

Intersecting Black Attractors in $8D \mathcal{N} = 1$ Supergravity

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Abstract

We study intersecting extremal black attractors in non chiral $8D \mathcal{N} = 1$ supergravity with moduli space $\frac{SO(2,N)}{SO(2) \times SO(N)} \times SO(1,1)$ and work out explicitly the attractor mechanism for various black p-brane configurations with the typical near horizon geometries $AdS_{p+2} \times S^m \times T^{6-p-m}$. We also give the classification of the solutions of the attractor equations in terms of the $SO(N-k)$ subgroups of $SO(2) \times SO(N)$ symmetry of the moduli space as well as their interpretations in terms of both heterotic string on 2-torus and its type IIA dual. Other features such as non trivial $SO(1,7)$ central charges $Z_{\mu_1 \dots \mu_p}$ in $8D \mathcal{N} = 1$ supergravity and their connections to p-form gauge fields are also given.

Key Words: $8D$ Supergravity, Superstring compactifications, Attractor Mechanism, Intersecting Attractors.

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1 Introduction

During the last decade, black attractor solutions in supergravity theories have been a subject of big interest; especially in connection with low energy $10D$ superstring and $11D$ M-theory compactifications [1]-[18]. Because of their specific properties [19]-[38], static, asymptotically flat and spherically symmetric extremal (vanishing temperature for non-zero entropy) black attractors have been investigated for supergravities in diverse

space time dimensions; with various numbers of conserved supersymmetries [39]-[56]. Guided by the new solutions on extremal BPS and non BPS black attractors in higher dimensional supergravity; in particular those on intersecting attractors obtained first by Ferrara et al. in [57], see also [58]; we focus in this paper on non chiral $8D$ $\mathcal{N} = 1$ supergravity with moduli space $\frac{SO(2,N)}{SO(2) \times SO(N)} \times SO(1,1)$ and study explicitly the attractor mechanism for various configurations of extremal black p-branes with the typical near horizon geometries $AdS_{p+2} \times S^m \times T^{6-p-m}$ where $p = 0, 1, 2, 3, 4$ and $m = 2, 3, 4, 5, 6$. Actually this analysis completes the results obtained in [58] for the case of *maximal* $\mathcal{N} = 2$ supergravity in $8D$; it also gives new solutions, along the line of [57], classified by $SO(N-k)$ subgroups of the $SO(2) \times SO(N)$ symmetry of the moduli space of the non chiral $8D$ $\mathcal{N} = 1$ supergravity.

The interest into this study is also motivated from the two following features: first because of its 16 conserved supersymmetries, extremal black attractors in this $8D$ supergravity may be viewed as the ancestor of an interesting class of black holes in $7D$, $6D$, $5D$ and $4D$ supergravities [1, 5]; in particular in $4D$ $\mathcal{N} = 2$ and $4D$ $\mathcal{N} = 4$ resulting from adequate compactifications of the $8D$ space time down to $4D$. It is also interesting from the view of higher dimensions since non chiral $8D$ $\mathcal{N} = 1$ supergravity may arise as low energy of heterotic string on T^2 and type IIA string on a real compact surface Σ that preserves half of the 32 conserved supercharges of the $10D$ type II superstrings. Black attractor solutions in $8D$ offers therefore a framework to explicitly check specific features of the heterotic/type IIA duality in $8D$ [59].

The paper is organized as follows. In section 2, we review the $8D$ $\mathcal{N} = 1$ supersymmetry algebra with central charges $Z_{\mu_1 \dots \mu_p}$. We derive the various $SO(1,7)$ charges of these $Z_{\mu_1 \dots \mu_p}$'s and give their connection with p-branes. It is also shown why the 3-form gauge field in $\mathcal{N} = 1$ theory should vanish. In section 3, we develop the study of the non chiral $\mathcal{N} = 1$ supergravity in $8D$ and its links with the low energy limit of the heterotic superstring on T^2 and its type IIA superstring dual. The various charges of the black attractors are also given. In section 4, we first give the effective potential and the attractor eqs; then we derive their solutions together with their classification in terms of $SO(N-k)$ subgroups of the $SO(2) \times SO(N)$ symmetry of the moduli space. In section 5, we study the intersecting attractors along the line of the approach of [57, 58] and in section 6, we give a conclusion.

2 Central charges in $8D$ $\mathcal{N} = 1$ supersymmetry

In this section, we identify the full set of the bosonic "central charges" $Z_{\mu_1 \dots \mu_p}$ involved in the generalized non chiral $8D$ $\mathcal{N} = 1$ superalgebra and give their links to black p-brane

attractors by using group theoretical methods.

To start it is interesting to recall that like in $4D$ space time, supersymmetry with sixteen supercharges may also live in other space times. In eight dimensions; this is precisely $\mathcal{N} = 1$ supersymmetry given by a graded superalgebra with both commutators and anticommutators; it exchanges $8D$ space time bosons into $8D$ space time fermions. In addition to the twenty eight $M_{[\mu\nu]}$ symmetry generators of $SO(1, 7)$, the standard (non extended) $\mathcal{N} = 1$ non chiral supersymmetry is moreover generated by the energy momentum vector operator P_μ and the fermionic generators Q_α and $\bar{Q}_{\dot{\alpha}}$ transforming as Weyl spinors under $SO(1, 7)$. To have more insight on the structure of this superalgebra and its connection with black branes in $8D$, we give below some useful details.

2.1 $\mathcal{N} = 1$ superalgebras in $8D$

We begin by recalling the group theoretical nature of the fermionic generators in $8D$ $\mathcal{N} = 1$ non chiral supersymmetry; these are $SO(1, 7)$ spinors with $2^4 = 16$ complex components that transform in the reducible $\mathfrak{8}_s \oplus \mathfrak{8}_c$ representation of the $8D$ Lorentz group respectively given by the eight component Weyl spinors Q_α^+ and $\bar{Q}_{\dot{\alpha}}^-$. These fermionic generators carry also charges under the $SO(2) \sim U_R(1)$ R-symmetry of the supersymmetric algebra.

Using general properties of tensor products of $SO(1, 7) \times U_R(1)$ representations, we learn that one may distinguish three kinds of $\mathcal{N} = 1$ anticommutation relations in $8D$; two chiral (complex) relations and a vector like (real) one:

(1, 0) *chiral relations in $8D$*

These are complex relations involving only the fermionic generator Q_α^+ ,

$$\{Q_\alpha^+, Q_\beta^+\} = Z_{(\alpha\beta)}^{++} \quad , \quad [Q_\alpha^+, Q_\beta^+] = Z_{[\alpha\beta]}^{++} \quad , \quad (2.1)$$

where the symmetric $Z_{(\alpha\beta)}^{++}$'s should be thought of as operators carrying charges of black branes in $8D$. The antisymmetric term $Z_{[\alpha\beta]}^{++}$'s, which may be expanded as $\sigma_{[\alpha\beta]}^{\mu\nu} M_{\mu\nu}$ may be interpreted in terms of $SO(1, 7)$ rotations in the Weyl representation.

(0, 1) *antichiral relations in $8D$*

These are the complex conjugate of (2.1); they involve the $\bar{Q}_{\dot{\alpha}}^-$ spinor

$$\{\bar{Q}_{\dot{\alpha}}^-, \bar{Q}_{\dot{\beta}}^-\} = \bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--} \quad , \quad [\bar{Q}_{\dot{\alpha}}^-, \bar{Q}_{\dot{\beta}}^-] = \bar{Z}_{[\dot{\alpha}\dot{\beta}]}^{--} \quad , \quad (2.2)$$

where $\bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--}$ and $\bar{Z}_{[\dot{\alpha}\dot{\beta}]}^{--}$ are the complex conjugate of $Z_{(\alpha\beta)}^{++}$ and $Z_{[\alpha\beta]}^{++}$.

$\mathcal{N} = (1, 1)$ *superalgebra in $8D$*

This is a *vector like* superalgebra with fermionic generators Q_α^+ and $\bar{Q}_{\dot{\alpha}}^-$ obeying the

following anticommutation relations,

$$\begin{cases} \{Q_\alpha^+, Q_\beta^+\} &= Z_{(\alpha\beta)}^{++} \quad , \\ \{Q_\alpha^+, \bar{Q}_{\dot{\beta}}^-\} &= Z_{\alpha\dot{\beta}}^0 \quad , \\ \{\bar{Q}_{\dot{\alpha}}^-, \bar{Q}_{\dot{\beta}}^-\} &= \bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--} \quad , \end{cases} \quad (2.3)$$

where the bosonic operators $Z_{(\alpha\beta)}^{++}$, $\bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--}$ are as before and where $Z_{\alpha\dot{\beta}}^0$ contains the usual energy momentum vector P_μ generating space time translations.

2.2 More on central charges in $8D \mathcal{N} = 1$ supersymmetry

The bosonic operators $Z_{(\alpha\beta)}^{++}$, $\bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--}$, $Z_{\alpha\dot{\beta}}^0$ capture several irreducible $SO(1,7)$ space time representations. To get their irreducible components, we use the correspondence¹

$$Q_\alpha^+ \sim (8_s, +1) \quad , \quad \bar{Q}_{\dot{\alpha}}^- \sim (8_c, -1) \quad (2.4)$$

and tensor product properties of the $SO(1,7) \times U_R(1)$ representations; in particular

$$\begin{aligned} (8_s, +1) \times (8_s, +1) &= (1, +2) + (28, +2) + (35_s, +2) \quad , \\ (8_s, +1) \times (8_c, -1) &= (8_v, 0) + (56_v, 0) \quad , \\ (8_c, -1) \times (8_c, -1) &= (1, -2) + (28, -2) + (35_c, -2) \quad . \end{aligned} \quad (2.5)$$

The symmetry property of the anticommutators of eqs(2.3) allows to read the group theoretical structure of the $Z_{(\alpha\beta)}^{++}$, $\bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--}$ and $Z_{\alpha\dot{\beta}}^0$; we have:

$$\begin{aligned} Z_{(\alpha\beta)}^{++} &\sim (1, +2) \oplus (35_s, +2) \quad , \\ Z_{\alpha\dot{\beta}}^0 &\sim (8_v, 0) \oplus (56_v, 0) \quad , \\ \bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--} &\sim (1, -2) \oplus (35_c, -2) \quad . \end{aligned} \quad (2.6)$$

Notice that the sub-index $i = v, s, c$ refer to the triality property of the $SO(1,7)$ symmetry which have three kinds of fundamental representations with same dimension.

