac{1}{2} H^a_{bc} e^b \wedge e^c. \tag{4.7}$$

It worthwhile to point out that the tensor $\mathcal{H}^a_{bc}$ can be interpreted as a non-metric connection [3, 19]. Since $\mathcal{H}^a_{bc}$ is skew-symmetric we have a torsion even though the geometric connection for the Eguchi-Hanson spaces is symmetric.
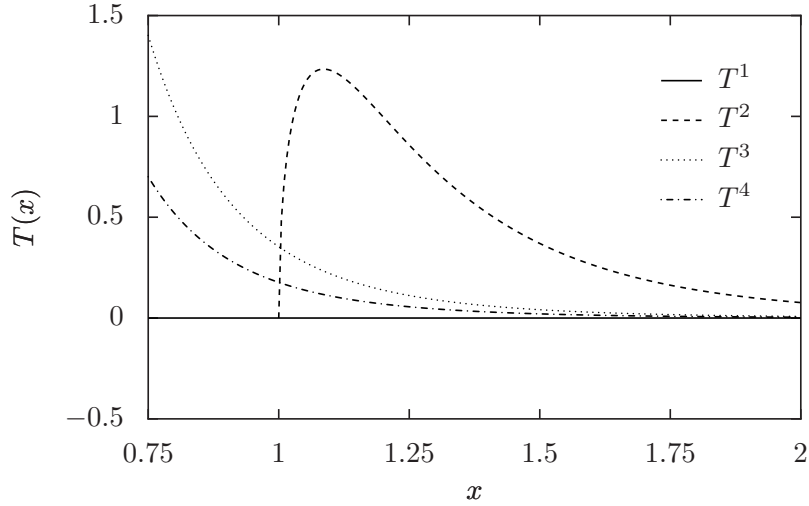
Hence, the components of torsion are

Figure 2: Graphic of components of torsion 2-form for $a = 1$. The component T^1 is identically null while the T^3 and T^4 components diverge at origin and vanish in UV regime. The T^2 component is defined only for $r \geq 1$ vanishing in this point and at infinity. Note that there is a neighborhood around $r = 1$ where the torsion is non-null.

$$T^1 \qquad\qquad\qquad = 0 \tag{4.8}$$

$$T^2 \quad = \tfrac{m^2}{a^4} r^{-5} H^{-\frac{3}{4}} W^{\frac{1}{2}} e^3 \wedge e^4 \tag{4.9}$$

$$T^3 \quad = \tfrac{m^2}{a^4} r^{-5} H^{-\frac{3}{4}} e^2 \wedge e^4 \tag{4.10}$$

$$T^4 = \tfrac{m^2}{a^4} r^{-5} (\sin\theta)^{-2} H^{-\frac{3}{4}} e^2 \wedge e^3 \tag{4.11}$$

In Fig. (4) we have plotted the graphic for the torsion above for $a = 1$. The last two above are defined for whole $r > 0$, diverging in $r \to 0$ and going quickly to zero when $r \to \infty$. The σ component is defined only for $r > a$ diverging in this point and vanishing at infinity. So the torsion diverges in the IR limit ($r \to 0$) and vanish in the UV limit ($r \to \infty$).

5. Resolution through BF term

Now we are interested in the resolution through a BF term. We will use an ansatz for B field previously used in Ref. [6], that have spherical symmetry and are defined only at spherical cycle:

$$B_2 = f(r) e^\theta \wedge e^\phi \tag{5.1}$$

This ansatz was studied by Klebanov and Tseytlin [28] in the context of conformal invariance of gauge-gravity duality on conifolds. A necessary condition is [29]:

$$4\pi^2 \left(\frac{1}{g_1^2} - \frac{1}{g_2^2} \right) = \frac{1}{g_s e^\phi} \left(\frac{1}{2\pi\alpha'} \int_{S^2} B_2 - \pi \right). \tag{5.2}$$

The equations for $\mathcal{H}_3$ are:

$$d(\star\mathcal{H}_3) = -k \star F_2 \qquad (5.3)$$

$$d\mathcal{H}_3 = B_2 \wedge F_2. \qquad (5.4)$$

Let us to extend the ansatz chosen in [1] to

$$e^{-\phi} \star_{10} \mathcal{H}_3 = (dH^{-1} + \omega_1) \wedge dx^6, \qquad (5.5)$$

where the 1-form ω_1 is such that

$$d\omega_1 \wedge dx^6 = - \star_{10} F_2. \qquad (5.6)$$

This is a system of differential equations relating the components of ω_1 to F_2 and so it has a solution. This ansatz yields the equation

$$d \star_4 dH + d \star_4 \omega_1 = B_2 \wedge F_2. \qquad (5.7)$$

Note that in this limit we have changed only the transgression of Bianchi identity compared with [1]. Assuming the same ansatz previously used for F_2 we have

$$B_2 \wedge F_2 = -mf(r)r^n e^r \wedge e^\sigma \wedge e^\theta \wedge e^\phi. \qquad (5.8)$$

For $||B_2|| >> ||F_2|| \Rightarrow d\omega_1 \to 0$ and this yields

$$\triangle H(r) = -mf(r)r^n. \qquad (5.9)$$

Let us take a power-dependence for f, like $f(r) = r^k$, thus,

$$\frac{d}{dr}\left(\frac{r^3 \beta\gamma^2 H'(r)}{\alpha}\right) = -m\alpha\beta\gamma^2 r^{n+k+3}. \qquad (5.10)$$