Moreover, using the $SO(1,7)$ Dynkin labels $(l_1 l_2 l_3 l_4)$, the three eight dimensional basic representations read as follows,

$$8_v = (1000), \quad 8_s = (0001), \quad 8_c = (0010). \quad (2.7)$$

With these basic representations, one can build the higher dimensional ones by taking tensor products. For the example of the leading lower dimensional representations, we have

$$\begin{aligned} 8_i \times 8_i &= 1 + 28 + 35_i \quad , \\ 8_i \times 8_j &= 8_k + 56_k \quad , \end{aligned} \quad (2.8)$$

¹viewed from $4D$, this corresponds to $\mathcal{N} = 4$ supersymmetry with fermionic generators Q_α^I in the $(2_s, 4)$ representation of $SO(1,3) \times SU(4)$.

with i, j, k cyclic and where

$$\begin{aligned} 35_v &= (2000), & 35_s &= (0002), & 35_c &= (0020), \\ 56_v &= (0011), & 56_s &= (1010), & 56_c &= (1001). \end{aligned} \quad (2.9)$$

Notice also that besides the real energy momentum vector $P_\mu \sim 8_v$ and complex singlets $Z_0^{++} = Tr(Z_{\alpha\beta}^{++}) \sim 1$, the bosonic operators $Z_{(\alpha\beta)}^{++}$, $\bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--}$, $Z_{\alpha\dot{\beta}}^0$ capture moreover $SO(1,7)$ higher dimensional representations namely the $35_s, 35_c, 56_v$.

In terms of $SO(1,7)$ vector indices, these representations may be decomposed by using *antisymmetric* products of the 8×8 Pauli-Dirac Γ^μ - matrices as follows

$$\begin{aligned} Z_{(\alpha\beta)}^{++} &= \delta_{\alpha\beta} Z_0^{++} + \Gamma_{(\alpha\beta)}^{\mu\nu\rho\sigma} Z_{[\mu\nu\rho\sigma]}^{++} \quad , \\ \bar{Z}_{(\dot{\alpha}\dot{\beta})}^{--} &= \delta_{\dot{\alpha}\dot{\beta}} \bar{Z}_0^{--} + \Gamma_{(\alpha\beta)}^{\mu\nu\rho\sigma} \tilde{Z}_{[\mu\nu\rho\sigma]}^{--} \quad , \\ Z_{\alpha\dot{\beta}}^0 &= \Gamma_{\alpha\dot{\beta}}^\mu Z_\mu^0 + \Gamma_{\alpha\dot{\beta}}^{\mu\nu\rho} Z_{[\mu\nu\rho]}^0 \quad , \end{aligned} \quad (2.10)$$

where antisymmetrization with respect to space time indices is understood. Notice that an antisymmetric rank 4-tensor type $Z_{[\mu\nu\rho\sigma]}$ has in general $\frac{8!}{4! \times 4!}$ degrees of freedom; but the 4- forms $Z_{[\mu\nu\rho\sigma]}^{++}$ and $\tilde{Z}_{[\mu\nu\rho\sigma]}^{--}$ involved in (2.10) capture each 35 degrees of freedom associated with the self dual and anti-self dual antisymmetric 4-rank tensors in $8D$ space time,

$$\begin{aligned} Z_{\mu_1\mu_2\mu_3\mu_4}^{++} &= \varepsilon_{\mu_1\dots\mu_8} Z^{++\mu_5\mu_6\mu_7\mu_8} \quad , \\ \tilde{Z}_{\mu_1\mu_2\mu_3\mu_4}^{--} &= -\varepsilon_{\mu_1\dots\mu_8} \tilde{Z}^{--\mu_5\mu_6\mu_7\mu_8} \quad . \end{aligned} \quad (2.11)$$

From this group theoretical analysis, it follows amongst others the two following features: **(1)** the simplest form of the non chiral $8D$ $\mathcal{N} = 1$ supersymmetric algebra reads as follows,

$$\begin{aligned} \{Q_\alpha^+, Q_{\dot{\beta}}^-\} &\sim \Gamma_{\alpha\dot{\beta}}^\mu P_\mu \quad , \\ \{Q_\alpha^+, Q_\beta^+\} &\sim \{Q_{\dot{\alpha}}^-, Q_{\dot{\beta}}^-\} = 0 \quad , \end{aligned} \quad (2.12)$$

and corresponds to switching off the p-forms Z_0^{++} , Z_μ^0 , $Z_{[\mu\nu\rho]}^0$ and $Z_{[\mu\nu\rho\sigma]}^{\pm\pm}$,

(2) there are no $Z_{[\mu\nu]}^{\pm\pm}$ components in eq(2.10); this means that in non chiral $8D$ $\mathcal{N} = 1$ supergravity we should have

$$Z_{[\mu\nu]}^{++} = 0 \quad , \quad Z_{[\mu\nu]}^{--} = 0 \quad , \quad (2.13)$$

showing in turn that the supergravity multiplet has 1-form and 2-form gauge fields; but no 3-form gauge field.

Below, we switch on these charges and study extremal black attractors in non chiral $8D$ $\mathcal{N} = 1$ supergravity arising from superstring compactifications.

2.3 Central charges and branes

From the above analysis, we learn that the bosonic Z - generators appearing in the generalized supersymmetric algebra (2.3) exhibit a set of remarkable properties; in particular the three following ones:

(1) to the bosonic operators $Z_{\mu_1 \dots \mu_p}$, which are charged under $SO(1,7) \times U_R(1)$, we associate a space time p-form operator density

$$\mathcal{Z}_p = \frac{1}{p!} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p} Z_{\mu_1 \dots \mu_p} \quad , \quad (2.14)$$

together with the charge,

$$\mathcal{J}_p = \int_{M_p} \mathcal{Z}_p \quad , \quad (2.15)$$

where M_p is a p-dimensional space time submanifold which may be thought of as the world volume of a p-brane.

(2) The $Z_{\mu_1 \dots \mu_p}$ operators have an interpretation in terms of fluxes of gauge fields in non chiral $8D$ supergravity. By using the usual relations $m = \frac{1}{4\pi} \int_{S^2} \mathcal{F}_2$ and $e = \frac{1}{4\pi} \int_{S^2} \tilde{\mathcal{F}}_2$ giving the magnetic and electric charges of particles coupled to $4D$ Maxwell gauge fields and thinking about the \mathcal{Z}_p 's in the same manner, we end with the following relations

$$\mathcal{Z}_0 \sim \int_{S^2} \mathcal{F}_2 \quad , \quad \mathcal{Z}_1 \sim \int_{S^2} \mathcal{F}_3 \quad , \quad \mathcal{Z}_2 \sim \int_{S^2} \mathcal{F}_4 \quad (2.16)$$

as well as their duals. In these relations, the \mathcal{F}_p 's stand for the gauge invariant p-forms,

$$\mathcal{F}_2 = d\mathcal{A}_1, \quad \mathcal{F}_3 = d\mathcal{A}_2, \quad \mathcal{F}_4 = d\mathcal{A}_3, \quad (2.17)$$

with Hodge duals

$$\tilde{\mathcal{F}}_4 = (*\mathcal{F}_4), \quad \tilde{\mathcal{F}}_5 = (*\mathcal{F}_3), \quad \tilde{\mathcal{F}}_6 = (*\mathcal{F}_2), \quad (2.18)$$

from which we learn

$$\mathcal{J}_p = \int_{M_p \times S^2} \mathcal{F}_{p+2} \quad , \quad (2.19)$$

teaching us that the \mathcal{Z}_p 's describe precisely charges of p-branes that couple to the $8D$ supergravity $(p+1)$ - form gauge fields \mathcal{A}_{p+1} with the field strengths \mathcal{F}_{p+2} and their magnetic duals $\tilde{\mathcal{F}}_{6-p}$.

(3) Using the relation (2.13) and eqs(2.15-2.19) it follows that $\mathcal{J}_2 = 0$ and

$$\int_{M_2 \times S^2} \mathcal{F}_4 = \int_{(\partial M_2) \times S^2} \mathcal{C}_3 = 0 \quad , \quad (2.20)$$

showing that, in non chiral $\mathcal{N} = 1$ supergravity, there is no magnetic nor electric charges associated with the dyonic 4-form gauge invariant field strength $\mathcal{F}_4 = d\mathcal{C}_3$. In other words there is no D2- brane in the type IIA set up of non chiral $8D$ $\mathcal{N} = 1$ supergravity.

3 Fluxes of black attractors in $8D$

In this section we study the non chiral $8D$ $\mathcal{N} = 1$ supergravity arising from low energy compactifications of $10D$ superstring that preserve sixteen supersymmetric charges.

We first study the case of $8D$ $\mathcal{N}=1$ supergravity embedded in heterotic string on T^2 with moduli space

$$\mathbf{M}_{8D-Het/T^2}^{N=1} = \frac{SO(2,r+2)}{SO(2) \times SO(r+2)} \times SO(1,1) \quad , \quad r \geq 0 \quad . \quad (3.1)$$

Then we develop the dual type IIA superstring on a compact real surface $\Sigma_2^{(r)}$. In this case, we will focus on the class of real surfaces given by the following union of irreducible 2-cycles (2-spheres) C_I

$$\Sigma_2^{(r)} = C_0 \cup \left(\bigcup_{I=1}^{r-1} C_I \right) \quad (3.2)$$

with intersection matrix

$$C_I \cdot C_I = -K_{IJ} \quad (3.3)$$

coinciding with the Cartan matrix of the of simply laced ADE Lie algebras. The moduli space of this theory is

$$\mathbf{M}_{8D-IIA/\Sigma}^{N=1} = \frac{SO(2,r+1)}{SO(2) \times SO(r+1)} \times SO(1,1) \quad , \quad r \geq 0 \quad . \quad (3.4)$$

The simplest surface $\Sigma_2^{(r)}$ corresponds obviously to taking $r = 0$ and its singular limit given by $vol(C_0) \rightarrow 0$ should be associated with a non abelian $SU(2)$ gauge symmetry. The similarity between $\mathbf{M}_{8D-Het/T^2}^{N=1}$ and $\mathbf{M}_{8D-IIA/\Sigma}^{N=1}$ shows precisely the duality between the two constructions; for details see [59].