Taking the Eguchi-Hanson space as a background for this defect, we obtain the differential equation for H as

$$\left(r^3 W(r)H'(r)\right)' = -mr^{n+k+3}, \qquad (5.11)$$

whose solution is:

$$H(r) = 1 + \frac{m}{(n+k+4)} \int \left[\left(\frac{r'^{n+k+5}}{r'^4 - a^4}\right) + c_1 \frac{r'}{(r'^4 - a^4)}\right] dr'. \qquad (5.12)$$

As seen before, there is a resolved solution for $n + k + 5 \leq -3$. Since $||B|| >> ||F||$ we can take the powers $n = -10$ and $k = 2$, for instance. Therefore

$$H(r) = 1 + \frac{m^2}{2a^4 r^2}. \qquad (5.13)$$

Although the resolution is given by the BF term instead of instanton one we have reached the same warp factor. Since the NS-NS field strength is given by the warp factor the

torsion has the same expression for the resolution obtained using the instanton defect that diverges only at origin. However, if we evaluate the non-topological NS-NS field strength using the ansatz for B_2 we obtain

$$B_2 = r^2 e^\theta \wedge e^\phi \Rightarrow H_3 = dB_2 = 2rW^{\frac{1}{2}}(r)e^r \wedge e^\theta \wedge e^\phi. \tag{5.14}$$

This ansatz yields a torsion in the form

$$T^1 = rW(r)^{\frac{3}{2}}H(r)^{-\frac{3}{4}}e^3 \wedge e^4 \tag{5.15}$$

$$T^2 \qquad = 0 \tag{5.16}$$

$$T^3 = r^{-1}H^{-\frac{3}{4}}(r)e^1 \wedge e^4 \tag{5.17}$$

$$T^4 = r^{-1}H^{-\frac{3}{4}}(r)e^1 \wedge e^3. \tag{5.18}$$

The component in r direction is defined only for $r > a$, diverging at infinity. The other directions are defined for all r vanishing in origin and diverging in UV regime. Since the whole topological NS-NS field strength $\mathcal{H}_3$ vanishes in UV limit we conclude that the topological 3-form added to H_3 cancels the diverging components resulting in an asymptotically torsion free manifold.

6. Conclusions

We have studied the process of resolution of a conifold by an instanton and a BF term. Since both belongs to same Chern character that appears in the anomaly term in type II string theories we heuristically have proposed the same change $\mathcal{F}_2 = B_2 + kF_2$ in the heterotic theory. Since k is related with the string scale l_s, the BF term is relevant for distances of string scale and the instanton term is relevant for distances greater than the string scale. The presence of the BF term yields a current proportional to $\star_{10}F_2$ and alter the Bianchi identity including the same $B_2 \wedge F_2$ besides of $F_2 \wedge F_2$. For the instanton term there is no current associated with this term and so we can choose an ansatz relating to dual of $\mathcal{H}_3$ with the derivative of warp factor. This ansatz yields a Poisson differential equation relating the laplacian to the instanton charge. Taking the Eguchi- Hanson space as background, we have found solutions previously studied by Cvetic, Lü and Pope [1] that do not have singularities and have a non-vanishing torsion that diverges in IR regime $(r \to 0)$ and vanishes in UV regime $(r \to \infty)$, so asymptotically the manifold is torsion free. For the BF term we have a current for $\mathcal{H}_3$ proportional to the dual of F_2 and so we must to add a new term in the differential equation for the warp factor. Assuming $||B_2|| \gg ||F_2||$ we can disregard this term and solve the Poisson equation previously studied. Since the warp factor is the same as in the instanton case the BF term resolves the singularity and has the same torsional behavior as instanton term. Evaluating only the dB_2 component of $\mathcal{H}_3$ we have found a solution with asymptotically divergent torsion and we conclude that the remainder topological term cancels the divergence.

   Nevertheless, we have reached a solution for the $||B_2|| \gg ||F_2||$ case where the term $d\omega_1$ is negligible and we could ask if a resolved solution can be reached for $||B_2|| \approx ||F_2||$ and

$||B_2|| \ll ||F_2||$. Moreover, we have studied only cases $B_2 \wedge F_2$ and $F_2 \wedge F_2$ separately and we could study a limit where both defects are working. We have limited us to an Abelian gauge group and it could be interesting to extend this work to full $E_8 \times E_8$ or $SU(32)$ groups as done in [17]. Another feature is to study the resolution of BF term taking into account the Gauss-Bonnet term $tr(R_2 \wedge R_2)$ in transgression of Bianchi identity. We could also extend this resolution taking 6-dimensional spaces instead of 4-dimensional ones, using the Strominger equations to find the (1,1)-fundamental form $J_{1,1}$ and the (3,0)-holomorphic form $\Omega_{3,0}$ as done in [15] for the instanton term. To do it, we must to extend the DUY (Donaldson-Uhlenbeck-Yau) equations, conditions for stability of the gauge bundle, to the BF term. In another perspective one can also consider the present study in M-theory following the lines of Ref. [30], where was pointed out that Chern-Simons corrections associated with anomaly on the M5-brane can resolve singularity on M2-brane.

Going to AdS-CFT correspondence, since the BF term resolves the conifold, an interesting question is to study the renormalization group flow under the BF term, for instance. In the context of AdS-QCD duality, since BF term resolves the singularity as well as the instanton term, we can ask what kind of fields are holographically related to BF term.

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