3.1 Heterotic string on \mathbf{T}^2

First recall that the massless bosonic fields of the $10D$ heterotic string belong to two representations of the $10D$ supersymmetric algebras; these are \mathcal{G}_{MN}^{10D} , \mathcal{B}_{MN}^{10D} , Φ_{dil}^{10D} of the supergravity multiplet and the typical gauge fields \mathcal{A}_M^I belonging to the Yang Mills multiplets. As we are interested in this study into black attractor solutions, we will restrict below to the abelian sector and think about \mathcal{A}_M^I as Maxwell gauge fields associated with the Cartan subalgebra of a given rank r gauge group; i.e: $I = 1, \dots, r$.

Under compactification of these bosonic fields on the two torus \mathbf{T}^2 , we get the following $8D$ ones

$$\mathcal{G}_{\mu\nu}, \quad \mathcal{B}_{\mu\nu}, \quad \sigma, \quad \mathcal{A}_\mu^I, \quad (3.5)$$

together with the four $8D$ gauge fields

$$\mathcal{G}_\mu^i, \quad \mathcal{B}_\mu^i, \quad (3.6)$$

as well as the $(4 + 2r)$ scalars

$$\mathcal{G}^{(ij)}, \quad \mathcal{B}^{[ij]} = \varepsilon^{ij} B, \quad \mathcal{A}^{iI}. \quad (3.7)$$

These fields combine into two $8D \mathcal{N} = 1$ supermultiplets namely:

- the $8D$ gravity multiplet with bosonic content

$$\mathcal{G}_{\mu\nu}, \quad \mathcal{B}_{\mu\nu}, \quad \mathcal{C}_\mu^i, \quad \sigma \quad (3.8)$$

containing the $8D$ graviton $\mathcal{G}_{\mu\nu}$, the $\mathcal{B}_{\mu\nu}$ antisymmetric field, two gauge fields $\mathcal{C}_\mu^i = (\mathcal{C}_\mu^1, \mathcal{C}_\mu^2)$ transforming as a real 2-vector under $SO(2)$ R-symmetry; and the $8D$ dilaton σ .

The total number of the degrees of freedom of this gravity multiplet is $48+48$; the other 48 superpartners come from the gravitino $\Psi_{\mu\alpha}$ and a photino χ_α carrying respectively 40 and 8 fermionic degrees of freedom.

- the $8D$ Maxwell multiplets whose bosonic fields are given by

$$\mathcal{A}_\mu^i, \quad \phi^{ij}, \quad \mathcal{A}_\mu^I, \quad \phi^{iI}. \quad (3.9)$$

These $8D \mathcal{N} = 1$ supermultiplets contain $(2 + r)$ Maxwell gauge fields $(\mathcal{A}_\mu^i, \mathcal{A}_\mu^I)$ which we denote collectively as \mathcal{A}_μ^a with $a = 1, \dots, r + 2$; and $2(r + 2)$ real scalars $\phi^{ia} \equiv (\phi^{ij}, \phi^{iI})$. Together with these bosons, we also have $r + 2$ gauginos λ_α^a given by pseudo-Majorana spinors in $8D$.

The moduli space of this $8D \mathcal{N} = 1$ supergravity that is embedded heterotic superstring on T^2 reads as follows

$$\mathbf{M}_{8D\text{-Het}/T^2}^{N=1} = \frac{SO(2,r+2)}{SO(2) \times SO(r+2)} \times SO(1,1) \quad (3.10)$$

where the extra factor $SO(1,1)$ refers to the dilaton σ and $\frac{SO(2,r+2)}{SO(2) \times SO(r+2)}$ for ϕ^{ia} . This real space has $(2r + 5)$ dimensions; it reduces for the particular case $r = 0$, to the five dimensional one

$$\frac{SO(2,2)}{SO(2) \times SO(2)} \times SO(1,1) \quad (3.11)$$

In addition to the dilaton, the four scalars ϕ^{ij} have geometric and stringy interpretations; three of them are given by the Kahler and complex structure of the 2-torus; the fourth is given by the value of the B_{NS} field on T^2 .

The field strengths associated with the various gauge fields of the $8D$ supergravity are given by the gauge invariant forms

$$\mathcal{F}_2^i = d\mathcal{C}_1^i, \quad \mathcal{F}_2^a = d\mathcal{A}_1^a, \quad \mathcal{F}_3 = d\mathcal{B}_2, \quad (3.12)$$

For later use, we give below the magnetic and electric charges associated with these field strengths as well as the brane interpretation; more details will be given when we consider the type IIA dual derivation. We have:

- a black hole and its 4-brane dual associated with the two graviphotons C_μ^i with magnetic and electric charges as follows

$$g^i = \int_{S^2} \mathcal{F}_2^i \quad , \quad e_i = \int_{\tilde{S}^6} \tilde{\mathcal{F}}_{6|i} \quad (3.13)$$

- a black hole and its 4-brane dual associated with the $(r+2)$ Maxwell fields \mathcal{A}_μ^a ; their magnetic and electric charges are given by

$$p^a = \int_{S^2} \mathcal{F}_2^a \quad , \quad q_a = \int_{\tilde{S}^6} \tilde{\mathcal{F}}_{6|a} \quad (3.14)$$

these two kinds of magnetic and electric charges of the black hole/black 4-brane combine into $SO(2, r+2)$ vector charges as given below

$$P^\Lambda = (g^i, p^a) \quad , \quad Q_\Lambda = (e_i, q_a) \quad (3.15)$$

- a black string and its 3-brane dual associated with the $B_{\mu\nu}$ -field; the corresponding charges are given by

$$p^0 = \int_{S^3} \mathcal{F}_3 \quad , \quad q_0 = \int_{\tilde{S}^5} \tilde{\mathcal{F}}_5 \quad (3.16)$$

All these electric and magnetic charges are linked by the usual Dirac quantization relation; they determine the effective potential

$$\mathcal{V}_{eff}^{het} = \mathcal{V}_{eff}^{het}(P, Q; p^0, q_0, \dots) \quad (3.17)$$

of the $8D$ black attractors to be considered later.

Notice that one of the remarkable features of this analysis is the absence of the 4-form field strengths $\mathcal{F}_4, \tilde{\mathcal{F}}_4$ as predicted from the group theory view. In what follows, we explore this issue by studying the type IIA dual compactification down to $8D$.

3.2 Black attractors in type IIA on $\Sigma_2^{(r)}$

In this subsection, we study the embedding of non chiral $8D$ $\mathcal{N} = 1$ supergravity in type IIA superstring on $\Sigma_2^{(r)}$. To that purpose, we first study the compactification of type IIA on the 2-sphere \mathcal{S}^2 corresponding to $\Sigma_2^{(0)}$. Then, we extend the analysis to the surface $\Sigma_2^{(r)}$ given by eq(3.2).

In type ten dimensional type IIA superstring with 32 supercharges, the massless bosonic particles of the perturbative spectrum, describing the low energy 10D type IIA supergravity, is given by

$$\text{bosons} : \mathcal{G}_{MN}, \mathcal{B}_{MN}, \Phi_{dil}, \mathcal{A}_M, \mathcal{C}_{MNK}, \quad (3.18)$$

with indices $M, N, K = 0, \dots, 9$ transforming as $SO(1, 9)$ vectors.

These fields capture 128 on shell degrees of freedom partitioned as follows

$$128 = 64 + 64 = (35 + 28 + 1) + (8 + 56) \quad , \quad (3.19)$$

with the first 64 coming from NS-NS sector ($\mathcal{G}_{MN}, \mathcal{B}_{MN}, \Phi_{dil}$) and the other 64 from RR-sector; i.e ($\mathcal{A}_M, \mathcal{C}_{MNK}$).

We also have a non perturbative sector with p-branes namely

$$\text{F1 string, NS 5-brane} \quad ; \quad \text{D0, D2, D4, D6.} \quad (3.20)$$

Some of these branes are the source of the gauge fields involved in the Maxwell sector of non chiral 8D $\mathcal{N} = 1$ supergravity. The fields are mainly similar to those given by eq(3.9); but here they should be thought of as the gauge fields associated with a D2-brane wrapped the irreducible 2-cycles of $\Sigma_2^{(r)}$.

3.2.1 Compactification of type IIA on S^2

After the space time compactification $R^{1,9} \rightarrow R^{1,7} \times S^2$ where the local coordinates (x^0, \dots, x^9) get split as (x^0, \dots, x^7) and $y = (z, \bar{z})$ with $z = x^8 + ix^9$ parameterizing the 2-sphere, the bosonic fields of the spectrum (3.18) reduces to:

$$\left\{ \begin{array}{l} \mathcal{G}_{\mu\nu} \\ \phi^1 \end{array} \right\} , \quad \left\{ \begin{array}{l} \mathcal{B}_{\mu\nu} \\ \phi^2 \end{array} \right\} , \quad \sigma \quad ; \quad \mathcal{A}_\mu, \mathcal{C}_{\mu\nu\rho}, \mathcal{C}_\mu, \quad (3.21)$$

with

$$\phi^1 = \mathcal{G}_{z\bar{z}} \quad , \quad \phi^2 = \mathcal{B}_{z\bar{z}} \quad , \quad \mathcal{C}_\mu = \mathcal{C}_\mu^{z\bar{z}} \quad (3.22)$$

respectively describing the Kahler modulus of the 2-sphere, the B_{NS} field on S^2 and the gauge particle associated with a D2-brane wrapping S^2 .

This field spectrum has 70 bosonic degrees of freedom; but only 48 of them combine with the 8D gravitino $\Psi_\mu = (\Psi_\mu^\alpha, \bar{\Psi}_{\mu\dot{\alpha}})$ and the graviphotino $\chi = (\chi^\alpha, \bar{\chi}_{\dot{\alpha}})$ to form the non chiral $\mathcal{N} = 1$ supergravity multiplet

$$\mathcal{G}_{\mu\nu}, \mathcal{B}_{\mu\nu}, \sigma, \mathcal{A}_\mu^i \quad ; \quad \Psi_\mu, \chi \quad (3.23)$$

where \mathcal{A}_μ^i stands for the $SO(2)$ doublet $(\mathcal{A}_\mu, \mathcal{C}_\mu)$. The on shell degrees of freedom are partitioned as $48_{\text{bose}} = (20 + 15 + 1) + 2 \times 6$ and $48_{\text{fermi}} = 40 + 8$. We also have the following branes,

$$\text{F1 string, (NS 5-brane/S}^2) \ ; \ \text{D0, (D6/S}^2), \ (\text{D2/S}^2), \ (\text{D4/S}^2). \quad (3.24)$$

satisfying the usual $8D$ electric magnetic duality relation between electric q-brane and its magnetic p-brane dual with the integers p and q constrained as $p + q = 4$.

3.2.2 Compactification of type IIA on $\Sigma_2^{(r)}$

The compactification of field content of the type IIA superstring on $\Sigma_2^{(r)}$ extends the case of the 2-sphere; it leads to the following:

- (1) a gravity multiplet; the same as in eq(3.23)
- (2) r Maxwell multiplets given by

$$\mathcal{A}_\mu^a, \ \phi^{ia}, \quad a = 1, \dots, r \quad (3.25)$$

where the ϕ^{1a} 's are the Kahler parameters of the irreducible 2-cycles C_a (2-spheres S_a^2) involved in $\Sigma_2^{(r)}$ and the ϕ^{1a} 's stand for the values of the B_{NS} fields on these S_a^2 's.

The gauge fields \mathcal{A}_μ^a are associated with the wrapping of D2-brane on the S_a^2 's of the compact surface $\Sigma_2^{(r)}$; i.e:

$$\mathcal{A}_\mu^a \quad : \quad (\text{D2/S}_a^2) \quad (3.26)$$

The moduli space $\mathbf{M}_{8D-IIA/\Sigma_2^r}^{N=1}$ of this $8D$ $\mathcal{N} = 1$ supergravity is parameterized by the $(2r + 1)$ scalars namely the dilaton σ and the $SO(2)$ doublets ϕ^{ia} ; it is given by

$$\mathbf{M}_{8D-IIA/\Sigma_2^r}^{N=1} = \frac{SO(2,r)}{SO(2) \times SO(r)} \times SO(1,1) \quad (3.27)$$

This space is comparable to $\mathbf{M}_{8D\text{-het}/T^2}^{N=1}$ eq(3.10); this is due to the string-string duality between the heterotic on T^2 and type IIA superstring on $\Sigma_2^{(r)}$ that follows from the well known duality relation in $6D$ space time,

$$\text{Het/T}^4 \quad \leftrightarrow \quad \text{Type IIA/K3} \quad (3.28)$$

The field strengths $\mathcal{F}_{p+1} = d\mathcal{A}_p$, associated with the various gauge fields of the $8D$ $\mathcal{N} = 1$ supergravity, are given by

$$\mathcal{F}_2^i = d\mathcal{A}^i \quad , \quad \mathcal{F}_2^a = d\mathcal{A}^a \quad , \quad \mathcal{F}_3 = d\mathcal{B}_2 \quad (3.29)$$

These gauge invariant fields transform under the $SO(2) \times SO(r)$ group as follows

$$\frac{\quad}{SO(2) \times SO(r)} \left| \begin{array}{c|c|c} \mathcal{F}_2^i & \mathcal{F}_2^a & \mathcal{F}_3 \\ \hline (2, 1) & (1, r) & (1, 1) \end{array} \right. \quad (3.30)$$

The corresponding magnetic and electric charges are as follows:

(a) the string and its dual 3-brane

$$p^0 = \int_{S^3} \mathcal{F}_3 \quad , \quad q_0 = \int_{\tilde{S}^5} \tilde{\mathcal{F}}_5 \quad (3.31)$$

the string is magnetically charged while the 3-brane is electrically charged.

(b) the black hole magnetic charges (g^i, p^a) and the electric duals (e_i, q_a) given by

$$\begin{aligned} g^i &= \int_{S^2} \mathcal{F}_2^i \quad , \quad e_i = \int_{\tilde{S}^6} \tilde{\mathcal{F}}_{6|i} \quad , \\ p^a &= \int_{S^2} \mathcal{F}_2^a \quad , \quad q_a = \int_{\tilde{S}^6} \tilde{\mathcal{F}}_{6|a} \quad . \end{aligned} \quad (3.32)$$

These are respectively $SO(2)$ and $SO(r)$ vectors.

We end this section by noting that one may write down the lagrangian densities of the various gauge fields. For the bosonic sector we have, in addition to the Einstein-Hilbert term $\frac{1}{16\pi G_8} \int_{\mathcal{M}_8} \sqrt{-\mathcal{G}} \mathcal{R}_{8D}$, two other contributions; the first one is given by the 1-form gauge fields $\mathcal{A}_\mu^\Lambda = (\mathcal{A}_\mu^i, \mathcal{A}_\mu^a)$

$$\mathcal{L}_{1\text{-form}} = \frac{1}{16\pi G_8} \int_{\mathcal{M}_8} \sqrt{-\mathcal{G}} [\mathcal{N}_{ab} \mathcal{F}_{\mu\nu}^a \mathcal{F}^{\mu\nu b} + \mathcal{N}_{ij} \mathcal{F}_{\mu\nu}^i \mathcal{F}^{\mu\nu j}] \quad , \quad (3.33)$$

where the field metric $\mathcal{N}_{\Lambda\Gamma} = \mathcal{N}_{\Lambda\Gamma}(\phi, \sigma)$ reads as $e^{2\sigma} L_\Lambda^i(\phi) \delta_{ij} L_\Gamma^j(\phi) - e^{2\sigma} L_\Lambda^a(\phi) \eta_{ab} L_\Gamma^b(\phi)$ with the field matrix $L_{\Lambda\Gamma}$ parameterizing the $SO(2, N)$ group [see eqs(4.4-4.5)] and $\phi \equiv \{\phi^{ia}\}$ being the free moduli that parameterize the moduli space $\frac{SO(2, N)}{SO(2) \times SO(N)}$. The second contribution comes from the \mathcal{B}_2 gauge field; it reads as follows

$$\mathcal{L}_{2\text{-form}} = \frac{1}{16\pi G_8} \int_{\mathcal{M}_8} \sqrt{-\mathcal{G}} \mathcal{N}(\sigma) \mathcal{F}_{\mu\nu\rho} \mathcal{F}^{\mu\nu\rho} \quad , \quad (3.34)$$

where now $\mathcal{N}(\sigma)$ is the metric associated with the $SO(1, 1)$ factor parameterized by the dilaton σ with no dependence in the ϕ^{ia} s.

4 Attractor equations and solutions

We first describe the effective potential of the black branes in non chiral $8D$ $\mathcal{N} = 1$ supergravity. Then, we study the attractor eqs and their solutions.

4.1 Effective potential

The effective potential \mathcal{V}_{eff} of the black attractors has contributions coming from the various gauge fields of the non chiral $8D$ $\mathcal{N} = 1$ supergravity. As there is no contribution coming from the D2-brane we have:

$$\mathcal{V}_{eff} = (\mathcal{V}_{BH} + \mathcal{V}_{4B}) + (\mathcal{V}_{string} + \mathcal{V}_{3B}) \quad , \quad (4.1)$$

with: (a) \mathcal{V}_{BH} is the effective potential of the $8D$ black hole associated with the charges of the two graviphotons \mathcal{A}_μ^i and the Maxwell gauge fields \mathcal{A}_μ^a . It is given by the usual Weinhold relation whose expression, in the flat coordinate frame, reads as:

$$\mathcal{V}_{BH} = \sum_{i,j=1}^2 \delta_{ij} X^i X^j + \sum_{a,b=1}^N \delta_{ab} Y^a Y^b \quad (4.2)$$

In this relation, X^i and Y^a , which respectively transform as $SO(2)$ and $SO(N)$ vectors, are the dressed central charges related to the magnetic bare $P^\Lambda = (g^i, p^a)$ of eqs(3.32) like,

$$X^i = e^\sigma L_\Lambda^i P^\Lambda \quad , \quad Y^a = e^\sigma L_\Lambda^a P^\Lambda \quad . \quad (4.3)$$

In these relations, e^σ and $L_{\Lambda\Upsilon}$ parameterize respectively the $SO(1,1)$ and $SO(2,N)$ group factors of the moduli space of the non chiral $8D$ $\mathcal{N} = 1$ supergravity. Notice that the $L_{\Lambda\Upsilon}$ matrix is a real $(2+N) \times (2+N)$ matrix

$$L_\Lambda^\Gamma = \begin{pmatrix} L_i^j & L_i^b \\ L_a^j & L_a^b \end{pmatrix} \quad , \quad (4.4)$$

satisfying the usual orthogonality relation $L^T \eta L = \eta$ which explicitly reads like

$$L_{\Gamma\Lambda} \eta^{\Gamma\Sigma} L_{\Sigma\Upsilon} = \eta_{\Lambda\Upsilon} \quad , \quad (4.5)$$

with $\eta_{\Lambda\Upsilon} = \text{diag}(+, +, -, \dots, -)$. A priori $L_{\Lambda\Gamma}$ has $(2+N)^2$ parameters; but this relation may be viewed as a constraint relation that reduces this number down to $\frac{1}{2}(N^2 + 3N + 2)$. Furthermore subtracting the $\frac{1}{2}(N^2 - N + 2)$ gauge degrees of freedom captured by the $SO(2) \times SO(N)$ symmetry of the moduli space, we end with $2N$ moduli parameterizing $\frac{SO(2,N)}{SO(2) \times SO(N)}$.

(b) \mathcal{V}_{4B} is the effective potential associated with the black 4D- branes dual the black holes,

$$\mathcal{V}_{4B} = \sum_{i,j=1}^2 \delta^{ij} \tilde{X}_i \tilde{X}_j + \sum_{a,b=1}^N \delta^{ab} \tilde{Y}_a \tilde{Y}_b \quad (4.6)$$

The \tilde{X}_i and \tilde{Y}_a are dressed central charges related to the electric $Q_\Lambda = (e_i, q_a)$ as follows

$$\tilde{X}_i = Q_\Lambda (L^{-1})_i^\Lambda e^{-\sigma} \quad , \quad \tilde{Y}_a = Q_\Lambda (L^{-1})_a^\Lambda e^{-\sigma} \quad (4.7)$$

(c) the term $(\mathcal{V}_{string} + \mathcal{V}_{3B})$ is the effective potential of the black string and its 3- brane (NS 5-brane/S²) dual; it is given by

$$\mathcal{V}_{string} = e^{4\sigma} p_0^2 \quad , \quad \mathcal{V}_{3B} = e^{-4\sigma} q_0^2 \quad (4.8)$$

where the magnetic charge p_0 and the electric q_0 one are as in eq(3.31).

Adding all terms, we get the total effective potential of the black attractors in non chiral $8D \mathcal{N} = 1$ supergravity

$$\begin{aligned} \mathcal{V}_{eff} = & + \sum_{i,j=1}^2 \left(X^i \delta_{ij} X^j + \tilde{X}_i \delta^{ij} \tilde{X}_j \right) + (e^{4\sigma} p_0^2 + e^{-4\sigma} q_0^2) \\ & + \sum_{a,b=1}^N \left(Y^a \delta_{ab} Y^b + \tilde{Y}_a \delta^{ab} \tilde{Y}_b \right) \end{aligned} \quad (4.9)$$

It is manifestly invariant under the $SO(2) \times SO(N)$ symmetry of the moduli space. Substituting the dressed central charges X^i , \tilde{X}_i , Y^a , \tilde{Y}_a by their explicit expressions in terms of the field moduli, we end with a function depending on the electric and magnetic charges as well as on the scalars σ and $L_{\Lambda M}$,

$$\mathcal{V}_{eff} = \mathcal{V}_{eff}(\sigma, L_{\Lambda M}; P^\Lambda, Q_\Lambda; p_0, q_0) \quad . \quad (4.10)$$

More explicitly, we have

$$\begin{aligned} \mathcal{V}_{eff} = & + \sum_{i,j=1}^2 \left(e^{2\sigma} P^\Lambda L_\Lambda^i \delta_{ij} L_\Upsilon^j P^\Upsilon + e^{-2\sigma} Q_\Lambda (L^{-1})_i^\Lambda \delta^{ij} (L^{-1})_j^\Upsilon Q_\Upsilon \right) \\ & + \sum_{a,b=1}^N \left(e^{2\sigma} P^\Lambda L_\Lambda^a \delta_{ab} L_\Upsilon^b P^\Upsilon + e^{-2\sigma} Q_\Lambda (L^{-1})_a^\Lambda \delta^{ab} (L^{-1})_b^\Upsilon Q_\Upsilon \right) \\ & + (e^{4\sigma} p_0^2 + e^{-4\sigma} q_0^2) \end{aligned} \quad (4.11)$$

with $L_{\Lambda\Upsilon}$ belonging to $SO(2, N)$ as given by eqs(4.5).

Notice that invariance of the effective potential \mathcal{V}_{eff} under the electric/magnetic duality symmetry between the charges of the black branes and their duals is captured by the relation $(M, \sigma) \rightarrow (E, -\sigma)$ with M standing form the magnetic charges and E for electric ones.

4.2 Attractor eqs

These are given as usual by minimizing the effective potential with respect to the field moduli σ and $L_{\Lambda\Upsilon}$;

$$\frac{\partial \mathcal{V}_{eff}}{\partial \sigma} = 0 \quad , \quad \frac{\partial \mathcal{V}_{eff}}{\partial L_{\Lambda\Upsilon}} = 0 \quad (4.12)$$

by taking into account the constraint relation $L^T \eta L = \eta$. This constraint relation may be implemented in the effective potential by using the Lagrange multiplier method; for technical details see [47] developed for the case of black attractors in $6D$ supergravity. We also need to compute the Hessian matrix

$$\frac{\partial^2 \mathcal{V}_{eff}}{\partial \sigma^2} = 0 \quad , \quad \frac{\partial^2 \mathcal{V}_{eff}}{\partial \sigma \partial L_{\Lambda\Upsilon}} = 0 \quad , \quad \frac{\partial^2 \mathcal{V}_{eff}}{\partial L_{\Lambda\Upsilon} \partial L_{\Gamma\Sigma}} = 0 \quad (4.13)$$

which needs to be positive definite for stable solutions.

Computing $\partial \mathcal{V}_{eff} / \partial \sigma = 0$

Now, using the fact that $X^i, \tilde{X}_i, Y^a, \tilde{Y}_a$ are eigenvectors of $\frac{\partial}{\partial \sigma}$; i.e $\frac{\partial X^i}{\partial \sigma} = X^i, \frac{\partial Y^a}{\partial \sigma} = Y^a, \frac{\partial \tilde{X}_i}{\partial \sigma} = -\tilde{X}_i, \frac{\partial \tilde{Y}_a}{\partial \sigma} = -\tilde{Y}_a$, the extremization with respect to the dilaton σ gives,

$$\begin{aligned} 0 = & + \sum_{i,j=1}^2 \left(X^i \delta_{ij} X^j - \tilde{X}_i \delta^{ij} \tilde{X}_j \right) \\ & + \sum_{a,b=1}^N \left(Y^a \delta_{ab} Y^b - \tilde{Y}_a \delta^{ab} \tilde{Y}_b \right) \\ & + 2 \left(e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2 \right) \quad . \end{aligned} \quad (4.14)$$

There are different ways to solve this attractor eq; one of them is to cast it as follows

$$\begin{aligned} X^i \delta_{ij} X^j - \tilde{X}_i \delta^{ij} \tilde{X}_j & = 0 \quad , \\ Y^a \delta_{ab} Y^b - \tilde{Y}_a \delta^{ab} \tilde{Y}_b & = 0 \quad , \\ e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2 & = 0 \quad , \end{aligned} \quad (4.15)$$

where summation on repeated indices is understood. An other way is to compensate the terms with X^i, \tilde{X}_i with the terms with Y^b, \tilde{Y}_a as follows,

$$\begin{aligned} X^i \delta_{ij} X^j + Y^a \delta_{ab} Y^b & = 0 \quad , \\ \tilde{X}_i \delta^{ij} \tilde{X}_j + \tilde{Y}_a \delta^{ab} \tilde{Y}_b & = 0 \quad , \\ e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2 & = 0 \quad , \end{aligned} \quad (4.16)$$

or like

$$\begin{aligned} X^i \delta_{ij} X^j - \tilde{Y}_a \delta^{ab} \tilde{Y}_b & = 0 \quad , \\ \tilde{X}_i \delta^{ij} \tilde{X}_j - Y^a \delta_{ab} Y^b & = 0 \quad , \\ e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2 & = 0 \quad . \end{aligned} \quad (4.17)$$

Further solutions are obtained by compensating $X^i, \tilde{X}_i, Y^b, \tilde{Y}_a$ with $e^{4\sigma} p_0^2$ and $e^{-4\sigma} q_0^2$; for instance as follows:

$$\begin{aligned} X^i \delta_{ij} X^j & = e^{-4\sigma} q_0^2 \quad , \\ e^{4\sigma} p_0^2 & = \tilde{X}_i \delta^{ij} \tilde{X}_j \quad , \\ Y^a \delta_{ab} Y^b & = \tilde{Y}_a \delta^{ab} \tilde{Y}_b \quad . \end{aligned} \quad (4.18)$$

We will give some explicit examples later on.

Substituting the dressed central charges by their field expressions back into (4.15), we get the following attractor eqs

$$\begin{aligned}
e^{2\sigma} P^\Lambda L_\Lambda^i \delta_{ij} L_\Upsilon^j P^\Upsilon - e^{-2\sigma} Q_\Lambda (L^{-1})_i^\Lambda \delta^{ij} (L^{-1})_j^\Upsilon Q_\Upsilon &= 0 \quad , \\
e^{2\sigma} P^\Lambda L_\Lambda^a \delta_{ab} L_\Upsilon^b P^\Upsilon - e^{-2\sigma} Q_\Lambda (L^{-1})_a^\Lambda \delta^{ab} (L^{-1})_b^\Upsilon Q_\Upsilon &= 0 \quad , \\
(e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2) &= 0 \quad .
\end{aligned} \tag{4.19}$$

Similar attractor eqs may be written down for the other cases given above.

Computing $\delta_L \mathcal{V}_{eff} = 0$

The extremization of the effective potential of the black attractors with respect to the field matrix $L_{\Lambda\Upsilon}$ is some how lengthy. Below, we give the main steps by using the expression of \mathcal{V}_{eff} in terms of X^i , \tilde{X}_i , Y^a , \tilde{Y}_a . First, we have

$$\begin{aligned}
\delta_{L_{\Lambda\Upsilon}} \mathcal{V}_{eff} = & +2 \sum_{i,j=1}^2 \left[(\delta_{L_{\Lambda\Upsilon}} X^i) \delta_{ij} X^j + (\delta_{L_{\Lambda\Upsilon}} \tilde{X}_i) \delta^{ij} \tilde{X}_j \right] \\
& +2 \sum_{a,b=1}^N \left[(\delta_{L_{\Lambda\Upsilon}} Y^a) \delta_{ab} Y^b + (\delta_{L_{\Lambda\Upsilon}} \tilde{Y}_a) \delta^{ab} \tilde{Y}_b \right]
\end{aligned} \tag{4.20}$$

where $\delta_{L_{\Lambda\Upsilon}} X^i$ and so on are the variation of the dressed central charges with respect to the field matrix $L_{\Lambda\Upsilon}$. These variations may be nicely expressed in terms of the Maurer-Cartan 1-form

$$\Omega = dLL^{-1} = -L(dL^{-1}) \quad , \tag{4.21}$$

of the orthogonal group $SO(2, N)$. Indeed, denoting $X^i = e^\sigma L_\Lambda^i P^\Lambda$ in a condensed manner as $X^i = e^\sigma (L.P)^i$ and similarly for the other dressed central charges, the variation with respect to L reads as $\delta X^i = e^\sigma (\delta L.P)^i$. Now inserting the relation $L^{-1}L = I$, we get $\delta X = e^\sigma (\delta L.L^{-1}.LP)^i$ where we recognize the Ω term. Doing the same for the other dressed central charges, we end with the following result:

$$\begin{aligned}
\delta X^i &= \Omega_k^i . X^k + \Omega_c^i . Y^c \quad , \quad \delta Y^a = \Omega_k^a . X^k + \Omega_c^a . Y^c \\
\delta \tilde{X}_i &= -\Omega_k^i . \tilde{X}_k - \Omega_c^i . \tilde{Y}_c \quad , \quad \delta \tilde{Y}_a = -\Omega_k^a . \tilde{X}_k - \Omega_c^a . \tilde{Y}_c
\end{aligned} \tag{4.22}$$

Putting back into (4.20), we get the vanishing condition of $\delta_L \mathcal{V}_{eff}$

$$\begin{aligned}
& \left(X\Omega X - \tilde{X}\Omega\tilde{X} \right) + \left(X\Omega Y - \tilde{X}\Omega\tilde{Y} \right) \\
& + \left(Y\Omega Y - \tilde{Y}\Omega\tilde{Y} \right) + \left(Y\Omega X - \tilde{Y}\Omega\tilde{X} \right) = 0
\end{aligned} \tag{4.23}$$

from which we can learn the associated attractor eqs. In this relation, the condensed terms are as follows

$$\begin{aligned}
X\Omega X &= +X^j \Omega_{jk} X^k \quad , \quad \tilde{X}\Omega\tilde{X} = +\tilde{X}_j \Omega^{jk} \tilde{X}_k \\
Y\Omega Y &= -Y^b \Omega_{bc} Y^c \quad , \quad \tilde{Y}\Omega\tilde{Y} = -\tilde{Y}_b \Omega^{bc} \tilde{Y}_c \\
X\Omega Y &= +X^j \Omega_{jc} Y^c \quad , \quad \tilde{X}\Omega\tilde{Y} = +\tilde{X}_j \Omega^{jc} \tilde{Y}_c \\
Y\Omega X &= -Y^b \Omega_{bk} X^k \quad , \quad \tilde{Y}\Omega\tilde{X} = -\tilde{Y}_b \Omega^{bk} \tilde{X}_k
\end{aligned} \tag{4.24}$$

where the i, j indices are raised and lowered by δ_{ij} and δ^{ij} while the indices a, b are raised and lowered by $-\delta_{ab}$ and $-\delta^{ab}$. Notice also that we have

$$\Omega_{\Lambda\Upsilon}^{(2+N, 2+N)} = \begin{pmatrix} \Omega_{ij}^{(2,2)} & \Omega_{ib}^{(2,N)} \\ \Omega_{aj}^{(N,2)} & \Omega_{ab}^{(N,N)} \end{pmatrix} \quad (4.25)$$

Notice as well that the attractor eqs of the black attractors (4.12) are given by eqs(4.14,4.23); a class of solutions of these eqs are given below.

4.3 Solutions of attractor eqs

We first solve the attractor eq $\partial\mathcal{V}_{eff}/\partial\sigma = 0$ (4.14) allowing to fix the dilaton in terms of the electric and magnetic bare charges. Then, we consider the case of the attractor eqs $\partial\mathcal{V}_{eff}/\partial L_{\Lambda\Upsilon} = 0$.

4.3.1 Solving $\partial\mathcal{V}_{eff}/\partial\sigma = 0$

Eq(4.14) may be solved in several ways:

(1) *no D-brane fluxes:* $P^\Lambda = 0, Q_\Lambda = 0$

This configuration corresponds to $X^i = 0, Y^a = 0; \tilde{X}_i = 0, \tilde{Y}_a = 0$. Substituting, eq(4.14) reduces to $(e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2) = 0$ whose solution is

$$\sigma_0 = \frac{-1}{4} \ln \frac{p_0}{q_0} \quad (4.26)$$

giving the value of the dilaton in terms of the magnetic charge of the string and the electric charge of the 3-brane. The near horizon geometry of this black attractor is given by $AdS_3 \times S^5$ and $AdS_5 \times S^3$ depending on the values of the magnetic and electric charges. Notice that for $p_0 = 0, \sigma_0 \rightarrow +\infty$ while for $q_0 = 0, \sigma_0 \rightarrow -\infty$.

(2) *general solutions*

These solutions correspond to compensate the contributions coming from the electric and magnetic sectors as in eqs(4.15,4.16,4.17,4.18). As an example, we consider the case

$$\begin{aligned} X^i \delta_{ij} X^j &= \tilde{X}_i \delta^{ij} \tilde{X}_j \quad , \\ Y^a \delta_{ab} Y^b &= \tilde{Y}_a \delta^{ab} \tilde{Y}_b \quad , \\ \sigma_0 &= \frac{1}{4} \ln \frac{q_0}{p_0} \quad . \end{aligned} \quad (4.27)$$

which may be solved in four ways by taking the dressed central charges as follows:

$$\begin{aligned} (i) & : X^i = +z^i \quad , \quad \tilde{X}_j = +z^i \delta_{ij} \quad , \quad Y^a = +y^a \quad , \quad \tilde{Y}_b = +y^a \delta_{ab} \\ (ii) & : X^i = +z^i \quad , \quad \tilde{X}_j = +z^i \delta_{ij} \quad , \quad Y^a = +y^a \quad , \quad \tilde{Y}_b = -y^a \delta_{ab} \\ (iii) & : X^i = -z^i \quad , \quad \tilde{X}_j = -z^i \delta_{ij} \quad , \quad Y^a = -y^a \quad , \quad \tilde{Y}_b = +y^a \delta_{ab} \\ (iv) & : X^i = -z^i \quad , \quad \tilde{X}_j = -z^i \delta_{ij} \quad , \quad Y^a = -y^a \quad , \quad \tilde{Y}_b = -y^a \delta_{ab} \end{aligned} \quad (4.28)$$

and σ_0 as before and z^i, y^a some constants. The above relations may also written as follows

$$\begin{aligned} (L^{-1})_i^\Lambda Q_\Lambda &= \pm e^{-2\sigma_0} (L)_\Upsilon^j P^\Upsilon \delta_{ij} \quad , \\ (L^{-1})_b^\Lambda Q_\Lambda &= \pm e^{-2\sigma_0} (L)_\Upsilon^a P^\Upsilon \delta_{ab} \quad , \\ L^T \eta^{SO(2,N)} L &= \eta^{SO(2,N)} \quad , \end{aligned} \quad (4.29)$$

with $e^{-2\sigma_0}$ given by eq(4.26) and whose solutions allow to express the field matrix $L_{\Lambda\Upsilon}$ in terms of Q_Λ, P^Υ as well as $\frac{p_0}{q_0}$.

Notice that the moduli space of solutions of (4.28) depends on the arbitrary values z^i and y^a . For instance taking

$$\tilde{Y}_b = \begin{pmatrix} y_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad Y^a \delta_{ab} = \begin{pmatrix} \pm y_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (4.30)$$

the $SO(2) \times SO(N)$ symmetry of the moduli space get reduced down to $SO(N-1)$. Generic solutions read as follows

$$\tilde{Y}_b = \begin{pmatrix} y_1 \\ \vdots \\ y_n \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad Y^a \delta_{ab} = \begin{pmatrix} \pm y_1 \\ \vdots \\ \pm y_n \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (4.31)$$

and have a $SO(N-n)$ symmetry.

4.3.2 Solving $\partial\mathcal{V}_{eff}/\partial\sigma = 0$ and $\partial\mathcal{V}_{eff}/\partial L_{\Lambda\Upsilon} = 0$

We give here below two classes of solutions; others solutions classified by the $SO(N-n)$ symmetries can be also written down.

Class I

The first class of solutions of the attractor eqs(4.12,4.14,4.23) is obtained by putting eqs(4.26) and (4.28) back into eq(4.23); then cast it as follows:

$$\begin{aligned} X\Omega X &- \tilde{X}\Omega\tilde{X} = 0 \\ Y\Omega Y &- \tilde{Y}\Omega\tilde{Y} = 0 \\ X\Omega Y &- \tilde{X}\Omega\tilde{Y} = 0 \\ Y\Omega X &- \tilde{Y}\Omega\tilde{X} = 0 \end{aligned} \quad (4.32)$$

Taking into account eqs(4.26, 4.28) solving $\partial\mathcal{V}_{eff}/\partial\sigma = 0$, it is not difficult to see that the solutions of eq(4.32) are classified as given below

$$\begin{aligned}
(i) \quad & : \quad X = +\tilde{X} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{pmatrix} \quad , \quad Y = +\tilde{Y} = \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} \\
(ii) \quad & : \quad X = -\tilde{X} = \begin{pmatrix} z'_1 \\ z'_2 \\ \vdots \\ z'_N \end{pmatrix} \quad , \quad Y = -\tilde{Y} = \begin{pmatrix} w'_1 \\ \vdots \\ w'_N \end{pmatrix}
\end{aligned} \tag{4.33}$$

together with $\sigma_0 = \frac{1}{4} \ln \frac{q_0}{p_0}$ and where z_i, w_a and z'_i, w'_a are some constant numbers. Notice that the terms $X\Omega X = X^i\Omega_{ij}X^j$ and $Y\Omega Y = Y^a\Omega_{ab}Y^b$ are symmetric quadratic forms; so there no contribution coming from the antisymmetric parts $\Omega_{[ij]} = (\Omega_{ij} - \Omega)_{ji}$ and $\Omega_{[ab]} = \Omega_{ab} - \Omega_{ba}$ of the Cartan-Maurer forms. This property captures precisely the $SO(2) \times SO(N)$ symmetry of the moduli space (3.10-3.27) of non chiral $8D \mathcal{N} = 1$ supergravity.

Notice also that for arbitrary values of z_i, w_a and z'_i, w'_a the symmetry group $SO(2) \times SO(N)$ of the effective potential is completely broken. The other possibilities where some of the parameters are zero or identical, the $SO(2) \times SO(N)$ symmetry of the moduli space is broken down to a subgroup G .

Class II

This class of solutions corresponds to solving the extremum of \mathcal{V}_{eff} (4.23) by compensating the X^i and \tilde{X}_i factors with the Y^a and \tilde{Y}_a as in (4.16-4.17). A way to do it is as follows:

$$\begin{aligned}
Y\Omega Y &= \tilde{X}\Omega\tilde{X} \quad , \quad \tilde{Y}\Omega\tilde{Y} = X\Omega X \\
Y\Omega X &= \tilde{X}\Omega\tilde{Y} \quad , \quad \tilde{Y}\Omega\tilde{X} = X\Omega Y
\end{aligned} \tag{4.34}$$

In this solution, the term $Y\Omega Y$ (resp $\tilde{Y}\Omega\tilde{Y}$) is compensated by $\tilde{X}\Omega\tilde{X}$ (resp $X\Omega X$); this corresponds to first breaking the moduli space $SO(2) \times SO(N)$ subsymmetry as

$$SO(2) \times SO(N) \quad \rightarrow \quad SO(2) \times SO(2) \times SO(N-2) \quad , \tag{4.35}$$

then compensate the terms associated with the two $SO(2)$ factor. An explicit solution

is given by:

$$\begin{aligned}
X &= \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, & \tilde{X} &= \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix}, \\
Y &= \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, & \tilde{Y} &= \begin{pmatrix} z_1 \\ z_2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}
\end{aligned} \tag{4.36}$$

Other configurations with symmetries $SO(N - m)$ with $m = 3, \dots, N$ may be also written down; one of them is given by X^i and \tilde{X}_i as in (4.36) and Y^a and \tilde{Y}_a like,

$$Y = \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ w \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \tilde{Y} = \begin{pmatrix} z_1 \\ z_2 \\ w \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4.37}$$

with $SO(N - 3)$ symmetry.

5 Intersecting attractors

Following [57, 58], one should distinguish two main classes of black p-brane solutions in higher dimensional supergravity. In non chiral $\delta D \mathcal{N} = 1$ supergravity we are considering here, these are:

- (1) the standard black p- brane solutions based on $AdS_{2+p} \times S^{6-p}$ with $p = 0, 1, 3, 4$, whose basic features have been given above.
- (2) the intersecting attractors with the typical near horizon geometries

$$AdS_{2+p} \times S^m \times M^{6-p-m} \tag{5.1}$$

where S^m is the real m-sphere and M^n stands for some manifolds; essentially a n -torus. Moreover, since there is no D2- brane flux in this theory; these geometries are restricted to black hole and black string geometries as well as their duals. As such, we have:

- (a) $AdS_3 \times S^m \times M^{5-m}$
- (b) $AdS_2 \times S^m \times M^{6-m}$

The novelty with these geometries is that they allow the two following features:

(i) a variety of irreducible sub-manifolds that support various kinds of branes and so a rich spectrum of electric and magnetic charges;

(ii) non trivial intersections between p_i -/ p_j - cycles of (5.1) leading to intersecting (BPS and non BPS) attractors.

To illustrate the first point, consider the example of the two compact manifolds S^{m+n} and $\mathcal{M}^{m+n} = S^m \times T^n$. While the sphere S^{m+n} supports only charges of $(m+n-2)$ -brane charges

$$\mathcal{F}_{n+m} = g \varpi_{n+m} \quad , \quad g = \int_{S^{m+n}} \mathcal{F}_{n+m} \quad , \quad (5.2)$$

and no $(m-1)$ - brane nor others, the manifold $S^m \times T^n$ allows however many possibilities. It has several irreducible k_i - cycles that support, in addition to $(m+n-2)$ -branes, other kinds; in particular n types of $(m-1)$ -branes with charges given by,

$$\begin{aligned} g^a &= \int_{\mathcal{C}_{m+1}^{(a)}} \mathcal{F}_{m+1} \quad , \quad \mathcal{F}_{m+1} = \sum_a g^a \varpi_{m+1|a} \quad , \\ \int_{\mathcal{C}_{m+1}^{(a)}} \varpi_{m+1|b} &= \delta_b^a \quad , \quad a = 1, \dots, n \quad , \end{aligned} \quad (5.3)$$

with

$$\mathcal{C}_{m+1}^{(a)} = \bigcup_{a=1}^n (S_a^1 \times S^m) \quad , \quad T^n = \bigotimes_{a=1}^n S_a^1 \quad . \quad (5.4)$$

The branes may be imagined as filling the fiber $F_{m-1}^{(a)}$ of these cycles $\mathcal{C}_{m+1}^{(a)}$ thought of in terms of the fibration $\mathcal{C}_{m+1}^{(a)} \sim F_{m-1}^{(a)} \times S^2$ with field strength

$$\mathcal{F}_{m+1} = \beta_{S^2} \wedge \left(\sum_a g^a \beta_{F_{m-1}^{(a)}} \right) \quad (5.5)$$

Using the anzats of [52], we focus below on the study of various examples of these typical horizon geometries and work out new and explicit solutions regarding intersecting attractors in the case of non chiral $8D$ $\mathcal{N} = 1$ supergravity. As the solutions are very technical, we will concentrate on drawing their main lines and giving the results.

5.1 Geometries with AdS_3 and AdS_5 factors

We distinguish several $AdS_3 \times S^m \times M^{5-m}$ and their $AdS_5 \times S^m \times M^{3-m}$ duals geometries; in particular:

- (a) $AdS_3 \times S^3 \times T^2$ with volume forms α_{AdS_3} , β_{S^3} and β_{T^2} ,
- (b) $AdS_3 \times S^2 \times T^3$ with volume forms α_{AdS_3} , β_{S^2} and β_{T^3} ,
- (c) $AdS_3 \times S^4 \times S^1$ with volume forms α_{AdS_3} , β_{S^4} and β_{S^1} .

Below, we study the two first ones.

5.1.1 $AdS_3 \times S^3 \times T^2$

On the geometry $AdS_3 \times S^3 \times T^2$ there is no irreducible 2-cycle nor irreducible 6-cycle that support the fluxes emanating from the D0 and D6- branes. As such the black attractor is given by,

	p-branes	(4 - p)- branes	
$p = 0$	$\mathcal{F}_2^\Lambda = 0$	$\tilde{\mathcal{F}}_{6 \Lambda} = 0$	(5.6)
$p = 1$	$\mathcal{F}_3 = p^0 \beta_{S^3}$	$\tilde{\mathcal{F}}_5 = q_0 \alpha_{AdS_3} \wedge \beta_{T^2}$	

from which we read the following effective potential,

$$\mathcal{V}_{eff} = e^{4\sigma} p_0^2 + e^{-4\sigma} q_0^2 \quad (5.7)$$

The extremization of this potential with respect to the dilaton leads to

$$e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2 = 0 \quad (5.8)$$

The solving of the above equation is given by $\sigma_0 = \frac{1}{4} \ln \frac{q_0}{p_0}$; it fixes the dilaton σ_0 at horizon in terms of the magnetic charge of the black string and the electric charge of the black 3-brane.

5.1.2 $AdS_3 \times S^2 \times T^3$

In this geometry which involve the volume forms α_{AdS_3} , β_{S^2} , β_{T^3} , the non vanishing field strength charges are given by

	p-branes	(4 - p)- branes	
$p = 0$	$\mathcal{F}_2^\Lambda = P^\Lambda \beta_{S^2}$	$\tilde{\mathcal{F}}_{6 \Upsilon} = Q_\Upsilon (\alpha_{AdS_3} \wedge \beta_{T^3})$	(5.9)
$p = 1$	$\mathcal{F}_3 = p^0 \alpha_{AdS_3}$	$\tilde{\mathcal{F}}_5 = q_0 (\beta_{S^2} \wedge \beta_{T^3})$	

where $\Lambda = (i, a)$ with $i = 1, 2$ and $a = 1, \dots, N$.

The effective potential \mathcal{V}_{eff} of these black attractor configuration reads as follows,

$$\begin{aligned} \mathcal{V}_{eff} = & + \sum_{i,j=1}^2 \left(X^i \delta_{ij} X^j + \tilde{X}_i \delta^{ij} \tilde{X}_j \right) + (e^{4\sigma} p_0^2 + e^{-4\sigma} q_0^2) \\ & + \sum_{a,b=1}^N \left(Y^a \delta_{ab} Y^b + \tilde{Y}_a \delta^{ab} \tilde{Y}_b \right) \end{aligned} \quad (5.10)$$

where the first term, which we write as $XX + \tilde{X}\tilde{X}$, is invariant under $SO(2)$ and the term $YY + \tilde{Y}\tilde{Y}$ is invariant under $SO(N)$. The extremization of \mathcal{V}_{eff} gives,

$$\begin{aligned} & \left(X\Omega X - \tilde{X}\Omega\tilde{X} \right) + \left(X\Omega Y - \tilde{X}\Omega\tilde{Y} \right) \\ & \left(Y\Omega Y - \tilde{Y}\Omega\tilde{Y} \right) + \left(Y\Omega X - \tilde{Y}\Omega\tilde{X} \right) = 0 \\ & e^{4\sigma} p_0^2 - e^{-4\sigma} q_0^2 = 0 \end{aligned} \quad (5.11)$$

where Ω is the Maurer Cartan 1-form of $SO(2, N)$ introduced previously.

The solutions of these attractor eqs may be realized in various ways; one of them is given by the following:

$$X = \pm\tilde{X} \quad , \quad Y = \pm\tilde{Y} \quad , \quad \sigma_0 = \frac{-1}{4} \ln \frac{p_0}{q_0} \quad (5.12)$$

These solutions correspond to diverse intersecting configurations composed of a black hole, a black 4-brane, a black string, and a black 3-brane.

Moreover, using eqs(4.22), we compute the following the Hessian matrix

$$\begin{aligned} \delta\delta\mathcal{V}_{eff} = & +16(e^{4\sigma}p_0^2 + e^{-4\sigma}q_0^2) \\ & +4\left(X\Omega^2X - \tilde{X}\Omega^2\tilde{X} + X\Omega^2Y - \tilde{X}\Omega^2\tilde{Y}\right) \\ & +4\left(Y\Omega^2Y - \tilde{Y}\Omega^2\tilde{Y} + Y\Omega^2X - \tilde{Y}\Omega^2\tilde{X}\right) \\ & +\left[X(\delta\Omega)Y - \tilde{X}(\delta\Omega)\tilde{Y}\right] + \left[Y(\delta\Omega)Y - \tilde{Y}(\delta\Omega)\tilde{Y}\right] \\ & +\left[Y(\delta\Omega)X - \tilde{Y}(\delta\Omega)\tilde{X}\right] + \left[X(\delta\Omega)X - \tilde{X}(\delta\Omega)\tilde{X}\right] \end{aligned} \quad (5.13)$$

For the case $X = +\tilde{X}$, $Y = +\tilde{Y}$ and $X = -\tilde{X}$, $Y = -\tilde{Y}$ the Hessian reduces to

$$\delta\delta\mathcal{V}_{eff} = +16(e^{4\sigma}p_0^2 + e^{-4\sigma}q_0^2) \quad (5.14)$$

which a positive definite quantity; it vanishes for $p_0 = q_0 = 0$; that is no black string nor 3-brane. This is clearly seen by using the identity $e^{4\sigma}p_0^2 = e^{-4\sigma}q_0^2$ and replacing $e^{4\sigma} = \frac{q_0}{p_0}$, we get

$$\delta\delta\mathcal{V}_{eff} = +32q_0p_0 \quad , \quad \mathcal{V}_{eff} = q_0p_0 \quad . \quad (5.15)$$

5.2 Geometries with AdS_2 factor

We study the following near horizon geometries.

- (a) $AdS_2 \times S^4 \times T^2$ with volume forms α_{AdS_2} , β_{S^4} and β_{T^2} ,
- (b) $AdS_2 \times S^3 \times T^3$ with volume forms α_{AdS_2} , β_{S^3} and β_{T^3} ,
- (c) $AdS_2 \times S^2 \times T^4$ with volume forms α_{AdS_2} , β_{S^2} and β_{T^4} .

5.2.1 $AdS_2 \times S^4 \times T^2$

Using the various n-cycles of $AdS_2 \times S^4 \times T^2$ and the corresponding n-forms that could live on, the general expressions of the field strengths on this geometry reads as follows,

	p-branes	(4-p)- branes	
$p = 0$	$\mathcal{F}_2^\Lambda = Q^\Lambda \alpha_{AdS_2}$	$\tilde{\mathcal{F}}_{6 \Lambda} = P_\Lambda (\beta_{S^4} \wedge \beta_{T^2})$	(5.16)
$p = 1$	$\mathcal{F}_3 = \sum_{k=1}^2 q_k^0 (\alpha_{AdS_2} \wedge \alpha_{S_k^1})$	$\tilde{\mathcal{F}}_5 = \sum_{k=1}^2 p_{0k} (\beta_{S^4} \wedge \alpha_{S_k^1})$	

where now the strings are charged electrically and the 3-branes magnetically.

The total effective potential \mathcal{V}_{eff} associated with this system is given as usual by the sum of the contribution of each extremal black-brane. The attractor equations following from the extremization of \mathcal{V}_{eff} are then given by:

$$e^{4\sigma} (p_{01}^2 + p_{02}^2) - e^{-4\sigma} (q_{01}^2 + q_{02}^2) = 0 \quad (5.17)$$

and

$$\begin{pmatrix} X\Omega X - \tilde{X}\Omega\tilde{X} \\ Y\Omega Y - \tilde{Y}\Omega\tilde{Y} \end{pmatrix} + \begin{pmatrix} X\Omega Y - \tilde{X}\Omega\tilde{Y} \\ Y\Omega X - \tilde{Y}\Omega\tilde{X} \end{pmatrix} = 0 \quad (5.18)$$

A class of solutions of (5.17-5.18) is given by,

$$\sigma_0 = \frac{1}{8} \ln \left(\frac{q_{01}^2 + q_{02}^2}{p_{01}^2 + p_{02}^2} \right) \quad , \quad X = \pm\tilde{X}, \quad Y = \pm\tilde{Y} \quad . \quad (5.19)$$

Other solutions like those given by eqs(4.36,4.37) may be also written down. Following the same method as before, we find in the case of (5.19) the following Hessian matrix at the horizon

$$\delta\delta\mathcal{V}_{eff} = +32\sqrt{(q_{01}^2 + q_{02}^2)(p_{01}^2 + p_{02}^2)} \quad (5.20)$$

5.2.2 $AdS_2 \times S^3 \times T^3$

The general form of the field strengths on this geometry reads as,

	p-branes	(4 - p)- branes
$p = 0$	$\mathcal{F}_2^\Lambda = Q^\Lambda \alpha_{AdS_2}$	$\tilde{\mathcal{F}}_{6 \Lambda} = P_\Lambda \beta_{S^3} \wedge \beta_{T^3}$
$p = 1$	$\mathcal{F}_3 = p_0 \beta_{S^3}$	$\tilde{\mathcal{F}}_5 = q^0 \left(\alpha_{AdS_2} \wedge \beta_{T^3} \right)$

(5.21)

The total effective potential reads, in terms of the dressed central charges of the black hole/4-brane, the black string/3-brane, as in (5.10) with typical solutions at horizon given by $X = \pm\tilde{X}$, $Y = \pm\tilde{Y}$, $\sigma_0 = \frac{-1}{4} \ln \frac{p_0}{q_0}$. Other solutions of type eqs(4.36,4.37) may be also written down. Notice also that the solutions with plus signs describe intersecting attractor involving string, 3-brane, D0- brane and D4- brane; those with minus signs are associated with the string, 3-brane, ant-D0 and anti D4- brane.

5.2.3 $AdS_2 \times S^2 \times T^4$

The associated field strengths on this geometry read as follows,

	p-branes	(4 - p)- branes
$p = 0$	$\mathcal{F}_2^\Lambda = P^\Lambda \beta_{S^2}$	$\tilde{\mathcal{F}}_{6 \Lambda} = Q_\Lambda \left(\alpha_{AdS_2} \wedge \beta_{T^4} \right)$
$p = 1$	$\mathcal{F}_3 = \sum_{k=1}^4 p_0^k \left(\beta_{S^2} \wedge \beta_{S_k^1} \right)$	$\tilde{\mathcal{F}}_5 = \sum_{l=1}^4 q_l^0 \varepsilon^{lijk} \left(\alpha_{AdS_2} \wedge \beta_{S_i^1} \wedge \beta_{S_j^1} \wedge \beta_{S_k^1} \right)$

(5.22)

Following the same approach we have been using, the effective potential \mathcal{V}_{eff} of these black brane configurations is given by,

$$\begin{aligned} \mathcal{V}_{eff} = & + \sum_{i,j=1}^2 \left(X^i \delta_{ij} X^j + \tilde{X}_i \delta^{ij} \tilde{X}_j \right) + \sum_{k=1}^4 \left(e^{4\sigma} p_{0k}^2 + e^{-4\sigma} q_{0k}^2 \right) \\ & + \sum_{a,b=1}^N \left(Y^a \delta_{ab} Y^b + \tilde{Y}_a \delta^{ab} \tilde{Y}_b \right) \end{aligned} \quad (5.23)$$

Here also there are various types of solutions describing intersecting attractors with the moduli space $SO(2) \times SO(N)$ symmetries broken down to subgroups; a class of them reads as:

$$\sigma_0 = \frac{1}{8} \ln \left(\frac{q_{01}^2 + q_{02}^2 + q_{03}^2 + q_{04}^2}{p_{01}^2 + p_{02}^2 + p_{03}^2 + p_{04}^2} \right) \quad , \quad X = \pm \tilde{X}, \quad Y = \pm \tilde{Y} \quad , \quad (5.24)$$

they correspond to the case where $SO(2) \times SO(N)$ is completely broken. Notice also that for the particular case $X = \pm \tilde{X} = 0$ and $Y = \pm \tilde{Y} \neq 0$, the moduli space symmetry is reduced to $SO(2)$ and in the case $X = \pm \tilde{X} \neq 0$ and $Y = \pm \tilde{Y} = 0$, it reduces to $SO(N)$.

6 Conclusion

In this paper, we have studied the attractor mechanism of intersecting black p-branes in non chiral $8D$ supergravity with 16 supercharges. Actually, this study completes previous results on black attractors in non chiral $8D$ supergravity with 32 supersymmetries [58] and agrees with the results on higher D -supergravities obtained in [57].

To do so, we have first studied the structure of non chiral $8D$ $\mathcal{N} = 1$ supersymmetric algebra with non trivial central charges $Z_{\mu_1 \dots \mu_p}$. Then we have given the link between these $Z_{\mu_1 \dots \mu_p}$ s and the fluxes $\int_{S^2} \mathcal{F}_{\mu_1 \dots \mu_{p+2}}$ of p-branes; in particular the D- branes of type IIA string on a compact real surface Σ given by eq(3.2). Using group theoretic method, we have shown that, besides the F-sting and the D0- brane, only the $(D2/\Sigma)$ -, $(D6/\Sigma)$ - and $(NS5/\Sigma)$ -branes wrapping 2-cycles of Σ which survive under compactification; no free D2- nor $(D4/\Sigma)$ -brane are allowed in non chiral $8D$ $\mathcal{N} = 1$ supergravity. This result has been also checked by using a field theoretical method by determining directly the fields content that follows from $10D$ type II spectrum on Σ .

We have also studied the attractor mechanism for both standard extremal black attractors in $8D$ supergravity with 16 supercharges as well as their intersections along the line of [57, 58]. We have worked out various classes of explicit solutions and shown that they are completely classified by the $SO(N - m)$ subgroups of the $SO(2) \times SO(N)$ symmetry of the moduli space $\frac{SO(2,N)}{SO(2) \times SO(N)} \times SO(1,1)$.

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